ENGR 105: Feedback Control Design
Winter 2013

Lecture 17 - Compensator Design

Wednesday, February 20, 2013

Today's Objectives

1. choosing a gain based on the root locus
2. definition, root locus, and implementation of a lead compensator
3. definition and example of notch compensator
4. definition of lag compensator

Reading: FPE Section 5.4

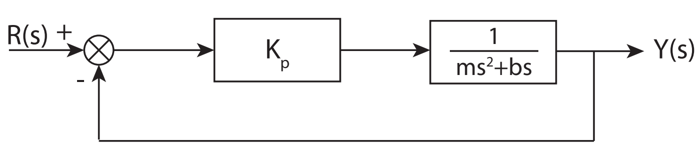
1 Choosing a controller gain based on the root locus

The root locus shows the different locations of the closed-loop poles as the gain K is varied. To figure out which value of K corresponds to a particular pole location, we can use the magnitude condition we derived back at the start of our discussion of the root locus:

$$1 + K\frac{b(s)}{a(s)} = 0$$

$$K = \left|\frac{a(s)}{b(s)}\right| = \frac{\text{product of distances to poles}}{\underbrace{\text{product of distances to zeros}}_{\text{This is 1 if there are no zeros}}}$$

Example



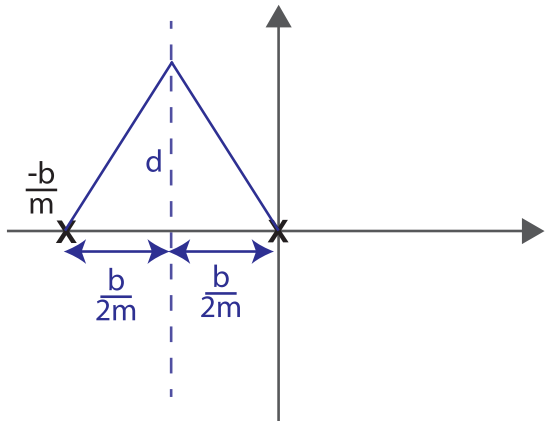
$$\frac{Y}{R} = \frac{K_P}{ms^2 + bs + K_P}$$

$$ms^2 + bs + K_P = 0$$

$$1 + K_P\frac{1}{ms^2 + bs} = 0$$

$$1 + \underbrace{\left(\frac{K_P}{m}\right)}_{K}\frac{1}{s^2 + \frac{b}{m}s} = 0$$

Say we wish to find the value of K_P for a point on the root locus that is on the vertical asymptote and a distance d from the real axis.



K is the product of distances from the point of interest to the poles

$$K = \sqrt{d^2 + \frac{b^2}{4m^2}} \cdot \sqrt{d^2 + \frac{b^2}{4m^2}}$$

$$K = d^2 + \frac{b^2}{4m^2} = \frac{K_P}{m}$$

$$\text{Check: } s = \frac{-b}{2m} \pm \underbrace{\frac{\sqrt{\frac{b^2}{4m^2} - \frac{K_P}{m}}}{2}}_{dj}$$

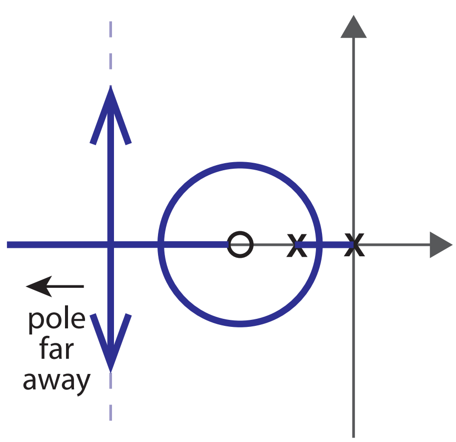
In practice this is rarely done by hand. But this example shows that, with a properly scaled root locus, gains can be calculated using a ruler.

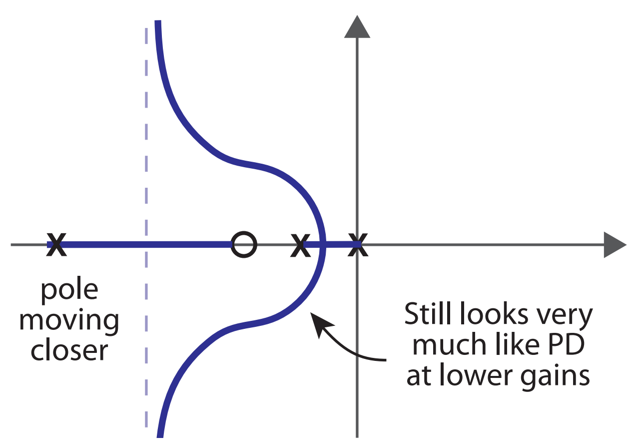
2 Lead Compensator

As described previously, a lead compensator is similar in function to PD control but is often more easily implemented. The transfer function for the lead compensator is

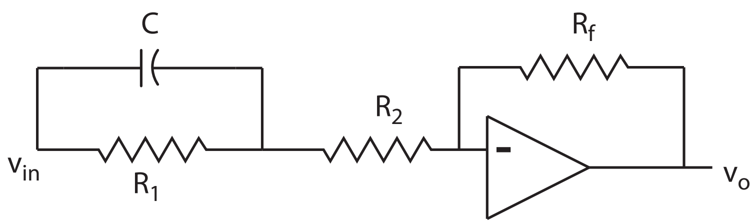
With a lead compensator, the zero is always to the right of the pole on the real axis. The further away from the origin the pole is, the more the lead compensator resembles PD control.

