ENGR 105: Feedback Control Design
Final Exam, Winter Quarter 2012
Monday, March 19, 2012, 8:30-11:30 am, Room 420-40

Name: ___Solutions_________

Exam Policies

- Allow one empty seat between yourself and the next person.
- No calculators, cell phones, or other electronic devices. Please leave these items in your bag or pocket throughout the exam. There is a clock in the room.
- The exam is closed book and closed notes, except you may use four sheets of paper with notes (writing on front and back is permitted).
- Simplify answers as fully as possible, box your answers, and show all work for full credit.

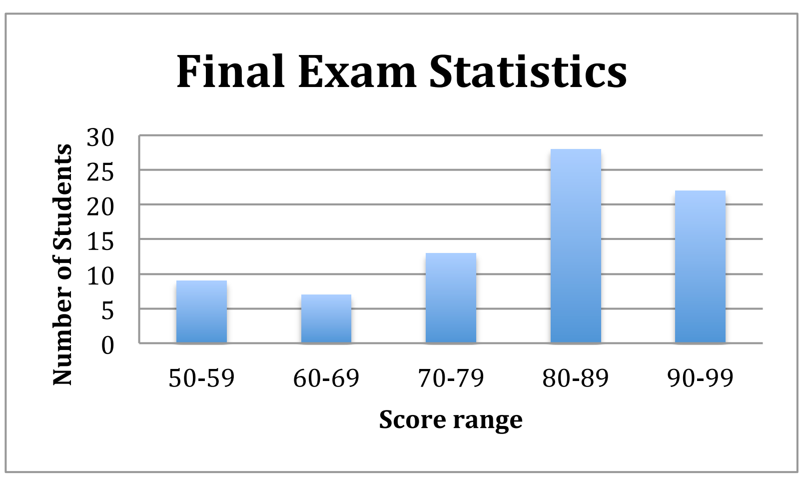
Stanford Honor Code

1. The Honor Code is an undertaking of the students, individually and collectively:
   a. that they will not give or receive aid in examinations; that they will not give or receive unpermitted aid in class work, in the preparation of reports, or in any other work that is to be used by the instructor as the basis of grading;
   b. that they will do their share and take an active part in seeing to it that others as well as themselves uphold the spirit and letter of the Honor Code.
2. The faculty on its part manifests its confidence in the honor of its students by refraining from proctoring examinations and from taking unusual and unreasonable precautions to prevent the forms of dishonesty mentioned above. The faculty will also avoid, as far as practicable, academic procedures that create temptations to violate the Honor Code.
3. While the faculty alone has the right and obligation to set academic requirements, the students and faculty will work together to establish optimal conditions for honorable academic work.

I acknowledge and accept the Honor Code on this exam and all other work associated with this class.

____
signature

Problem	Mean / Std. Dev.
1 (10 pts.)	8.4 / 1.4
2 (15 pts.)	11.6 / 3.7
3 (20 pts.)	16.8 / 3.6
4 (15 pts.)	13.3 / 1.8
5 (20 pts.)	15.6 / 3.8
6 (20 pts.)	14.9 / 5.6
Total (100 pts.)	80.6 / 13.3



Problem 1. (10 pts.)

Mark the following True (T) or False (F):

__T___ a. (1 pt.) A pure time delay has no effect on the magnitude of the open-loop transfer function.

__F___ b. (1 pt.) For an equation to be in root-locus form, the polynomial $a(s)$ must be of lower degree than $b(s)$.

__F___ c. (1 pt.) Stability margins can be found from the Bode plot of any system.

__T___ d. (1 pt.) As the poles of a system move further away from the origin of the real-imaginary plane, the system response gets faster.

__F___ e. (1 pt.) Lead compensation resembles a PI controller.

__T___ f. (1 pt.) A lead compensator can be implemented as an analog circuit.

__T___ g. (1 pt.) A magnitude of 1000 is equivalent to 60 decibels.

Provide a short narrative answer to each of the following questions:

h. (1 pt.) What is a physical interpretation of the poles of a guitar?

The poles describe the vibrations of the strings. Harmonics, pitch, or other answers relating directly to vibrations are also okay.

i. (1 pt.) Why might you want to determine stability from a Nyquist plot rather a Routh Array?

