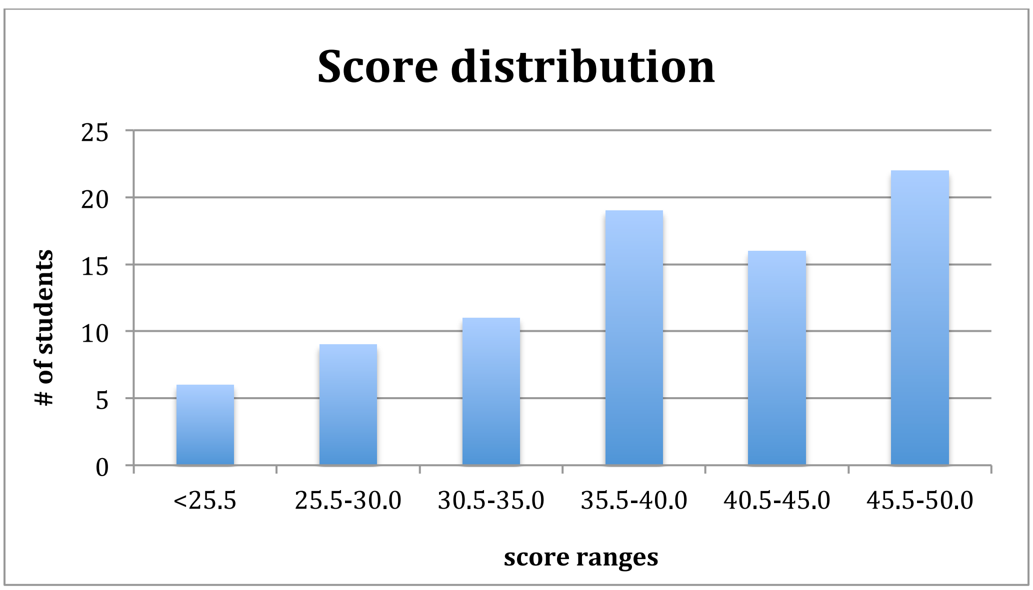
ENGR 105: Feedback Control Design
Midterm Exam, Winter Quarter 2013
Friday, Feb. 15, 2013, 9:00-9:50 am, Room 420-40

Name: ___SOLUTIONS_________

Problem	Mean / Std. Dev.
1 (5 pts.)	4.4 / 0.8
2 (15 pts.)	11.5 / 2.9
3 (15 pts.)	10.7 / 5.0
4 (15 pts.)	12.2 / 2.9
Total (50 pts.)	38.8 / 8.4



Problem 1. (5 pts.)

Mark the following True (T) or False (F):

T a. (1 pt.) The definition of peak overshoot, $M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$, applies only to the step response of second order systems.

T b. (1 pt.) Simulink allows you to simulate the behavior of nonlinear systems.

see example in Lecture 14

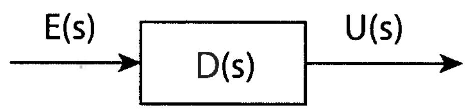
F ← but T okay!

c. (1 pt.) Consider a transfer function $\frac{Y(s)}{R(s)} = H(s)$. The impulse response of that transfer function is equal to $Y(s)1(s)$.

→ no such thing as $1(s)$ → after seeing so many people miss it, I decided it was a silly question & so either answer gets credit.

Circle the correct answer for the following questions:

d. (1 pt.) Consider the block diagram below. Which of the following statements is *not* true?



(i) $u(t) = d(t)e(t)$, for $t > 0$

(ii) $u(t) = d(t) * e(t)$, where $*$ represents the convolution for $t > 0$

(iii) $u(t) = \int_0^\infty d(t-\tau)e(\tau)d\tau$

↑ limits of integration confused some people. If you circled (i) & (iii), that was okay. Limits depend on t under consideration

e. (1 pt.) Consider a system for which the characteristic equation is written as $a(s) + Kb(s) = 0$. Which of the following equations *alone* will allow you to test whether a point lies on the root locus?

