Problem 1

part a

The characteristic equation is

$$\begin{aligned} s^2 + cs + s + 1 &= 0 \\ \Big(s^2 + 1\Big) + c\Big(s + 1\Big) &= 0 \end{aligned}$$

So then

$$\begin{aligned} K &= c \\ a(s) &= s^2 + 1 \\ b(s) &= s + 1 \end{aligned}$$

Note that both $a(s)$ and $b(s)$ are monic polynomials.

part b

part i

The characteristic equation is

$$1 + K_P G(s) + \frac{K_I}{s} G(s) + \frac{K_D s G(s)}{\tau s + 1} = 0$$

Note that this is not even a polynomial, so we first make it a polynomial, then extract K_D:

$$\begin{aligned} 1 + K_P \frac{Ac(s)}{d(s)} + \frac{K_I Ac(s)}{sd(s)} + K_D \frac{sAc(s)}{(\tau s + 1)d(s)} &= 0 \\ \Big(sd(s)(\tau s + 1) + (\tau s + 1)K_P Asc(s) + (\tau s + 1)K_I Ac(s)\Big) + K_D\Big(s^2 Ac(s)\Big) &= 0 \end{aligned}$$

So we have candidates for $a(s)$ and $b(s)$, but they aren't monic, so to use Evans' method directly, we have to divide by τ (to make the first polynomial monic) and A (to make the second polynomial monic). Then

$$\begin{aligned} K &= K_D A/\tau \\ a(s) &= sd(s)(s + 1/\tau) + K_P(s + 1/\tau)Asc(s) + (s + 1/\tau)K_I Ac(s) \\ b(s) &= s^2 c(s \end{aligned}$$

Based on our assumption on $G(s)$, both polynomials are monic

part ii

The characteristic equation is

$$1 + K_P G(s) + \frac{K_I}{s} G(s) + \frac{K_D s G(s)}{\tau s + 1}$$

Note that this is not even a polynomial, so we first make it a polynomial, then extract τ:

$$\begin{aligned}
1 + K_P G(s) + \frac{K_I}{s} G(s) + \frac{K_D s G(s)}{\tau s + 1} &= 0 \\
(\tau + 1) + (\tau + 1) K_P G(s) + (\tau + 1) \frac{K_I}{s} G(s) + K_D s G(s) &= 0 \\
\tau \left(s + K_P \frac{Ac(s)}{d(s)} s + K_I \frac{Ac(s)}{d(s)} \right) + \frac{K_I Ac(s)}{s d(s)} + \frac{K_P s Ac(s)}{d(s)} + K \frac{K_P Ac(s)}{d(s)} + 1 &= 0 \\
\tau \left(s^2 d(s) + K_P A s^2 c(s) + K_I A s c(s) \right) + \left(K_I Ac(s) + K_D s^2 Ac(s) + K_P A s c(s) + s d(s) \right) &= 0 \\
\left(s^2 d(s) + K_P A s^2 c(s) + K_I A s c(s) \right) + \frac{1}{\tau} \left(K_I Ac(s) + K_D s^2 Ac(s) + K_P A s c(s) + s d(s) \right) &= 0
\end{aligned}$$

So then

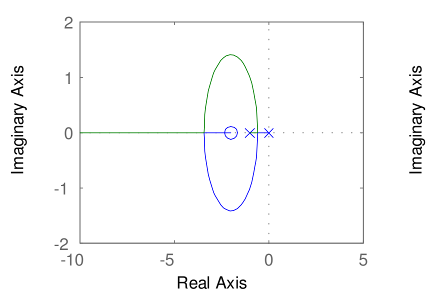
$$\begin{aligned}
K &= \frac{1}{\tau} \\
b(s) &= K_I Ac(s) + K_D s^2 Ac(s) + K_P A s c(s) + s d(s) \\
a(s) &= s^2 d(s) + K_P A s^2 c(s) + K_I A s c(s)
\end{aligned}$$

Based on our assumption on $G(s)$, both polynomials are monic. The reason we had to take $K = 1/\tau$ is that the order of b must be less than or equal to the order of a.