Many possible answers here. The Routh Array requires that you compute the closed-loop poles, whereas the Nyquist plot can be generated from the open-loop transfer function, which for some systems may be easier. Nyquist also provides more information, e.g. regarding stability margins. Nyquist plots can also be derived from experimentally obtained Bode plots.

j. (1 pt.) For the Phantom Omni robots/haptic devices shown in class, name two assumptions we made about the system dynamics to permit use of the analysis techniques learned in this course.

Many possible answers here. The main point is that we assume the system is linear and time-invariant. Possible takes on this answer include that there are no Coriolis, centripetal, or Coulomb friction effects. Also, the inertia does not change over the workspace. We only considered one degree of freedom, assuming that the degrees of freedom are uncoupled.

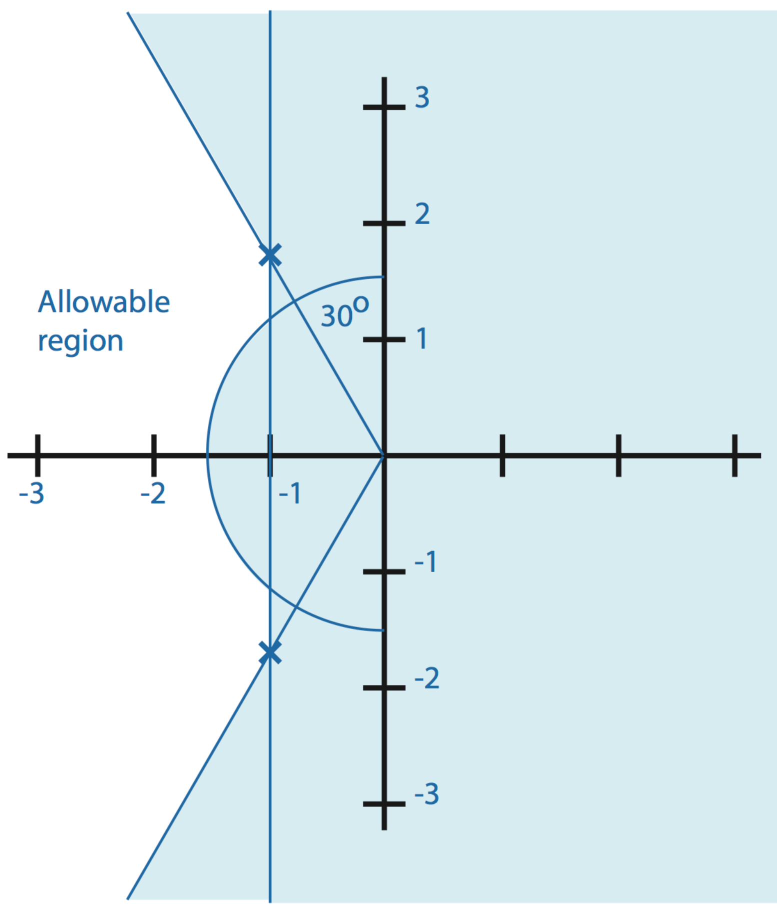
Problem 2. (15 pts.)

A certain system has dynamics dominated by a complex pair of poles and no finite zeros. The time-domain specifications are:

$$t_r \leq 1.2 \text{ sec}$$
$$M_p \leq 17\%$$
$$t_s \leq 4.6 \text{ sec (estimate)}$$

a. (10 pts.) Sketch the region in the s-plane where the poles could be placed so the system will meet these specifications. Use the axes given below.

b. (5 pts.) Indicate on your sketch the specific locations that will have the *smallest rise time* and also meet the settling time estimate exactly.



