Assignment 8: Nyquist Plots and Stability Margins

ENGR 105: Feedback Control Design
Winter Quarter 2013
Due no later than 4:00 pm on Wednesday, Mar. 13, 2013
Submit in class or in the box outside the door to area of Room 107, Building 550

Problem 1. (20 pts.)

For the two systems below,

(i) Sketch the Bode plots of $KG(s)$ for $K = 1$.

(ii) Sketch the Nyquist plot (of $KG(s)$ for $K = 1$) based on the Bode plots.

(iii) Ensure that your sketched Nyquist plot is correct by comparing it with that obtained using the Matlab command `nyquist` (submit your Matlab code and the resulting plot).

(iv) Using the Nyquist plot, estimate the range of $K > 0$ for which the system is stable.

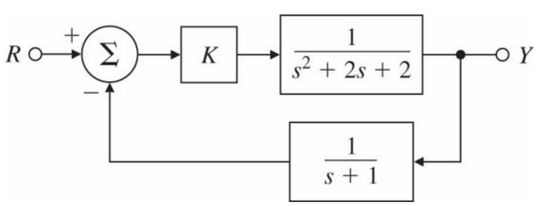
(v) Verify your result for the range of K by using MATLAB to create the root-locus plot.

a. $KG(s) = \frac{K(s+2)}{s+20}$

b. $KG(s) = \frac{K}{(s+10)(s+1)^2}$

Problem 2. (5 pts.)

Draw the Nyquist plot for the system in the figure below. Using the Nyquist stability criterion, determine the range of K for which the system is stable. Consider both positive and negative values of K. Note: make sure to use the correct characteristic equation for the closed-loop system, which here contains a transfer function in the feedback loop. You can use MATLAB for this problem, but you may also want to sketch by hand for practice.



Problem 3. (10 pts.)

A non-minimum phase system is one that is causal and stable, and its inverse is causal and unstable. This occurs when there is a zero in the right half plane (RHP). Non-minimum phase behavior can be ascertained from the Bode plots: when evaluated for frequency inputs between zero and infinity, there is a net change in phase which, for an associated magnitude plot, is greater than if all poles and zeros were in the left half place (LHP). Consider the following two systems:

A minimum-phase system: $G_1(s) = \frac{s+1}{s+10}$ A non-minimum-phase system: $G_2(s) = \frac{s-1}{s+10}$

Note that when $s = j\omega$, the non-minimum system phase system's numerator phase $\angle(j\omega - 1)$ decreases with ω rather than increasing.

a. Sketch the phase portion of the Bode plot of $G(s)$ for both the minimum-phase and non-minimum-phase systems. Use an input of ω = 0.1 to 100 rad/sec.

b. Sketch the Nyquist plot for each of these systems. Does a RHP zero affect the relationship between the -1 encirclements on the polar plot and the number of unstable closed-loop roots?

c. Sketch the phase portion of the Bode plot for the following unstable system for $\omega = 0.1$ to 100 rad/sec.

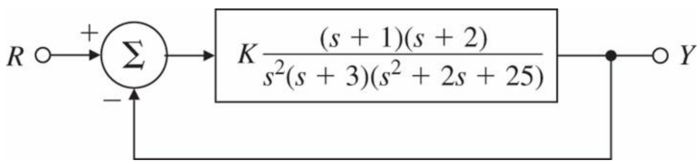
$$G_3(s) = \frac{s+1}{s-10}$$

d. Consider that these plants are used in a unity feedback system with proportional controller $K > 0$. Check the stability of all three systems using the Nyquist criterion on $KG(s)$. Determine the range of K for which the closed-loop system is stable.

e. Check your results for the range of K using a root locus or finding the roots of the characteristic equation using MATLAB.

Problem 4. (5 pts.)

For the system in the figure below, examine stability margins. You can use MATLAB for this problem, but you may also want to sketch by hand for practice.



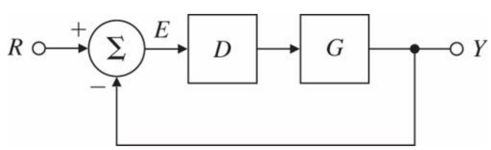
a. Is this a valid system to determine gain and phase margin from the Bode plots?

b. Use Bode plots to determine the gain at which instability occurs. Submit your plots and show how you determined this.

c. What gain (or gains) gives a PM of 20°? What is the gain margin when PM = 20°?

Problem 5. (10 pts.)

Consider the general form of the unity negative feedback system shown below.



where $G(s) = \frac{5}{s(s+1)\left(\frac{s}{5}+1\right)}$

a. Use Bode plot sketches to design a lead compensation $D(s)$ with unity DC gain so that PM (phase margin) $\geq 40°$.

b. Verify and refine your design using Matlab. (Describe your process, and include final code and plot.)

c. Bandwidth is defined as the maximum frequency at which the output of the system will track an input sinusoid in a satisfactory manner. Here, I define "satisfactory" to mean that the output is attenuated to a factor of no more than 0.707 times the input. What is the approximate bandwidth of your compensated system?