ENGR 105: Feedback Control Design
Winter 2013

Lecture 18 - Frequency Response Basics

Monday, February 25, 2013

Today's Objectives

1. review the derivation of the the frequency response of a transfer function
2. revisit RC circuit example from a frequency response perspective
3. example comparing PD control and lead compensation

Reading: FPE Section 6.1

1 Review of Frequency Response

Thinking back to the concept of Fourier Transforms, a physical signal can be decomposed into a series of sinusoids at different frequencies. If we can describe what happens to each one of these sinusoids as it goes through a linear system, we can fully describe what happens to the signal. Thus, we can think of describing a linear system by its frequency response.

When we discussed transfer functions, we derived the sinusoidal response of a linear system:

Once the response from the stable poles dies out, This can be rearranged into a more insightful form:

$$
\begin{aligned}
a &= (s + j\omega)\, Y(s)\Big|_{s=-j\omega} \\
&= (s + j\omega)\, H(s) \frac{A\omega}{s^2 + \omega^2}\Bigg|_{s=-j\omega} \\
&= H(s) \frac{\omega A}{s - j\omega}\Bigg|_{s=-j\omega}
\end{aligned}
$$

Also,

Remember, $H(j\omega)$ is just a complex number. It has real and imaginary components, which can also be described by a magnitude and phase:

$$y(t) = -\frac{1}{2j} \left| H(j\omega) \right| A e^{-j\phi} e^{-j\omega t} + \frac{1}{2j} \left| H(j\omega) \right| A e^{j\phi} e^{j\omega t}$$

$$= A \left| H(j\omega) \right| \frac{1}{2j} \left[e^{j(\omega t + \phi)} - e^{-j(\omega t + \phi)} \right]$$

So a sine wave passed into a linear system produces a sine wave of the same frequency but different magnitude and phase (once transients have died out). The transfer function describes the change in the magnitude and phase.

If I have a more complicated input that I can write as a sum of sinusoids, the output is a simple sum of sinusoids:

We can think of this in two ways:

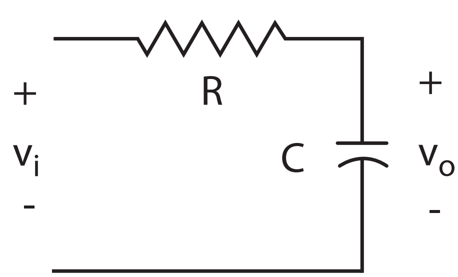
1. the transfer function fully describes the frequency response of the system
2. if we know the frequency response of the system, we can build the transfer function

In frequency-domain system identification, sinusoidal inputs are used to build an empirical transfer function experimentally. This can be an extremely useful method of obtaining a system model.

The frequency domain can be very useful in thinking about systems with vibrating or oscillations at specific frequencies. Audio systems and structural vibrations (in a car or bridge, for instance) are good examples.

We can use these techniques to gain insight into systems we have already studied.

2 Example: RC circuit revisited



Magnitude:

Phase:

So our system response looks like:

3 Example: PD control and lead compensation compared

Lead compensator: $D(s) = K\dfrac{Ts+1}{\alpha Ts+1}$ $\qquad {\scriptstyle 0<\alpha<1} \quad {\scriptstyle K>0}$

$$|D(j\omega)| = |K|\frac{|Tj\omega+1|}{|\alpha Tj\omega+1|} = |K|\frac{\sqrt{1+(\omega T)^2}}{\sqrt{1+(\alpha\omega T)^2}}$$

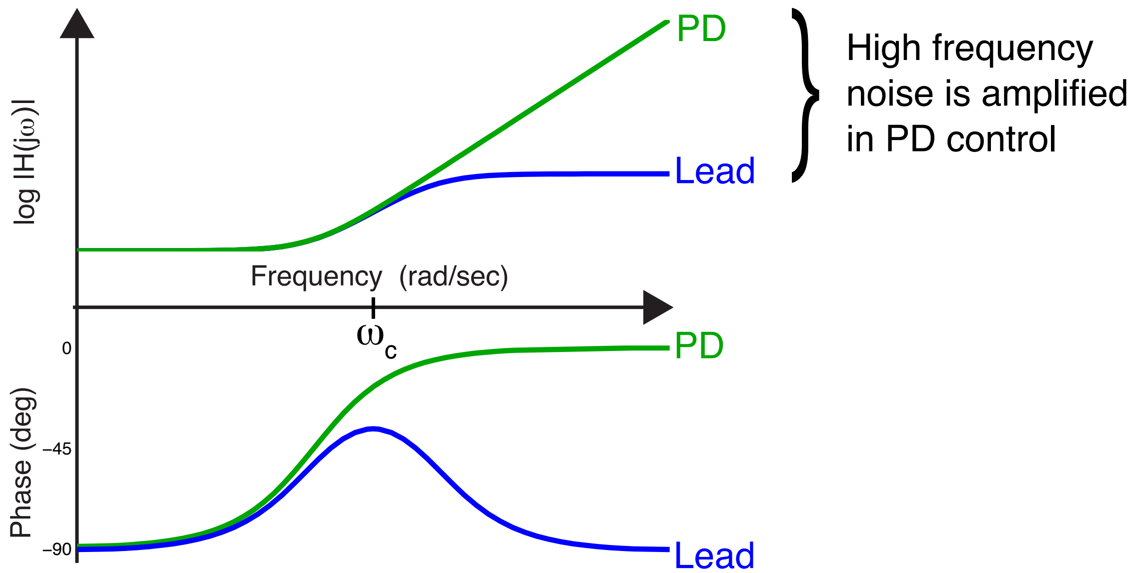
$$\phi = \angle(1+j\omega T) - \angle(1+\alpha j\omega T)$$

$$= \tan^{-1}(\omega T) - \tan^{-1}(\alpha\omega T)$$

PD Control: $D(s) = K(Ts+1)$

$D(j\omega) = K(Tj\omega+1)$ same as lead compensator when $\alpha = 0$

Graphically:



This gives a clearer explanation of lead compensation as a combination of PD control and a low-pass filter. This is literally true. The lead compensator does not amplify high-frequency signals as much as the PD controller. By changing the value of α, we can make the lead compensator match the response (and behavior) of the PD controller over a certain frequency range.

Our intuition is that signals beyond the frequency range of interest represent "noise" that we do not want to amplify.