Problem 2

part a

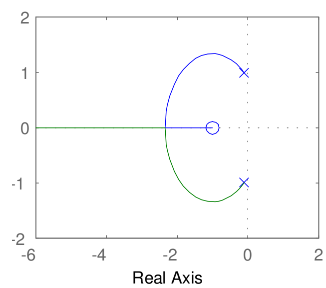
We know where the locus will be on the real axis; the break-in and break-out angles are $\pm 90°$.



part b

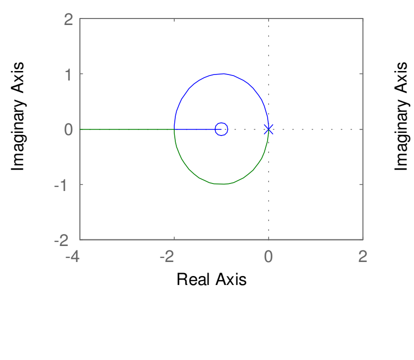
We know the locus will be left of the zero. Looking at the top pole, the departure angle is roughly given by

$$\begin{aligned} 45^\circ - 90^\circ - \phi &= 180^\circ \\ \phi &= 135^\circ \end{aligned}$$



part c

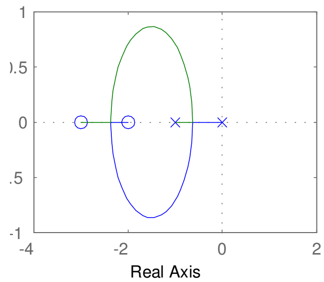
We know the locus will be left of the zero. The break-in and break-out angles are $\pm 90^\circ$.



part d

We know the locus will be between the two zeros, and between the two poles. The break-in and break-out angles are $\pm 90^\circ$.

$$\begin{aligned} 45^\circ - 90^\circ - \phi &= 180^\circ \\ \phi &= 135^\circ \end{aligned}$$

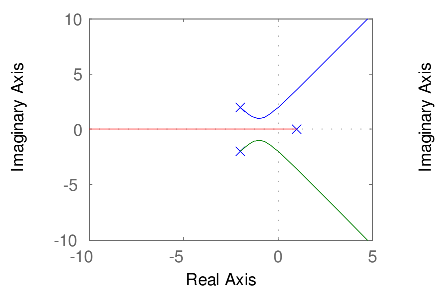


part e

We know the locus will be left of the real pole. Looking at the top pole, the departure angle is roughly given by

$$\begin{aligned}-90^\circ - 135^\circ - \phi &= 180^\circ \\ \phi &= -45^\circ\end{aligned}$$

The asymptotic angles are $180^\circ, \pm 60^\circ$. The center of the asymptotes is at the average of the poles, so on the real line, a bit left of the origin.

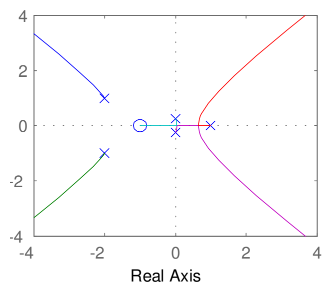


part f

We know the locus will be between the zero and the real pole. Looking at the top right pole, the departure angle is roughly given by

$$\begin{aligned} 0 - 180^\circ - 90^\circ - (-45)^\circ - 45^\circ - \phi &= 180^\circ \\ \phi &= -90^\circ \end{aligned}$$

So the poles move *toward* the real axis. One of the three closed-right-half-plane poles goes to the zero, and the other two break out at $\pm 90^\circ$. The asymptotic angles are $\pm 45^\circ, \pm 135^\circ$. The center of the asymptotes is roughly around the origin.



