Assignment 5: System Type, Routh Array, and PI/PD/PID control

ENGR 105: Feedback Control Design
Winter Quarter 2013
Due no later than 4:00 pm on Wednesday, Feb. 13, 2013
Submit in class or in the box outside the door to area of Room 107, Building 550
Note: This assignment cannot be submitted late!
(We will post solutions on Feb. 13 in preparation for the midterm exam on Feb. 15)

Problem 1. (5 pts.)

Use Routh's stability criterion to determine how many roots with positive real parts the following equations have:

a. $s^4 + 8s^3 + 32s^2 + 80s + 100 = 0$

b. $s^4 + 6s^2 + 25 = 0$

Problem 2. (10 pts.)

Consider a system with the following characteristic equation:

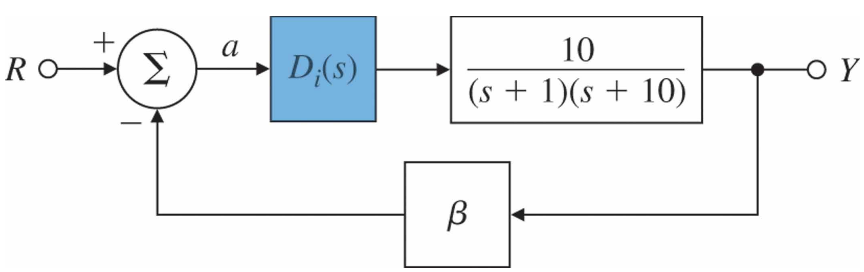
$$s^5 + 5s^4 + 10s^3 + 10s^2 + 5s + K = 0$$

c. Find the possible values of K for which all roots of this characteristic equation are in the left half plane (LHP).

d. Create a plot using Matlab that plots the roots of the characteristic equation in the s-plane for a range of values of K (both inside and outside the stable range). You can use the Matlab function `roots` for a few values of K. Annotate the plot to show the limits of stability and the values of K at that location(s). Verify that the result matches your answer to part a. Include your Matlab code and plot.

Problem 3. (10 pts.)

Consider the system shown in the figure below, where the feedback gain β is subject to variations. You are to design a controller for this system so that the output $y(t)$ accurately tracks the reference input $r(t)$.



a. Let $\beta = 1$. Here are three options for the controller $D_i(s)$:

$$D_1(s) = k_P \qquad D_2(s) = \frac{k_P s + k_I}{s} \qquad D_3(s) = \frac{k_P s^2 + k_I s + k_2}{s^2}$$

Choose the controller (including particular values for the controller constants) that will result in a Type 1 system with a steady-state error to a unit reference ramp of exactly 0.1.

b. Next, suppose that there is some attenuation in the feedback path that is modeled by $\beta = 0.9$. Find the steady-state error due to a ramp input for your choice of $D_i(s)$ in part a. What is the system type?

Problem 4. (10 pts.)

Consider the second-order system $G(s) = \frac{1}{s^2+2\zeta s+1}$. We would like to add a transfer function of the form $D(s) = K\frac{s+a}{s+b}$ in series with $G(s)$ in a unity feedback structure.

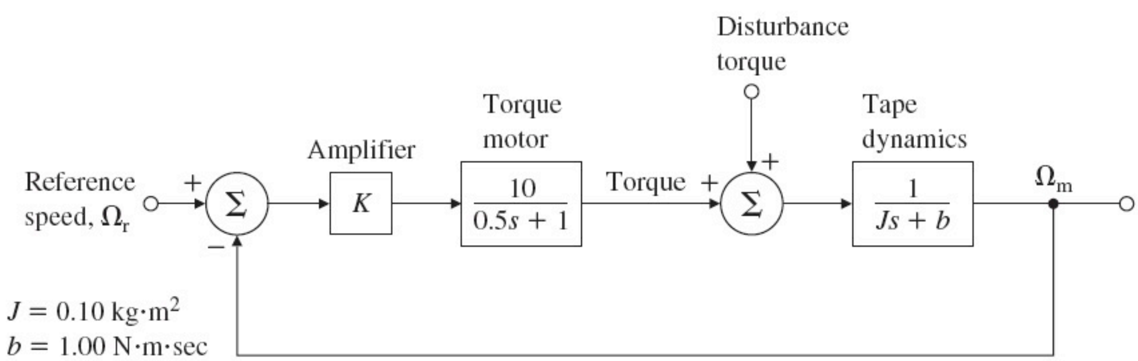
a. Ignoring stability for now, what are the constraints on K, a, and b so that the system is Type 1?

b. What are the constraints on K, a, and b so that the system is both stable and Type 1?

c. What are the constraints on a and b so that the system is both Type 1 and remains stable for every positive value of K?

Problem 5. (15 pts.)

A system for speed control of a magnetic tape drive is shown in the figure below. The speed sensor is fast and accurate enough that its dynamics and noise can be neglected.



a. Assuming the reference input is zero, what is the steady-state error due to a step disturbance torque of 1 N-m? What must the amplifier gain K be in order to make the steady-state error to a step disturbance torque $e_{ss} \leq 0.01$ rad/sec?

b. Plot the roots of the closed-loop system in the complex plane.

c. Now, assuming there is no disturbance, accurately sketch (by hand, using calculations of performance specifications to help you) the time response of the output for a step reference input using the gain K computed in part a.

d. Plot the region in the complex plane of acceptable closed-loop poles corresponding to the specifications of a 1% settling time of $t_s \leq 0.1$ sec and an overshoot of $M_p \leq 5\%$.

e. Give the values for k_P and k_D for a PD controller that will meet these specifications.

f. How would the disturbance-induced steady-state error change with the new control scheme in part (f)? How could the steady-state error to a disturbance torque be eliminated entirely?