1 Feedthrough

part a

Consider the transfer function

$$H(s) = \frac{2s+1}{s+2}.$$

Find the response of this system to a step function.

part b

Find the response of the system given in part a to a general input $u(t)$ (yes, you must find an explicit representation in the time domain involving $u(t)$. Simplify as much as possible).

part c

In part b, you decomposed $H(s)$ into two terms (using partial fraction expansion). One of these terms is the *feedthrough* (or *memoryless*) component of $H(s)$. Sketch the step response of each component, and guess which is the feedthrough component.

part d

i

Let's say you want the final value of the output to be -3. Find an input that achieves this. **Remark:** This course is about designing *closed-loop* ("feedback") controllers, but here we skip the controller entirely and design the input ourselves, which we then apply to the system without using feedback. This is called *open-loop* control.

ii

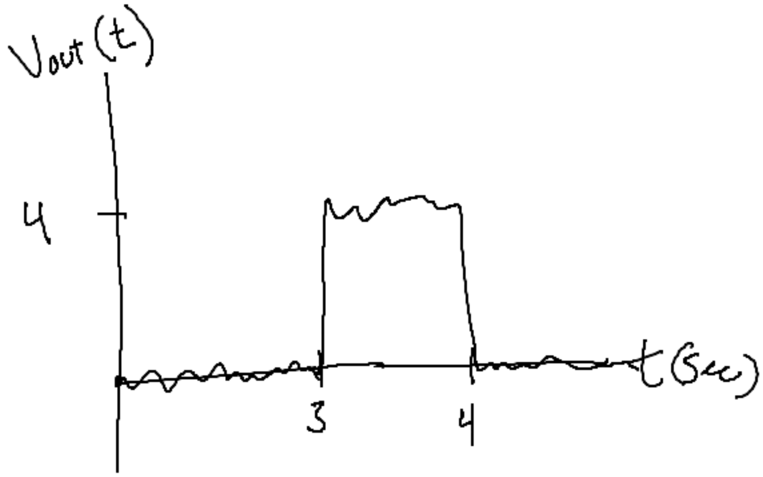
Use the final value theorem to confirm that the output is indeed -3 when your input is applied.

1.1 part e

For a rational transfer function $G(s)$, if the order of the numerator is equal to the order of the denominator, the system has a feedthrough component. If $G(s)$ has the order of the numerator is less than the order of the denominator, the system has no feedthrough component, and is called *strictly causal.* Why do you think we don't see many transfer functions with the order of the numerator *greater* than the order of the denominator?

2 Black box LTI system

You find a box lying by the side of the road. It has two wires coming out of one side, and two more coming out the other, and a mysterious sticker that reads "WARNING: CONTAINS LINEAR, TIME-INVARIANT SYSTEM." You apply a DC voltage of 2V to one side at time $t = 0$, and sketch what happens to the voltage across the wires on the other side:



You then plan to apply an AC current to the same side as before, with peak-to-peak amplitude of A, and frequency $\frac{\omega}{2\pi}$ Hz. What do you expect the response will be?

It looks like

$$v_{out}(t) = 4 \cdot 1(t-3) - 4 \cdot 1(t-4)$$

Laplace:

$$V_{out}(s) = 4e^{-3s}\frac{1}{s} - 4e^{-4s}\frac{1}{s}$$
$$V_{out}(s) = \frac{4}{s}(e^{-3s} - e^{-4s})$$

What we see is the step response, so $v_{out}(t) = h(t) * v_{in}(t)$ *or equivalently* $V_{out}(s) = H(s)V_{in}(s)$*. So then*

$$\frac{4}{s}(e^{-3s} - e^{-4s})\frac{1}{s} = H(s)\frac{1}{s}$$
$$H(s) = 4(e^{-3s} - e^{-4s})$$

Now we just need to apply the new input signal. A good model for an AC input is

$$v_{AC} = A\sin(\omega t)$$
$$v_{AC} = 2\sin(\omega t)$$

So then

$$V_{AC}(s) = \frac{A\omega}{s^2 + \omega^2}$$

If we apply this to the black box, we get

$$\begin{aligned}
V_{out}(s) &= H(s)V_{AC}(s) \\
&= 4(e^{-3s} - e^{-4s})\frac{A\omega}{s^2 + \omega^2} \\
&= e^{-3s}\frac{4A\omega}{s^2 + \omega^2} - e^{-4s}\frac{4A\omega}{s^2 + \omega^2}
\end{aligned}$$

Take the inverse Laplace transform:

$$\begin{aligned}
v_{out}(t) &= 4A\mathcal{L}^{-1}\left(e^{-3s}\frac{\omega}{s^2 + \omega^2}\right) - 4A\omega\mathcal{L}^{-1}\left(e^{-4s}\frac{\omega}{s^2 + \omega^2}\right) \\
&= 4A\sin(\omega(t-3)) - 4A\sin(\omega(t-4))
\end{aligned}$$

If we want, we can use $\sin(u) + \sin(v) = 2\sin(\frac{u+v}{2})\cos(\frac{u-v}{2})$ *to get*

$$v_{out}(t) = 8A\cos(\omega)\sin\left(\omega t - \frac{7\omega}{2}\right)$$