ENGR 105: Feedback Control Design
Winter 2013

Lecture 19 - Bode Plots

Wednesday, February 27, 2013

Today's Objectives

1. show how the component signals of a Bode plot arise
2. describe component 1: poles and zeros at the origin
3. describe component 2: real poles and zeros
4. describe component 3: complex poles and zeros

Reading: FPE Section 6.1

1 Bode Plot derivation

It is very easy to get a feel for the frequency response of a system from a sketch based on a few simple rules. These plots are easiest if made on a log-log scale for magnitude and log-linear for phase. Plots of the frequency response in this form are known as Bode plots.

The advantages of working with Bode plots (over other analysis techniques and graphical descriptions such as root locus and Nyquist plots) are:

1. dynamic compensator design (i.e., the lead compensator we discussed earlier) can be based entire on Bode plots
2. Bode plots can be determined experimentally
3. Bode plots of systems in series simply add
4. The use of a log scale permits display of a wide range of frequencies

For the root locus, we put the transfer function in the form:

$$KG(s) = K\frac{(s-z_1)(s-z_2)\dots(s-z_m)}{(s-p_1)(s-p_2)\dots(s-p_n)}$$

For Bode plots, we rearrange this slightly to get:

$$KG(j\omega) = K_0\frac{(j\omega\tau_1+1)(j\omega\tau_2+1)\dots(j\omega\tau_m+1)}{(j\omega\tau_a+1)(j\omega\tau_b+1)\dots(j\omega\tau_*+1)}$$

This form is especially useful because K_0 is related to the magnitude scaling of the transfer function at low frequencies. It also allows us to consider input frequencies of value ω (we assigned $s = j\omega$), which we recognized in the previous lecture as providing a useful description of how a transfer function transforms an input signal.

Magnitude:

If we take the logarithm (base 10) of the magnitude:

$$\begin{aligned}\log|KG(j\omega)| = \log|K_0| &+ \log|j\omega\tau_1 + 1| + \log|j\omega\tau_2 + 1| + \dots \\ &- \log|j\omega\tau_a + 1| - \log|j\omega\tau_b + 1| - \dots\end{aligned}$$

everything simply adds!

Phase

For the phase angle:

$$\begin{aligned}\angle KG(j\omega) = \angle K_0 &+ \angle(j\omega\tau_1 + 1) + \angle(j\omega\tau_2 + 1) + \dots \\ &- \angle(j\omega\tau_a + 1) - \angle(j\omega\tau_b + 1) - \dots\end{aligned}$$

This is also additive.

We can therefore sketch the frequency response of the system as a sum of individual elements:

1. $K_0(j\omega)^n$: for poles and zeros at the origin

2. $(j\omega\tau + 1)^{\pm 1}$: for real poles and zeros

3. $\left[\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_n}\right) + 1\right]^{\pm 1}$: for complex poles and zeros

When plotting the magnitude, it is common to use dB.