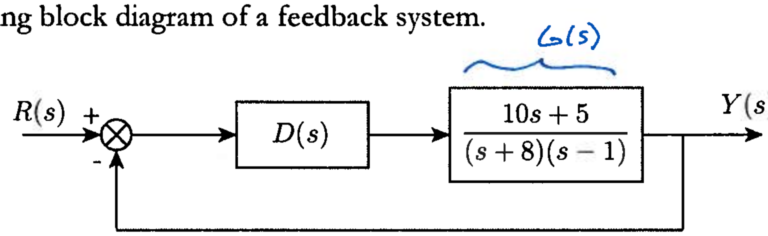
(i) $\left|K\frac{b(s)}{a(s)}\right| = 1$ ← you can always find a "K" to make this work, so this is not a test.

(ii) $\angle\left(K\frac{b(s)}{a(s)}\right) = 180^\circ + 360^\circ n$ (where n is an integer)

(iii) $a(s) = b(s)$

Problem 2. (15 pts.)

Consider the following block diagram of a feedback system.



a. (3 pts.) Assume that $D(s) = 1$. Find the transfer function $T(s)$ from $R(s)$ to $Y(s)$. Simplify your answer.

Answer: $T(s) = \dfrac{10s+5}{s^2+17s-3}$

$$Y = DG(R-Y)$$

$$\frac{Y}{R} = \frac{DG}{1+DG}$$

$$\frac{Y}{R} = \frac{\frac{10s+5}{(s+8)(s-1)}}{1+\frac{10s+5}{(s+8)(s-1)}} = \frac{10s+5}{(s+8)(s-1)+10s+5}$$

$$= \frac{10s+5}{s^2+17s-3}$$

b. (3 pts.) Is the system stable with this $D(s)$? Give the basis for your answer.

Yes ______

No __X__

look at the roots of the characteristic eqn. (these are the poles):

$$s^2+17s-3 = 0$$

$$s = \frac{-17 \pm \sqrt{17^2-4(-3)}}{2} = \frac{-17 \pm \sqrt{17^2+12}}{2}$$

since $\sqrt{17^2+12} > 17$,

one of the roots will be a positive real number.

$\therefore$ unstable

c. (3 pts.) Now consider $D(s) = \frac{1}{s^n}$, where n is an integer greater than or equal to zero. For what value(s) of n is this system stable?

Answer: $n = 1$

$$\frac{Y}{R} = \frac{DG}{1+DG} = \frac{10s+5}{s^n(s+8)(s-1)+10s+5} = \frac{10s+5}{s^{n+2}+7s^{n+1}-8s^n+10s+5}$$

for $n=0$, $D(s)=1$ $\rightarrow$ this is unstable from part b.

for $n=1$, characteristic eqn. is $s^3+7s^2-8s+10s+5$
$= s^3+7s^2+2s+5$

Routh
s^3 : 1 2
s^2 : 7 5
s^1 : 9/7
s^0 : 5

$\frac{-\det\begin{vmatrix}1 & 2\\ 7 & 5\end{vmatrix}}{7} = \frac{9}{7}$

$\rightarrow$ all positive, so stable for $n=1$

need to check $n>1$!

for $n=2$, characteristic eqn. is $s^4+7s^3-8s^2+10s+5$

Routh
s^4 : 1 −8 5
s^3 : 7 10
s^2 : $\frac{-56-10}{7} < 0$ $\longrightarrow$ negative, so unstable (can stop this Routh array here)

for $n=3$ and higher $s^{5+\cdots}+7s^{4+\cdots}-8s^{3+\cdots}+10s+5$

Routh
s^{n+2} : 1 −8 ...
s^{n+1} : 7 0 ...
s^n : * $\longleftarrow$ is always $\frac{7(-8)}{7} = -8$ negative, so unstable

d. (3 pts.) For the *lowest* value of n you give for part b, determine the system type (with respect to reference input) as defined in class.

Answer: The system is Type $\underline{\ \ I\ \ }$.

note: if you got part c wrong, but did parts d & e correctly given that wrong answer, we gave you credit.

