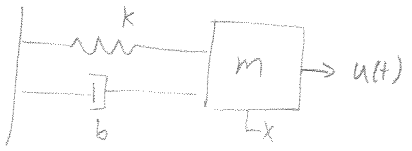
Problem 1



$$\sum F = m\ddot{x} = -Kx - b\dot{x} + u(t)$$

$$ms^2X(s) + bX(s)s + KX(s) = U(s)$$

$$\frac{X(s)}{U(s)} = \frac{1}{ms^2 + bs + k}$$

$$= \frac{1/m}{s^2 + \frac{b}{m}s + \frac{k}{m}}$$

Suppose $m = 1/4$ kg $\quad b = 2 \frac{N\cdot s}{m} \quad k = 1 \frac{N}{m}$

$$\frac{X(s)}{U(s)} = \frac{4}{s^2 + 2s + 4}$$

$$\omega_n = \sqrt{4} = 2$$

$$2\zeta\omega_n = 2$$

$$2\zeta(2) = 2$$

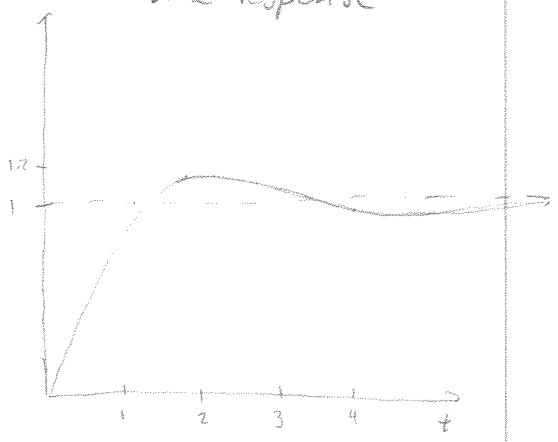
$$\zeta = 1/2$$

$\underline{\omega_n = 2} \qquad \underline{\zeta = 1/2}$

Suppose we give the system a step input, what response do we expect to see?

$$X(s) = U(s)\ \frac{4}{s^2+2s+4}$$

time response



final value:

$$X_{ss} = \lim_{s\to 0} s \cdot \frac{1}{s}\ \frac{4}{s^2+2s+4} = \frac{4}{4} = 1$$

overshoot

$$OS = \exp\left(\frac{-(1/2)\pi}{\sqrt{1-(1/2)^2}}\right) = .163 = 16.3\%$$

peak time

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{2\sqrt{1-(1/2)^2}} = \frac{\pi}{\sqrt{3}} \approx 1.81\ s$$

settling time

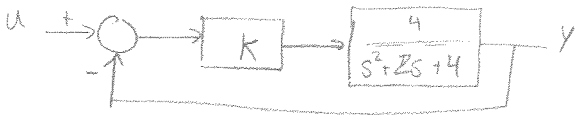
$$t_s \approx \frac{4}{\zeta\omega_n} = \frac{4}{(1/2)(2)} = 4s$$

rise time

$$t_r \approx \frac{1.8}{\omega_n} = \frac{1.8}{2} = .9s$$

Problem 2

Designing a proportional controller



with no controller (open-loop) how long to settle?

$\omega_n = 2 \qquad 2 = 2\zeta\omega_n$

$\zeta = 1/2$

$t_s \approx \frac{4}{(2)(\frac{1}{2})} \approx 4s$

what is the new transfer function with k?

$$\frac{K\left(\frac{4}{s^2+2s+4}\right)}{1+K\left(\frac{4}{s^2+2s+4}\right)} = \frac{4K}{s^2+2s+4(1+K)}$$

$\omega_n' = 2\sqrt{1+K} \qquad 2\zeta'\omega_n' = 2$

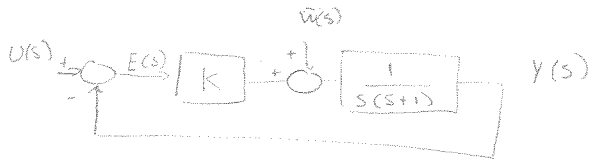
$2\zeta' 2\sqrt{1+K} = 2$

$\zeta' = \frac{1}{2\sqrt{1+K}}$

From ω_n' and ζ' we can select, ss. error, peak time, overshoot, etc.

Problem 3

Disturbance rejection



$$\frac{E(s)}{U(s)} = \frac{1}{1 + \frac{K}{s(s+1)}} = \frac{s(s+1)}{s(s+1)+K}$$

Error to reference

$$e_{ss} = \lim_{s \to 0} s\,U(s) \cdot \frac{s(s+1)}{s(s+1)+K}$$

error to step

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{1}{s} \cdot \frac{s(s+1)}{s(s+1)+K} = \boxed{0}$$

error to ramp

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{1}{s^2} \cdot \frac{s(s+1)}{s(s+1)+K} = \boxed{\frac{1}{K}}$$

Error to disturbance

$$\frac{E(s)}{W(s)} = \frac{\frac{1}{s(s+1)}}{1 + \frac{K}{s(s+1)}} = \frac{1}{s(s+1)+K}$$

error to step

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{1}{s} \cdot \frac{1}{s(s+1)+K} = \boxed{\frac{1}{K}}$$

error to ramp

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{1}{s^2} \cdot \frac{1}{s(s+1)+K} = \boxed{\infty}$$