$t_r \leq 1.2$ sec, $t_r = \frac{1.8}{\omega_n}$ results in $\omega_n \leq 1.5$ rad/sec

$M_p \leq 17\%$ results in $\zeta \geq 0.5$ (from Fig. 3.18b, which is provided), which corresponds to $\theta = \sin^{-1}\zeta \geq 30°$

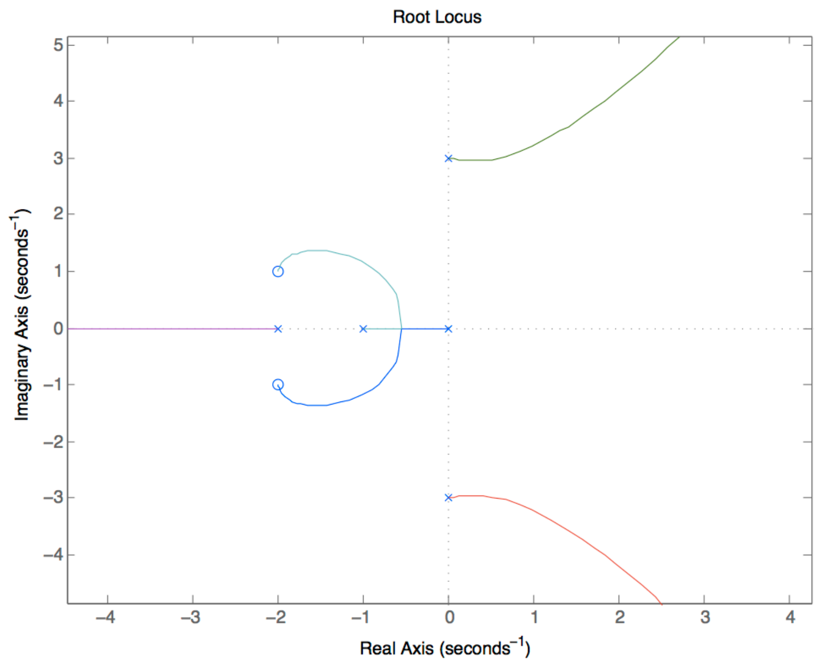
$t_s \leq 4.6$ sec, $t_s = \frac{4.6}{\sigma} = \frac{4.6}{\zeta\omega_n}$ results in $\sigma \geq 1$ rad/sec

We choose the points farther away from the real axis in order to have a smaller rise time.

Problem 3. (20 pts.)

Consider the transfer function $G(s) = \frac{s^2+4s+5}{s(s+1)(s+2)(s^2+9)}$

a. (3 pts.) Enter the poles and zeros of $G(s)$ on the s-plane below.



b. (12 pts.) Calculate the following guidelines for sketching the root locus $1 + KG(s)$.

The angles of the asymptotes are: __60, 300, and 180______________________________

The center of the asymptotes is: __+1/3_________________________________

The arrival angles at the complex zeros are: __-90, 90______________________

The departure angles from the complex poles are: _-19.5, 19.5__________________

$$\phi_i = \frac{180+360(i-1)}{3} \Rightarrow 60, 300, 180 \qquad \alpha = \frac{\sum p_i - \sum z_i}{3} = \frac{1}{3}$$

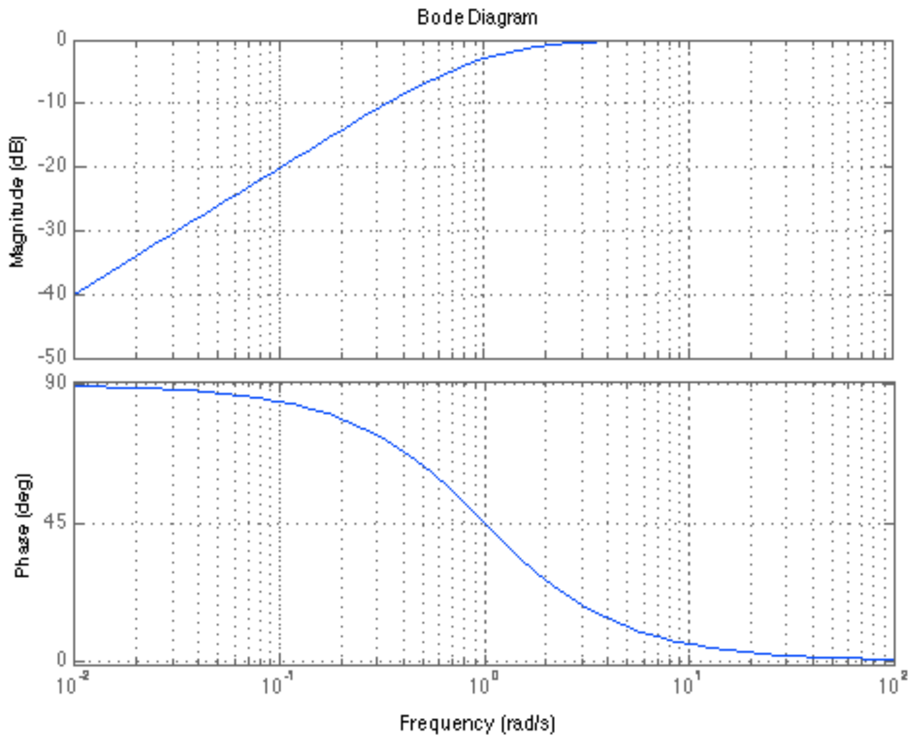
$$\psi_{1arr} = \sum \phi_i - \sum_{i\neq 1} \psi_i = 90 \qquad \psi_{2arr} = -90$$