Problem 3

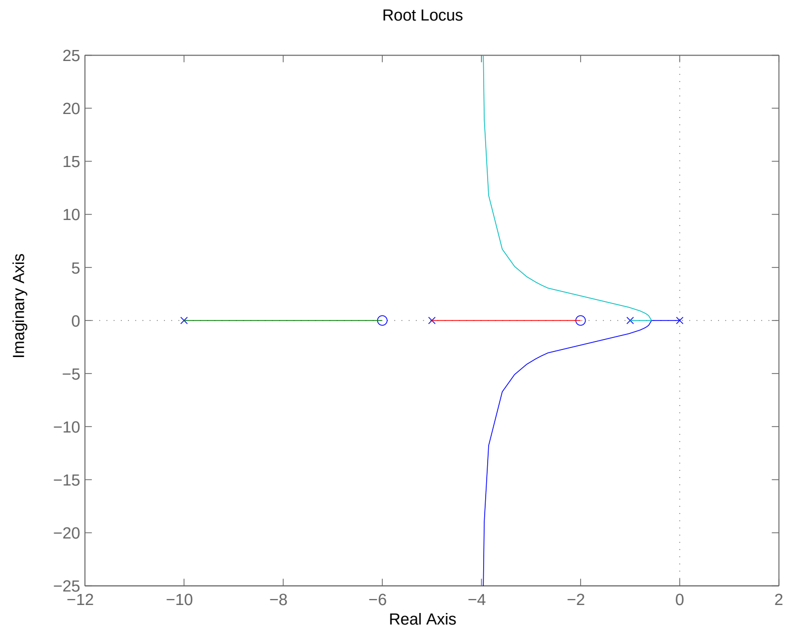
part a

$$\alpha = \frac{(0 - 1 - 5 - 10) - (-2 - 6)}{4 - 2} = -4$$

The poles at 0 and -1 approach each other and branch at $\pm 90^\circ$. There are two more poles than zeros, so the asymptotic angles are $\pm 90^\circ$.

MATLAB:

```
s=tf('s');
P=(s+2)*(s+6)/(s*(s+1)*(s+5)*(s+10));
rlocus(P)
print -dpdf ~/Dropbox/E105-Win2013/Assignments/assignment6-nick/solutions
```

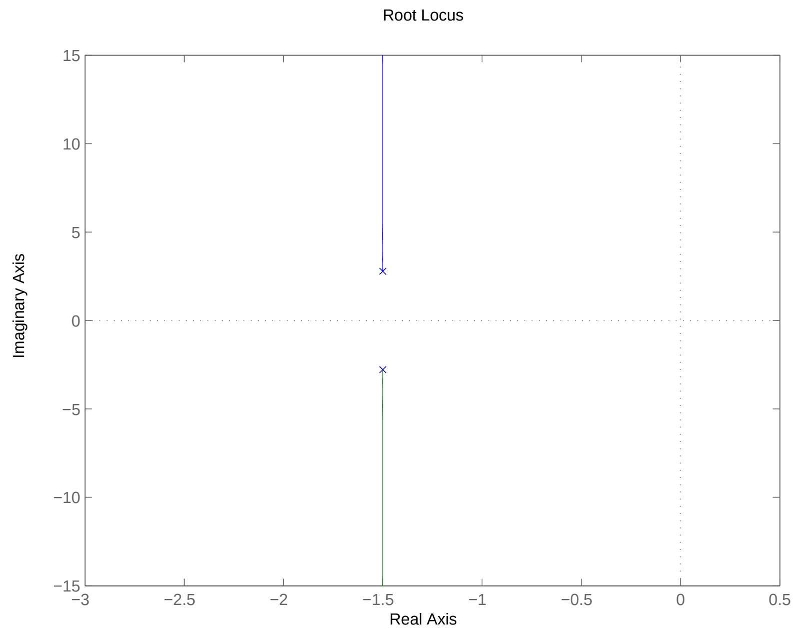


part b

The two poles are at $-3/2 \pm 3j$. There are no zeros, so the center of the asymptote is the average of the poles, which is $-3/2$. There are two poles, so the asymptotic angles are $\pm 90°$.