Magnitude in dB $= 20\log_{10}(G(j\omega))$

Magnitude of $1 = 0$ dB

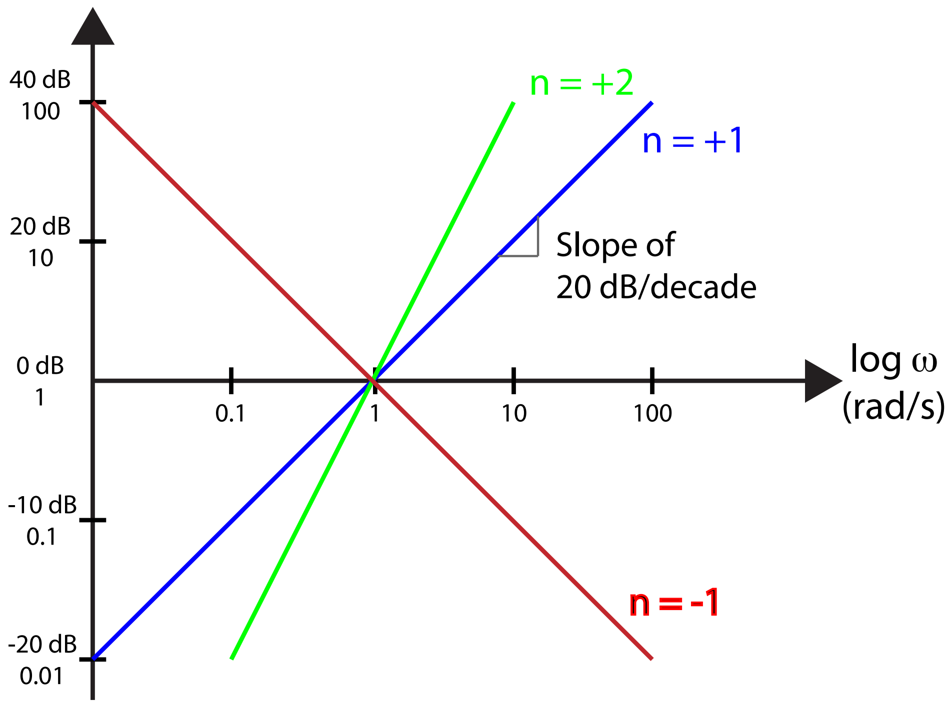
An order of magnitude is 20 dB (from 0.1 to 1, for instance)

2 Component 1: poles and zeros at the origin

Poles and zeros at the origin contribute to the Bode plot in terms that look like $K_0(j\omega)^n$.

Looking at the magnitude: $\log K_0|(j\omega)^n| = \log K_0 + n\log|j\omega|$

Magnitude plot of $n\log|j\omega|$:

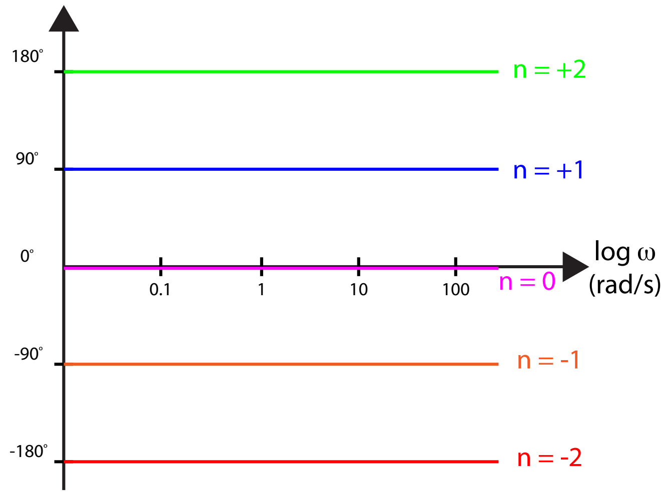


The magnitude is just a straight line on the log-log plot. The slope is +20 dB/decade/zero or −20 dB/decade/pole.

The value of K_0 just shifts the plot up or down (the magnitude at 1 rad/s is K_0 – or $\log K_0$ in dB).

Looking at the phase: $\angle K_0(j\omega)^n = \underbrace{\angle K_0}_{\substack{0^\circ \text{ if } K_0>0 \\ 180^\circ \text{if } K_0<0}} + \underbrace{\angle(j\omega)^n}_{n\times 90^\circ}$

Phase plot of $n \log |j\omega|$:

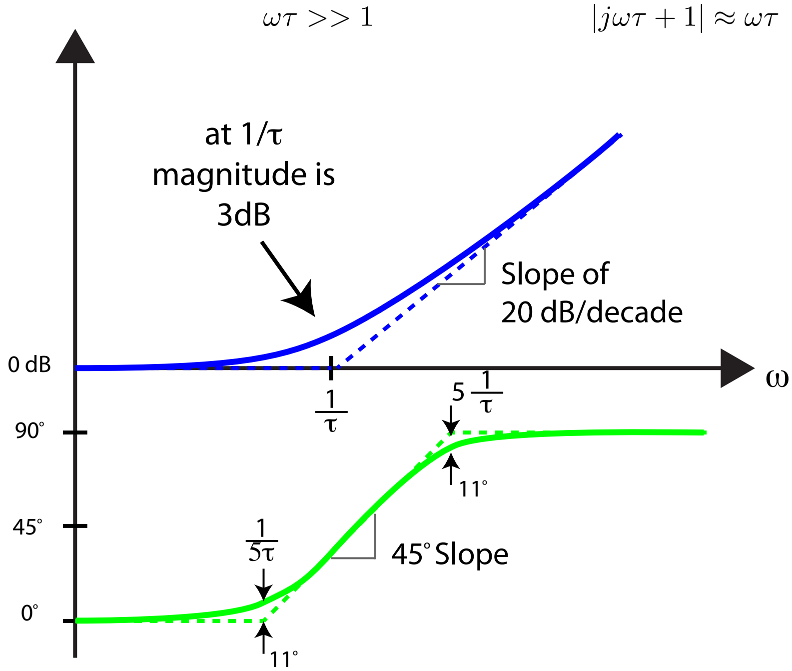


3 Component 2: Real poles and zeros

Real poles and zeros contribute to the Bode plot in terms that look like $(j\omega\tau + 1)^{\pm 1}$.

Let's first consider the zeros, where the contributing term is $(j\omega\tau + 1)$.

When	$\omega\tau << 1$	$\lvert j\omega\tau + 1\rvert \approx 1$	$\angle(j\omega\tau + 1) \approx 0^\circ$
	$\omega\tau >> 1$	$\lvert j\omega\tau + 1\rvert \approx \omega\tau$	$\angle(j\omega\tau + 1) \approx 90^\circ$