$D = \frac{1}{s}$

$\frac{E}{R} = \frac{1}{1+DG} = \frac{s(s+8)(s-1)}{s(s+8)(s-1)+10s+5}$

$e_{ss\,step} = \lim_{s \to 0} s \cdot \frac{E}{R} \cdot \frac{1}{s} = 0$

$e_{ss,\,ramp} = \lim_{s \to 0} s \cdot \frac{E}{R} \cdot \frac{1}{s^2} = \frac{-8}{5} \rightarrow$ constant

finite error to a ramp, so system is Type I

another method: loop transfer fn. $L(s) = D(s)\,G(s) = \frac{1}{s} \frac{10s+5}{(s+8)(s-1)}$

$= \frac{L_0(s)}{s^n}$ n is sytem type

here, mly m independent integrator on be found

$\Rightarrow \frac{1}{s^1} \frac{10s+5}{(s+8)(s-1)}$ $\therefore$ Type I

e. (3 pts.) Let k be the number of the system type (e.g., 0, 1 or 2) that you found in part d. What is the value of the steady-state error for a reference input of $\frac{1}{s^{k+1}}$ to this system?

Answer: $e_{ss} = \underline{\ \ -8/5\ \ }$

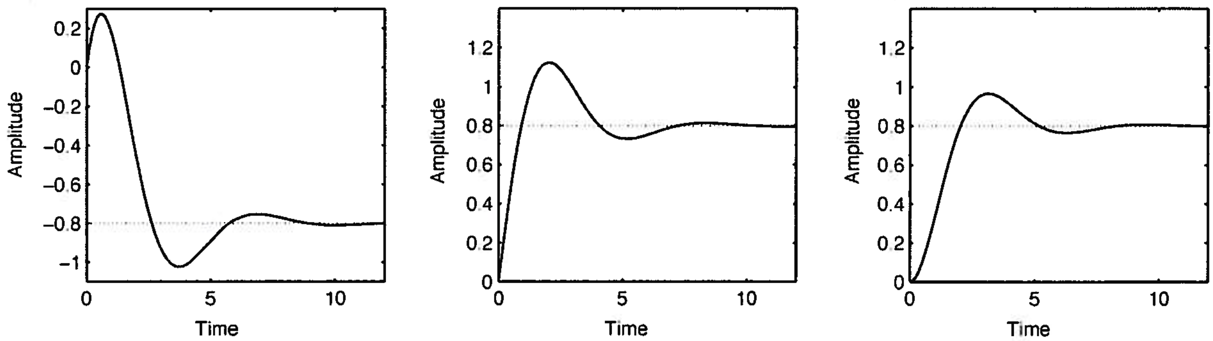
$R = \frac{1}{s^{k+1}} = \frac{1}{s^2}$ since $k = 1$

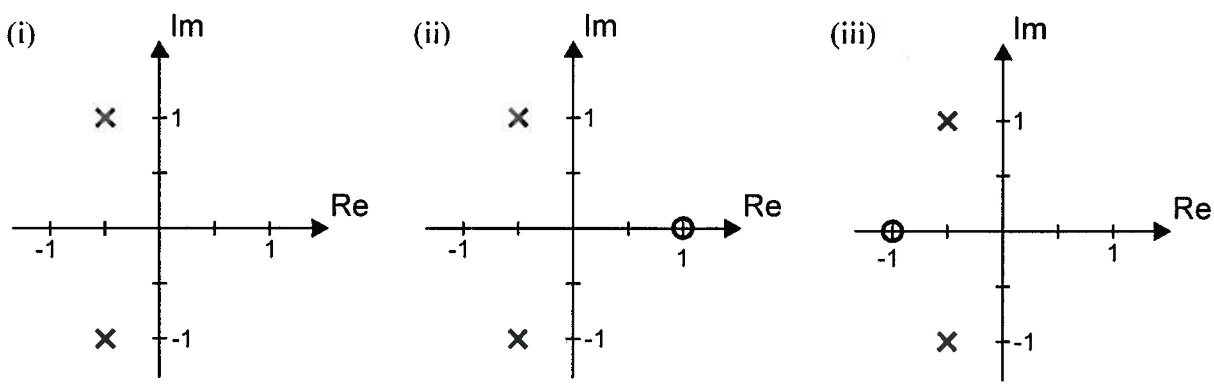
see method in part d. above to find $e_{ss,\,ramp}$

Problem 3. (15 pts.)

