ENGR 105: Feedback Control Design
Winter 2013

Lecture 8 - Closed-Loop Systems

Friday, January 25, 2013

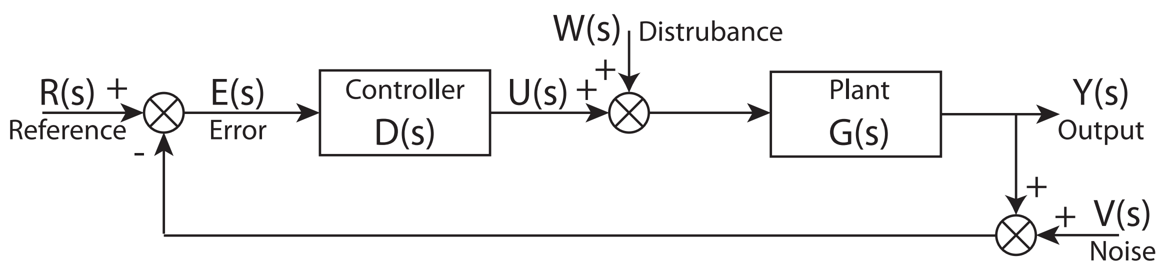
Today's Objectives

1. develop closed-loop system transfer functions
2. examine the behavior of closed-loop poles

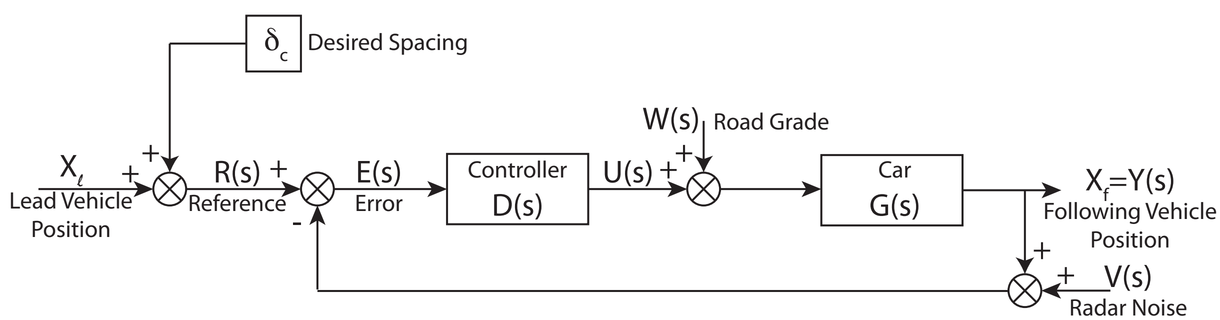
Reading: FPE Section 4.1

1 Closed-loop system transfer functions

The payoff for studying Laplace transforms and transfer functions is the ability to design controllers that can achieve desired performance characteristics. We will study this over the next few lectures for a system in the form:



To make the concepts more concrete, we will use the example of vehicle following. Here, a following vehicle tries to maintain a constant distance behind a lead vehicle. This is one suggested scenario for autonomous cars.



We will model the following vehicle as follows (where we use x instead of x_f for clarity):

$$m\ddot{x} = u - b\dot{x}$$

$$ms^2X(s) = U(s) - bsX(s)$$

$$\Rightarrow \frac{X(s)}{U(s)} = \frac{1}{ms^2 + bs} = G(s)$$

This can be re-written as: $G(s) = \frac{\frac{1}{m}}{s(s+\frac{b}{m})}$

The open loop system poles are at $s = 0$ and $s = -\frac{b}{m}$. What does the pole at $s = 0$ imply?

We can look at the response of Y, E or U to R, W or V:

$$Y(s) = \frac{DG}{1+DG}R(s) + \frac{G}{1+DG}W(s) - \frac{DG}{1+DG}V(s)$$

$$E(s) = \frac{1}{1+DG}R(s) - \frac{G}{1+DG}W(s) - \frac{1}{1+DG}V(s)$$

$$U(s) = \frac{D}{1+DG}R(s) - \frac{DG}{1+DG}W(s) - \frac{D}{1+DG}V(s)$$

The first thing to notice is that all nine transfer functions contained in these equations have the *same denominator*. This means that all of the responses have the same poles.

2 Closed-loop poles

The poles are just the solution to the characteristic equation:

$$1 + D(s)G(s) = 0$$

Stability of the system is determined by the poles. There are different definitions of stability but the book defines stability as having all poles in the left half plane. Under this definition, the initial conditions will decay to zero for a stable system. Systems with poles on the imaginary axis are unstable under this definition but can also be referred to as *marginally stable*. Note that marginally stable does not actually mean stable. Remember that the impulse response of a stable system always goes to zero as $t \to \infty$. This might not be true with a marginally stable system.

In terms of stability, there is no difference between the response of the output to a reference, the response of the error to a disturbance, or any other combination. If the system is stable, all of these transfer functions are stable.

For our example, we will begin with a proportional gain controller, $D(s) = K$:

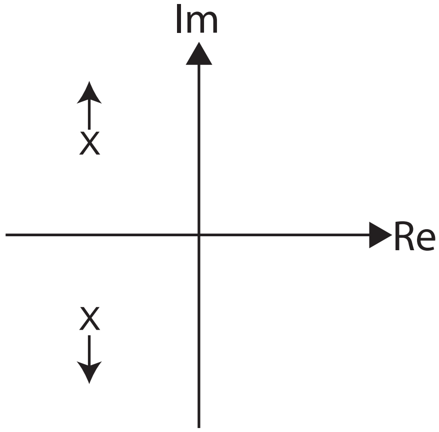
$$1 + D(s)G(s) = 1 + \frac{K}{ms^2 + bs} = 0$$

$$\Rightarrow ms^2 + bs + K = 0$$

$$\text{Open loop poles at } s = 0, s = \frac{-b}{m}$$

$$\text{Closed loop poles at } s = \frac{-b \pm \sqrt{b^2 - 4mK}}{2m}$$

$$\text{These are complex when } K > \frac{b^2}{4m}$$



For this system, when the poles are complex, the poles have a constant real part (-$\frac{b}{2m}$), but an increasing imaginary part, as the gain K increases.

What is a good location for the closed-loop poles? To answer this, we need to decide what we want the system to do. One approach is to place some specifications on the step response of the system, and we will examine this in the next lecture.