$$\phi_{1dep} = \sum \psi_i - \sum_{i\neq 1} \phi_i = -180 + 45 + \tan^{-1} 2 - \tan^{-1}\frac{3}{2} - \tan^{-1} 3 - 90 - 90 = -19.5$$

c. (5 pts.) Sketch the root locus on the s-plane above.

Problem 4. (15 pts.)

The Bode plot shown below was determined from a transfer function $G(s)$.



a. (8 pts.) Find the simplest $G(s)$ that could result in this Bode plot.

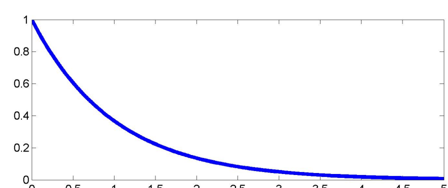
At low frequencies, the magnitude has a slope of +20 dB/decade and a phase of 90 degrees. This indicates a zero at the origin.

At high frequencies, the magnitude has a slope of zero and a phase of zero degrees, so a pole must have added a magnitude with a slope of -20 dB/decade and a phase of -90 degrees.

Since the break point is at 1, the pole must be at -1. Thus, $G(s) = \frac{s}{s+1}$.

b. (7 pts.) Solve for the response of $G(s)$ to a unit step input, assuming zero initial conditions, and sketch the response versus time. Explain how this result is intuitive, given the zero and pole locations.

$$Y(s) = G(s) \cdot R(s) = \frac{s}{s+1} \cdot \frac{1}{s} = \frac{1}{s+1}$$

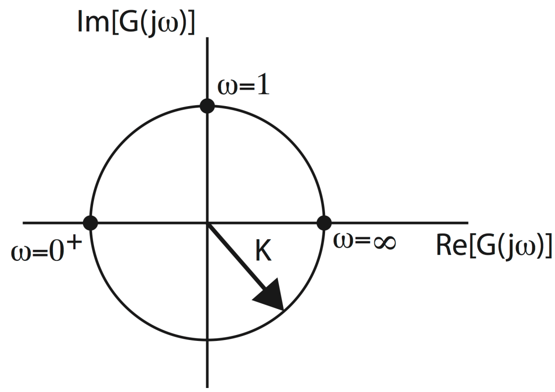


$y(t) = e^{-t}$ (starts at 1 and decays to zero)

This makes sense, as the zero makes the response start instantaneously at one, and the pole at -1 results in an exponential decay with time constant of one.

Problem 5. (20 pts.)

