Problem Session

Bode plots

$u(t) = \sin(\omega t) \longrightarrow [\,H(s)\,] \longrightarrow y(t) = |H(j\omega)| \sin(\omega t + \phi)$

gain plot — phase plot

consider $H(s) = \frac{100}{(s+30)}$

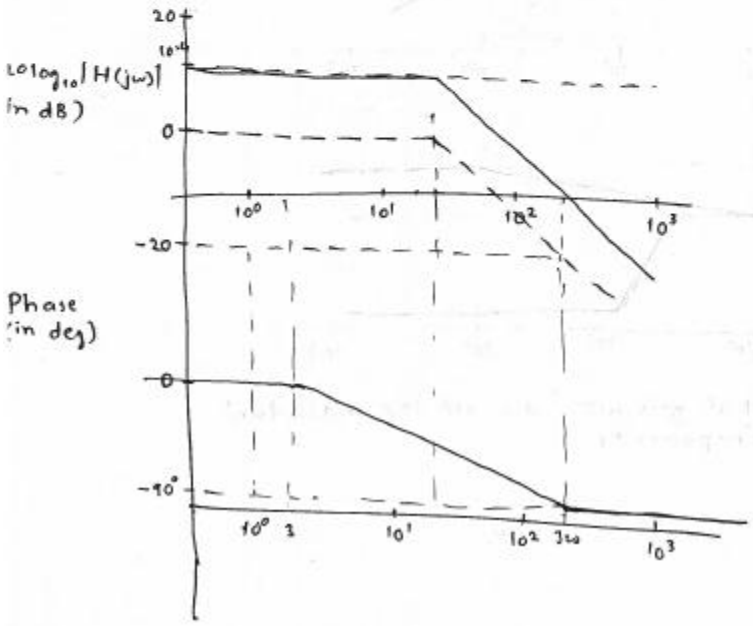
Step 1) Rewrite transfer function $\frac{100}{30} \frac{1}{\left(\frac{s}{30}+1\right)}$

constant = 3.3 $\longrightarrow$ 10.4 dB

Step 2) look at poles and zeros

one pole at (−30)

after 30 rad/s we have a slope of −20 dB/dec



$$H(s) = \frac{30(s+10)}{s^2+3s+50} = \frac{6\left(\frac{s}{10}+1\right)}{\left(\frac{s^2}{50}+\frac{3s}{50}+1\right)}$$

constant $\omega \to 0$ $\quad H(j\omega) \to 6 \longrightarrow 15.5$ dB

$\omega_0 = \sqrt{50} = 7.07 \qquad \zeta = \frac{3\sqrt{50}}{2 \times 50} = 0.21$

peak height $= -20\log_{10}(2\zeta\sqrt{1-\zeta^2})$

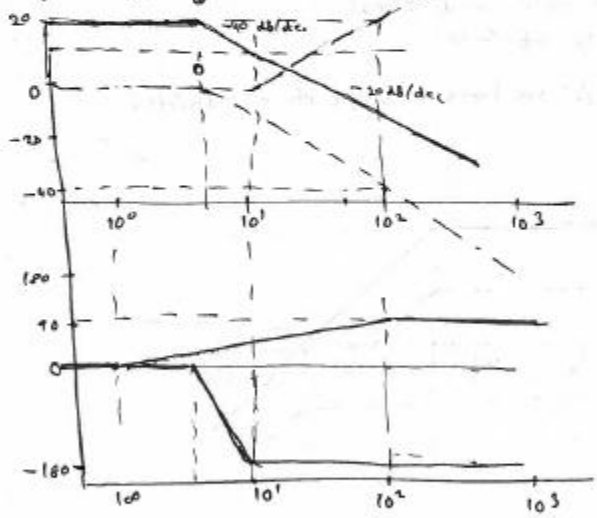
$= 7.9$ dB

~~$\omega_0 \sqrt{1-2\zeta^2} = 6.91$ rad/sec~~

$\omega = \frac{\omega_0}{5\zeta} = 5$ rad/sec

low freq

high freq $\omega_0 \cdot 5\zeta = 9.9$ rad/sec



Note that you will add all the individual components

consider complex conjugate terms

ex $(1+j\omega)(1-j\omega) = z_1$

$(1+j\omega)^2 = z_2$

note $|z_1| = |z_2| = 1+\omega^2$ gain plot is same but the phase plot is different for the two

Notch filter

example

$$H(s) = \frac{s^2+0.5s+100}{s^2+5s+100}$$

Bode plot looks like

dip in gain plot leads to large attenuation at that freq. and it is filtered out.