Problem 1) $KG(s)$ is the given open loop system
unity feedback is applied.
Using Routh's criterion determine whether the resulting closed loop system is stable

$$KG(s) = \frac{4(s+2)}{s(s^3+2s^2+3s+4)}$$

solution

a) Closed loop system is $\dfrac{KG}{1+KG}$

$$= \frac{4(s+2)}{s(s^3+2s^2+3s+4)+4(s+2)}$$

$$= \frac{4(s+2)}{s^4+2s^3+3s^2+8s+8}$$

Look at the denominator & construct the Routh arra

Array:

s^4:	1	3	8
s^3:	2	8	
s^2:	a	b	
s^1:	c		
s^0:	d		

$$a = \frac{2\times 3 - 8\times 1}{2} = (-1) \qquad b = \frac{2\times 8 - 0\times 1}{2} = 8$$

$$c = \frac{8a-2b}{a} = \frac{(-8)-2(8)}{-1} = 24$$

$$d = \frac{c\times b - a\times 0}{c} = b = 8$$

so there are two sign changes in the first column

$\Rightarrow$ 2 roots not in the LHP

$\Rightarrow$ unstable system

Problem 2) Modify the Routh's criterion so that it applies to the case in which all the poles are to the left of $(-\alpha)$ when $\alpha > 0$.

Apply the modified test to the polynomial

$$s^3 + (6+K)s^2 + (5+6K)s + 5K = 0$$

find those values of K for which all poles have a real part $< (-1)$.

<u>Solution</u>

We will shift the polynomial function by variable substitution & apply the standard test on the new variable.

let $p = s + \alpha$ i.e. $s = (p - \alpha)$

Note that $s = 0$ is eqvlt to $p = \alpha$

or $s = (-\alpha)$ is eqvlt to $p = 0$

$\Rightarrow$ the new Im. axis for p is now at $s = (-\alpha)$

So we can apply Routh's criterion on p to get K

$$\therefore \; (p-1)^3 + (6+K)(p-1)^2 + (5+6K)(p-1) + 5K = 0$$

$$\Rightarrow \; p^3 + (3+K)p^2 + (4K-4)p + 1 = 0$$

Routh array is

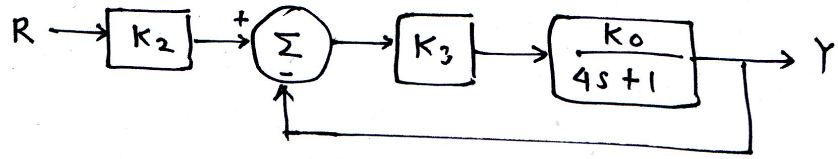
$$\begin{array}{lll} p^3 : & 1 & (4K-4) \\ p^2 : & (3+K) & 1 \\ p^1 : & \dfrac{(3+K)(4K-4)-1}{(3+K)} & 0 \\ p^0 : & 1 & \end{array}$$

first element of column 1 > 0

$\Rightarrow$ for stability all elements > 0

$3 + K > 0 \Rightarrow K > (-3)$

Problem 3)



a) find K_2 & K_3 so that

i) zero steady state error to step input

ii) static velocity error constant $K_v = 1$ when $K_0 = 1$

b) Suppose small perturbation in K_0

$K_0 \rightarrow K_0 + \delta K_0$. Analyze effect on $e_{ss,step}$

Is the system robust?

Solution find error to input tf from block diagram

$$\frac{E(s)}{R(s)} = \frac{4s+1+K_3K_0(1-K_2)}{4s+1+K_3K_0} \qquad (K_0 = 1)$$

apply FVT $\Rightarrow$ $e_{ss,step} = \dfrac{1+K_3K_0(1-K_2)}{1+K_3K_0} = 0$

$$\Rightarrow \boxed{1 + K_3(1-K_2) = 0} \qquad (\because K_0 = 1)$$

①

$$e_{ss,ramp} = \lim_{s\to 0} s \cdot \frac{4s + (1 + K_3(1-K_2))}{4s+1+K_3K_0} \cdot \frac{1}{s^2}$$

(since $1+K_3(1-K_2) = 0$ from ①)

$$\Rightarrow e_{ss,ramp} = \lim_{s\to 0} \cancel{s} \cdot \frac{4\cancel{s}}{4s+1+K_3} \cdot \frac{1}{\cancel{s^2}}$$

$$= \frac{4}{K_3+1} = \frac{1}{K_v}$$

$K_v = 1 \Rightarrow K_3 = 3$ & $1+K_3(1-K_2) = 0$

$$\Downarrow$$

$$K_2 = \frac{4}{3}$$

Problem 3) contd..

let $k_0 \to k_0 + \delta k_0$

$$e_{ss,\,step} = \lim_{s \to 0} \frac{4s + 1 + k_3(k_0 + \delta k_0)(1 - k_2)}{4s + 1 + k_3(k_0 + \delta k_0)} \quad \left|\begin{array}{l} k_0 = 1 \\ 1 + k_3(1 - k_2) = 0 \\ k_3 = 3,\ k_2 = \frac{4}{3} \end{array}\right.$$

$$= \frac{-\delta k_0}{1 + 3(1 + \delta k_0)} \neq 0$$

Thus we can see that small changes in the parameters of the plant can cause the control design to lose its effectiveness.