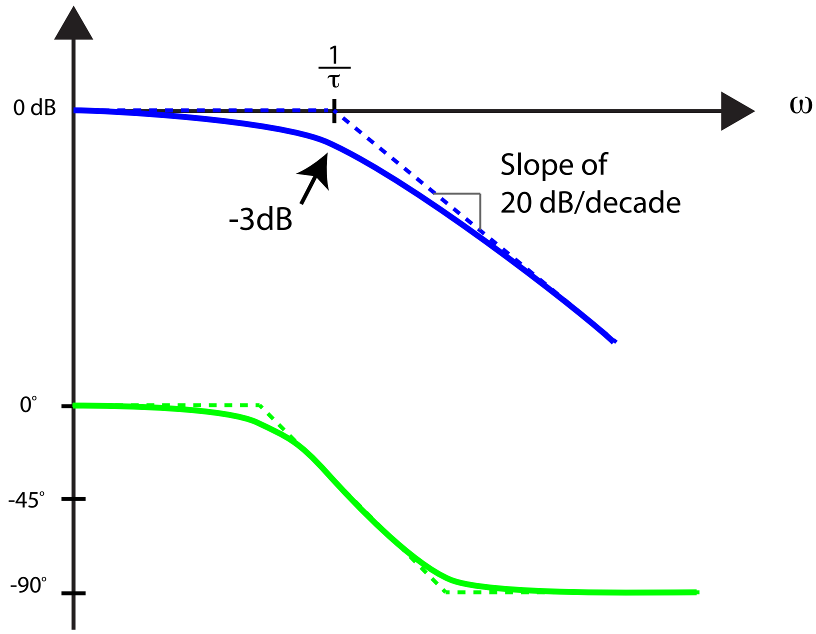
@ $\omega\tau = 1$,

$|j\omega\tau + 1| = \sqrt{2}$ or $+3$dB

Can draw phase with asymptotes as well

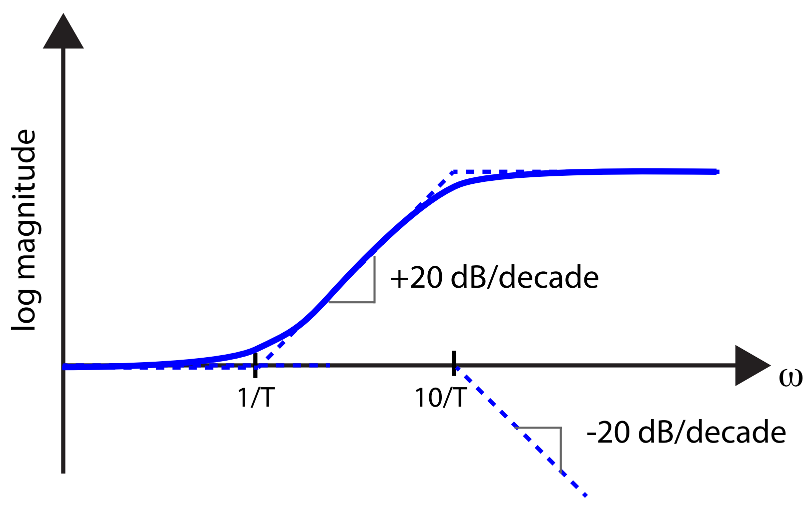
Now let's consider the poles, where the contributing term is $\frac{1}{(j\omega\tau+1)}$.

This is easy – just flip sign on phase and magnitude (on logarithmic scale).



With these rules, we could sketch our lead compensator very easily.

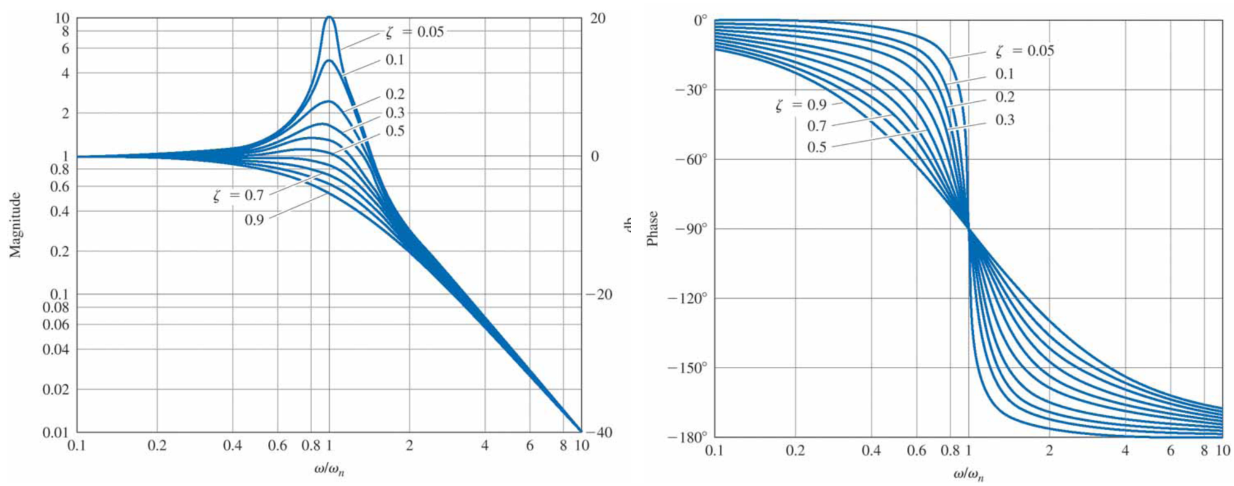
$$D(s) = K\frac{Ts+1}{0.1Ts+1} \qquad \alpha = 0.1 \text{ in this example}$$



4 Component 3: Complex poles and zeros

Complex poles and zeros contribute to the Bode plot in terms that look like $\left[\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_n}\right) + 1\right]^{\pm 1}$

The plots below (Figure 6.3 from FPE) shows $\left[\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_n}\right) + 1\right]^{-1}$ (complex poles)



To sketch, the plot is centered around ω_n, with a resonance peak and phase angle slope that are determined by the damping ratio.

Away from ω_n (by a factor of 10 or more in frequency): this look similar to the contribution of the two real poles:

$\omega << \omega_n$	0 dB and $0°$ phase
$\omega >> \omega_n$	-40 dB/dec slope and $-180°$ phase

To find the response for $\left[\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\left(\frac{j\omega}{\omega_n}\right) + 1\right]$ (a zero), just invert the plot.