MATLAB:

```
s=tf('s');
P=1/(s^2+3*s+10);
rlocus(P)
print -dpdf ~/Dropbox/E105-Win2013/Assignments/assignment6-nick/solutions
```

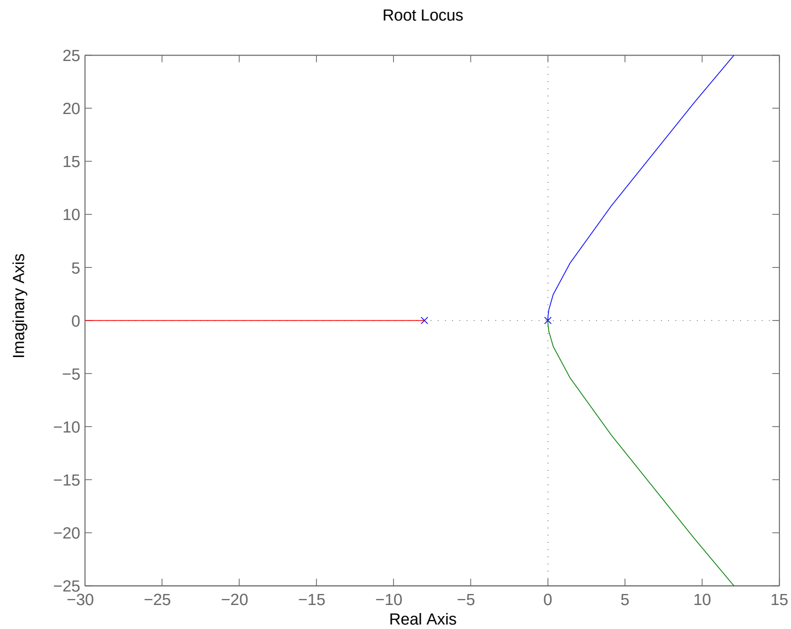


part c

There are no zeros, so the center of the asymptote is the average of the poles, which is $-8/3$. There are three poles, so the asymptotic angles are $180°, \pm 60°$. The set $(\infty, -8]$ is on the locus, so we can infer that the pole at -8 goes to $-\infty$. The poles at zero branch with angles $\pm 90°$.

MATLAB:

```
s=tf('s');
P=1/(s^2*(s+8));
rlocus(P)
print -dpdf ~/Dropbox/E105-Win2013/Assignments/assignment6-nick/solutions
```