Below are shown plots of ~~four~~ 3 pole-zero constellations in the s-plane (i, ii, and iii) and ~~four~~ 3 time responses to step inputs to a plant with these poles and zeros (a, b, and c). The poles are shown as × and the zeros as O. Match them up and give brief reasons (required) for your choices in each case.

(a) (b) (c)





Circle the correct match for (a): (i) (ii) (iii)

Reason: The system initially moves in the opposite direction of the final ss value. This is non-minimum phase behavior due to the positive zero.

Circle the correct match for (b): (i) (ii) (iii)

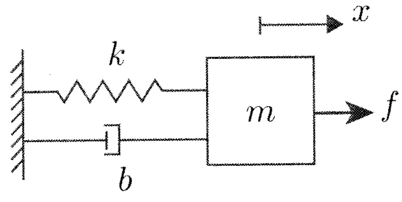
Reason: In comparison to (c), (b) responds more quickly. This is due to the zero (in left half plane), which causes a system to respond more quickly to inputs.

Circle the correct match for (c): (i) (ii) (iii)

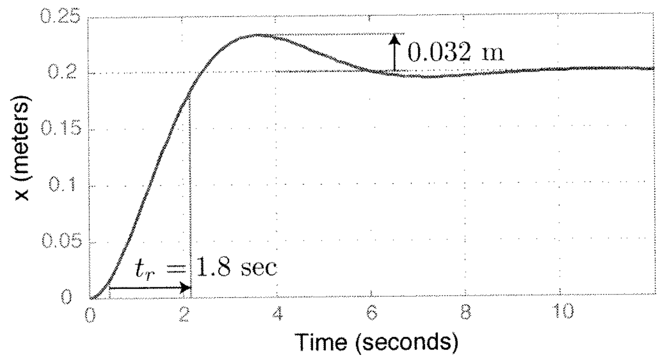
Reason: See reason above. (Process of elimination)

Problem 4. (15 pts.)

A simple mechanical system is shown below. The parameters are k (spring constant), b (damping constant), and m (mass).



A step of 2 Newtons is applied as $f = 2 \cdot 1(t)$. The resulting step response is sketched below with the final value, overshoot, and rise time as shown.



What are the values of k, b, and m? Show all your work.

Answers:

$k =$ 10

$b =$ 10

$m =$ 10

equation of motion:

$$m\ddot{x} + b\dot{x} + kx = f$$

$$\frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + k} \qquad F(s) = \frac{2}{s}$$

Steady-state value: Final value theorem

$$X_{ss} = \lim_{t \to \infty} x(t) = \lim_{s \to 0} sX(s) = \lim_{s \to 0} s \frac{1}{ms^2 + bs + k} \cdot \frac{2}{s} = \frac{2}{k}$$

Since $X_{ss} = 0.2$ m from plot, $0.2 = \frac{2}{k} \rightarrow k = \frac{2}{0.2} = \boxed{10 = k}$

rise time: $t_r = \frac{1.8}{\omega_n} = 1.8$ sec from plot

$\therefore \omega_n = 1 = \sqrt{m/k} \rightarrow k = \boxed{m = 10}$

maximum overshoot is 0.032 m. Since ss value is 0.2, % max overshoot $M_p = \frac{0.032}{0.2} = 0.16 = 16\%$. From table, $\zeta = 0.5$

$$\zeta = \frac{b}{2\sqrt{mk}} \Rightarrow b = 2\zeta\sqrt{mk} = 2 \cdot 0.5\sqrt{100}$$

$\boxed{b = 10}$