Assignment 6: Root Locus

ENGR 105: Feedback Control Design
Winter Quarter 2013
Due no later than 4:00 pm on Wednesday, Feb. 27, 2013
Submit in class or in the box outside the door to area of Room 107, Building 550

Problem 1. (10 pts.)

For the following closed-loop characteristic equations, set up the equation in the form suited to the Evans's root-locus method (see Lecture 15). Give $a(s)$, $b(s)$, and K in terms of the original parameters in each case. Be sure to select K such that $a(s)$ and $b(s)$ are monic, and the degree of $b(s)$ is not greater than $a(s)$.

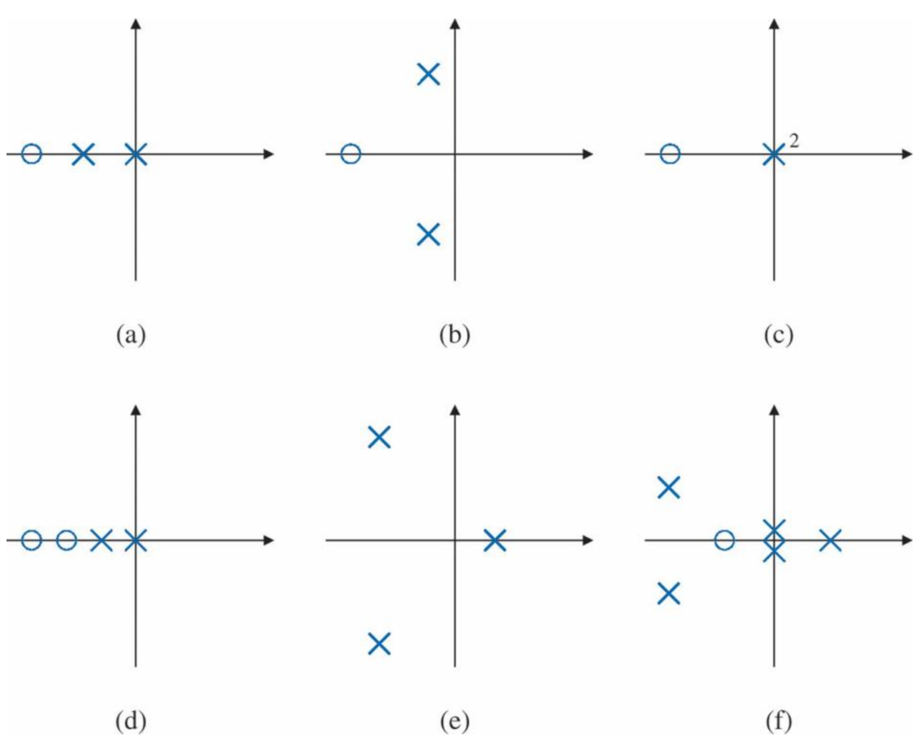
a. $s^2 + cs + c + 1 = 0$ relative to c.

b. $1 + \left[k_p + \frac{k_I}{s} + \frac{k_D s}{\tau s+1}\right] G(s) = 0$. Assume that $G(s) = A\frac{c(s)}{d(s)}$, where $c(s)$ and $d(s)$ are monic polynomials with the degree of $d(s)$ greater the degree of $c(s)$ plus 1.

i. relative to k_D
ii. relative to τ

Problem 2. (18 pts.)

Sketch the (positive) root loci for the pole-zero maps shown below without the aid of a computer, and briefly summarize your reasoning. (When it is necessary to estimate center and angles of the asymptotes, arrival and departure angles for complex poles, and/or break-in and break-out points, show how you get your estimates). If you want, you may use MATLAB to check your answers *after* attempting to draw the root loci by hand – but this is not required, as there are no numbers on the axes!



Problem 3. (12 pts.)

For each of the $L(s)$ shown below, sketch the root locus with respect to K for the equation $1 + KL(s) = 0$. Give the asymptotes and the arrival and departure angle for any complex zero or pole. After completing each hand sketch, verify your results using the MATLAB function `rlocus`. (Be sure to read the MATLAB instructions for using `rlocus` carefully.) Turn in your hand sketches and MATLAB plots on the same scales. Also, submit your MATLAB code.

a. $L(s) = \frac{(s+2)(s+6)}{s(s+1)(s+5)(s+10)}$ (real poles and zeros)

b. $L(s) = \frac{1}{s^2+3s+10}$ (complex poles)

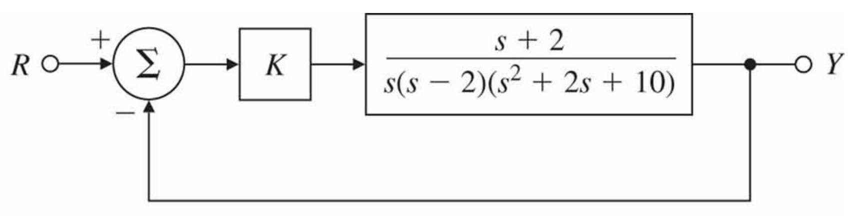
c. $L(s) = \frac{1}{s^2(s+8)}$ (multiple poles at the origin)

d. $L(s) = \frac{s+2}{s(s+10)(s^2+2s+2)}$ (mixed real and complex poles)

Sketching the root locus can be tedious, but practice is the only way to learn!

Problem 4. (10 pts.)

Consider the system shown below.



a. Use Routh's stability criterion to determine all values of K for which the system is stable.

b. Use MATLAB to draw the root locus versus K and find the values of K at the imaginary axis crossings. Submit your MATLAB plot and code. How does your MATLAB plot compare with your answer to part a?