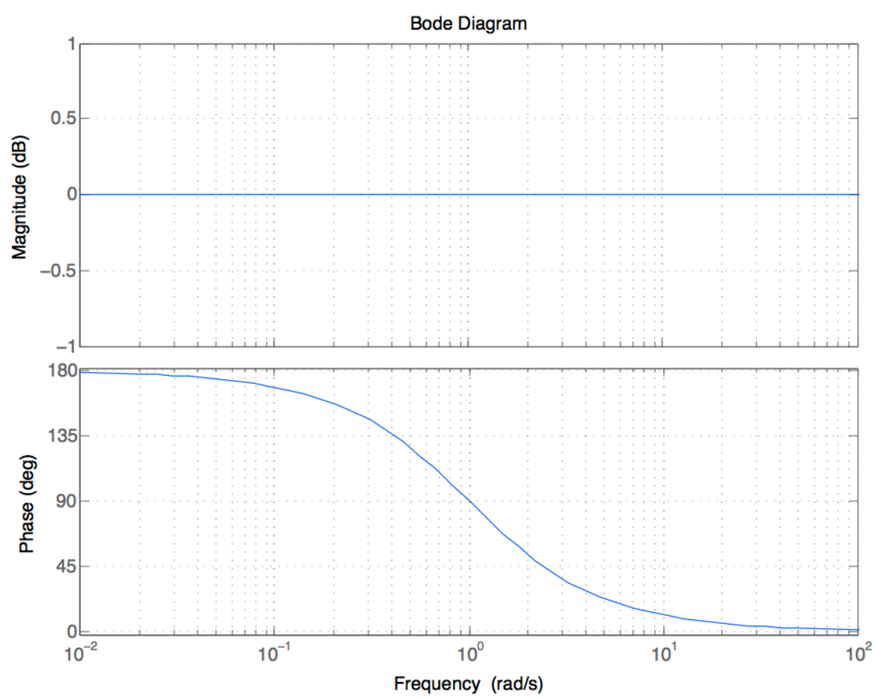
Consider the Nyquist plot shown below.



a. (7 pts.) Sketch the corresponding Bode plot (magnitude and phase).

Magnitude will be constant at a value of K. (Plot shown below is for K=1, which is a magnitude in dB of 0, but you must show K, or 20*log(K) if in dB.)

Phase will start at a 180 degree asymptote and end at a 0 degree asymptote. At ω = 1, the phase is 90 degrees.



b. (7 pts.) Suppose you know the transfer function is of the form $G(s) = A\frac{s+z}{s+p}$.

Find values for A, z, and p that correspond to the given Nyquist plot.

A = K

z = -1

p = 1

c. (3 pts.) For what values of K will the corresponding unity feedback system be stable?

|K| < 1

d. (3 pts.) Is the transfer function of part b unique in corresponding to the given Nyquist diagram? Explain.

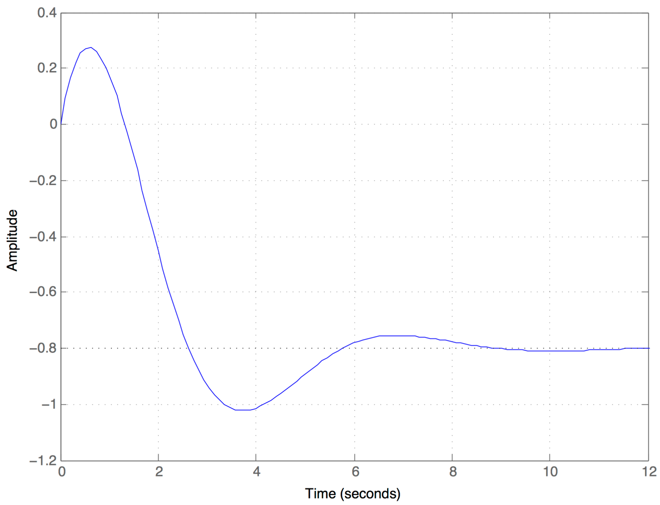
No. The phase is not known between the shown values of w on the Nyquist plot. One could add new pairs of poles and zeros (with large ω) and still maintain the same Nyquist diagram.

Problem 6. (20 pts.)

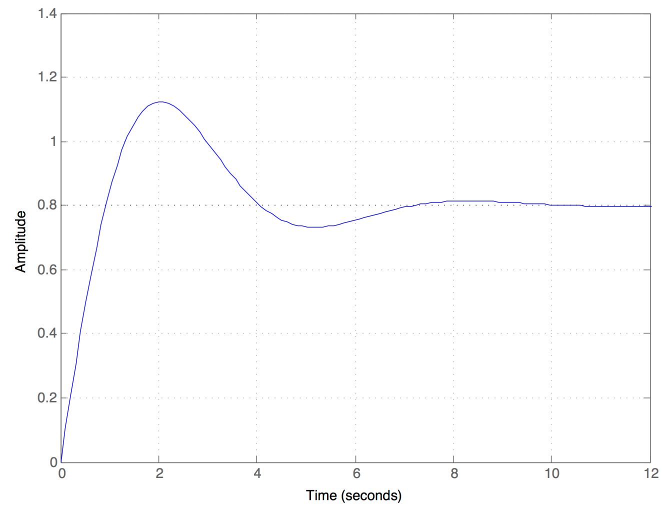
Plots of four time responses to a unit step (a, b, c, d), four s-plane plots of open-loop poles and zeros (i, ii, iii, iv), and four Nyquist plots (i, ii, iii, iv) are shown below and on the next page. Match them up on the page after the next, and give reasons for your choice in each case.