Consider our favorite plant with characteristic equation $ms^2 + bs = 0$. If the controller $D(s)$ is a lead compensator, the root locus will look like one of the two examples below, depending on how far away (how large) p is.





Lead compensators can be used to specify the location of the dominant pair of closed loop poles in order to meet performance specifications like rise time and overshoot. How do we build a lead compensator? One implementation is in the form of an analog circuit.



(This also inverts)

Analog implementation has the advantage of simplicity and speed (bandwidth). However, the controller characteristics are tied to these specific component values.

More commonly, compensators are implemented using digital control and microprocessors. To get a sense of how this works, consider what the transfer functions means:

$$D(s) = \frac{U(s)}{E(s)} = K\frac{s+z}{s+p}$$

To implement this control law digitally, we need a way represent differentiation with differences between individual samples. There are a number of ways to do this discretization. The book shows the Bilinear Transformation. The simplest to use as an example is Euler.

$$\dot{x} \approx \frac{x(t) - x\left(t - \Delta T\right)}{\Delta T}$$

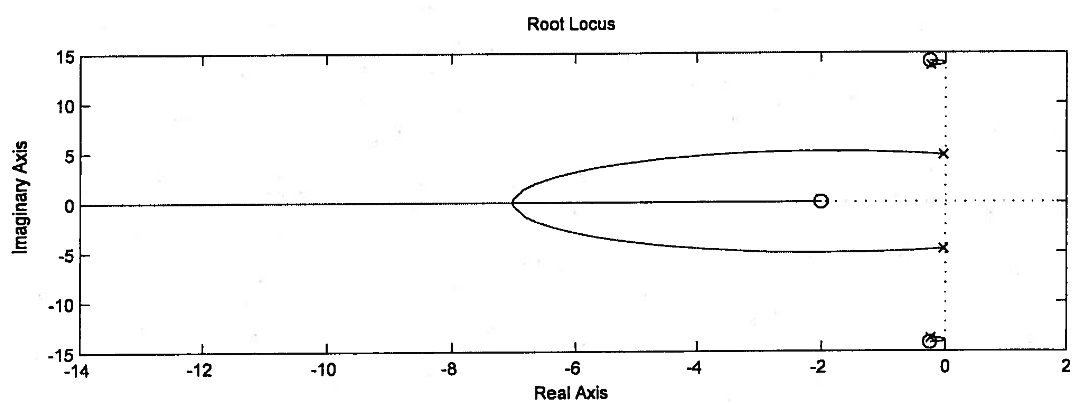
$$\Rightarrow \frac{u(t) - u\left(t - \Delta T\right)}{\Delta T} + pu(t) = K\left[\frac{e(t) - e\left(t - \Delta T\right)}{\Delta T} + ze(t)\right]$$

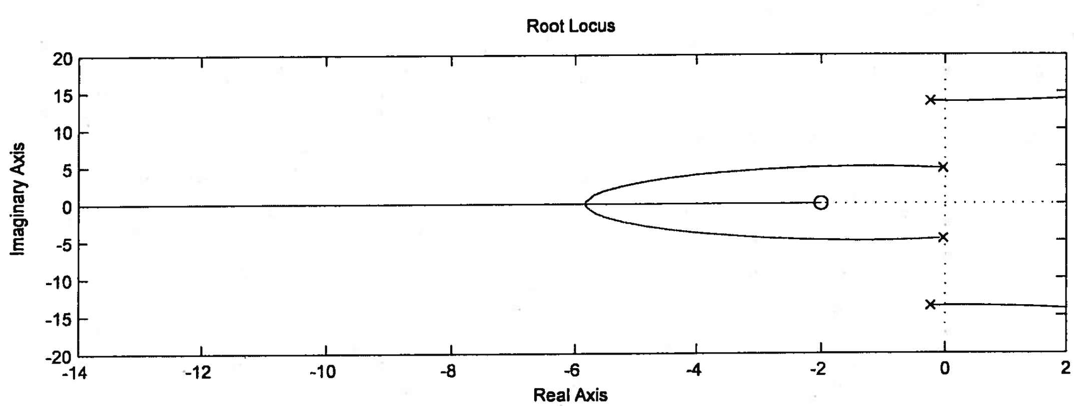
So given choices of gain, zero, and pole locations, this is an algorithm that will implement the lead compensator on a micro controller with sampling time ΔT. This algorithm requires knowing the error at the current time step and the prior values of both the error and the input. Different methods of discretizing will give different values for these coefficients but generally tie the same form (needing to know current information and one previous time step).

3 Notch Compensation

Consider the difference between a co-located and non-colocated control input:

The root locus shows the difference–the lightly damped poles corresponding to the system resonance go either to zeros or infinity.





We can intentionally add zeros near lightly damped poles to get the same effect. This is notch compensation:

We add two real poles to the system when we do this since we cannot add pure zeros. This works very well for high frequency, lightly damped poles so that the added poles are far to the left. They will still result in asymptotic behavior similar to the original system but hopefully at a higher gain than the designer will use.

Be careful with this technique – it is not very robust if your system is uncertain! You are always better off designing the system correctly to avoid such problems if you can.

4 Lag Compensation

If $z > p$ in $D(s) = K\frac{s+z}{s+p}$ you have a lag compensator. This resembles PI control but is more easily understood after some discussion of frequency domain techniques.