part d

There are poles at $0, -10, 1 \pm j$. The center of the asymptote is

$$\alpha = \frac{(0 - 10 - (1+j) - (1-j)) - (-2)}{4-1} = -10/3$$

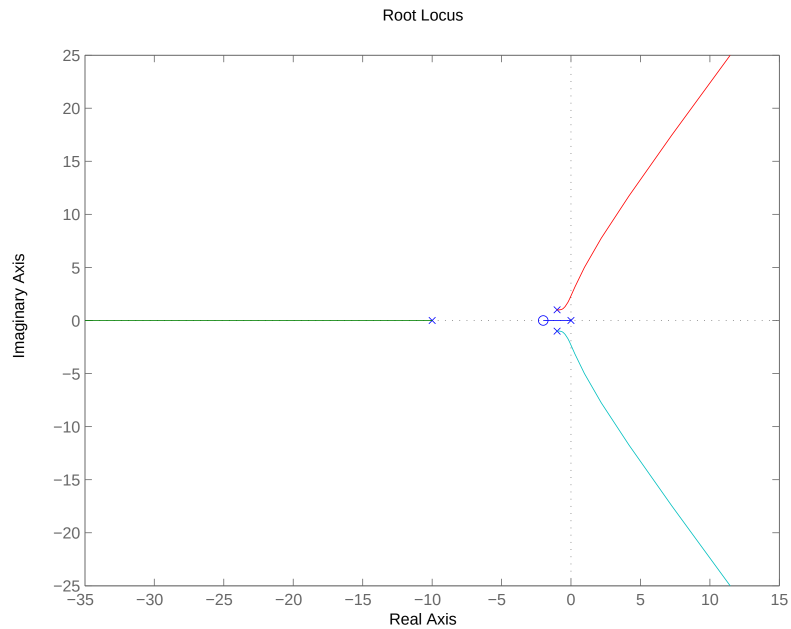
There are three more poles than zeros, so the asymptotic angles are $180°, \pm 60°$. The set $(\infty, -10] \cup [-2, 0]$ is on the locus, so we can infer that the pole at -10 goes to $-\infty$ and the pole at 0 goes to -2. The poles at zero branch with angles $\pm 90°$. The departure angle for first complex pole is

$$\begin{aligned}
&180° - \angle(-2 - (-1+j)) + \angle(0 - (1+j)) + \angle((1+j) - (1+j)) + \angle(-10 - (1+j)) \\
&= 180° - 45° + 135° + 90 + 6.34 \\
&= 6.34
\end{aligned}$$

And for the other complex pole, it is -6.34.

MATLAB:

```
s=tf('s');
P=(s+2)/(s*(s+10)*(s^2+2*s+2));
rlocus(P)
print -dpdf ~/Dropbox/E105-Win2013/Assignments/assignment6-nick/solutions
```



Problem 4

part a

The characteristic equation is

$$s^3 + 2s^2 + 10s + K$$

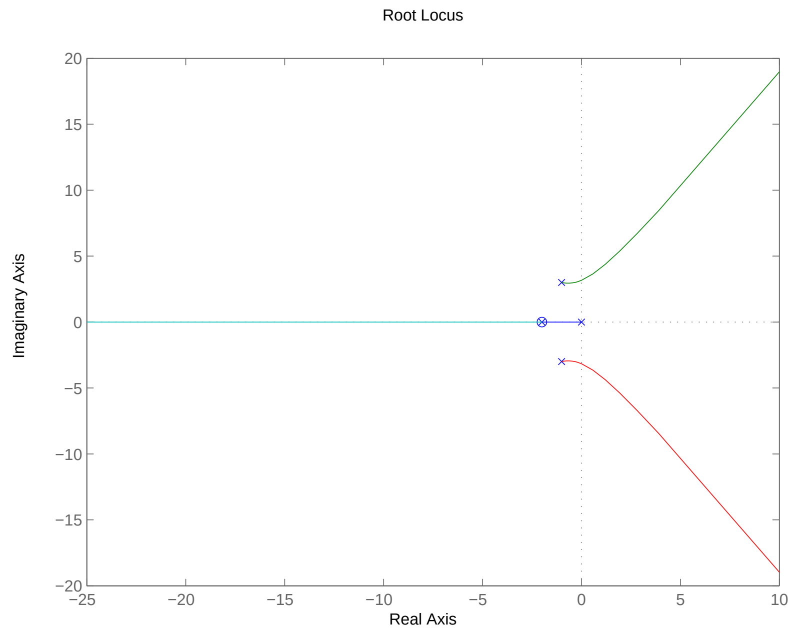
The Routh array is

$$\begin{array}{cc} 1 & 10 \\ 2 & K \\ \frac{K-20}{-2} & \\ -K & \end{array}$$

So the conditions for stability are $K > 0$ and $K < 20$.

part b

The root locus is




```
s=tf('s');
P=(s+2)/(s*(s+2)*(s^2+2*s+10));
rlocus(P)
print -dpdf ~/Dropbox/E105-Win2013/Assignments/assignment6-nick/solutions
```