Step responses:

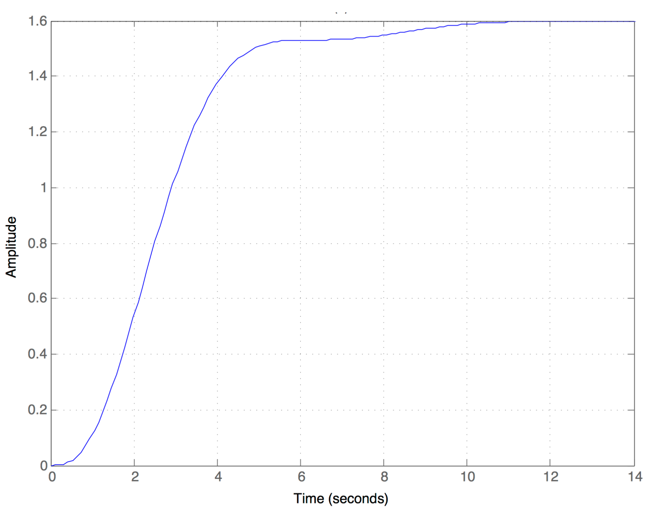
(a)



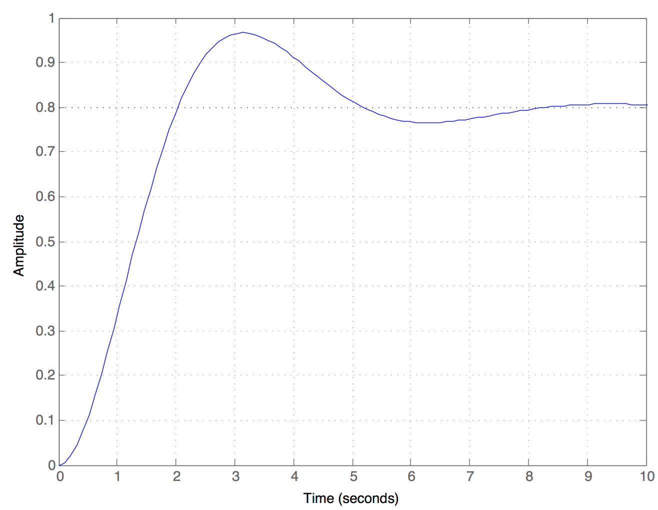
(b)



(c)

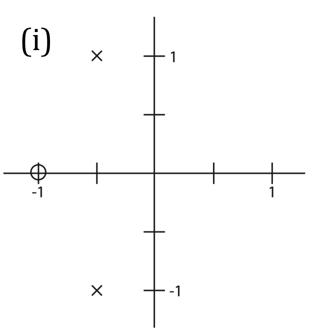


(d)

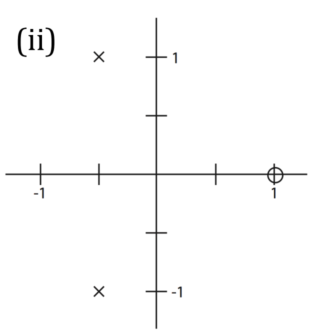


S-plane plots:

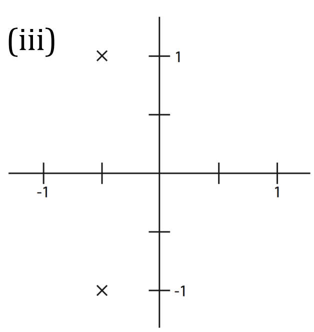
(i)



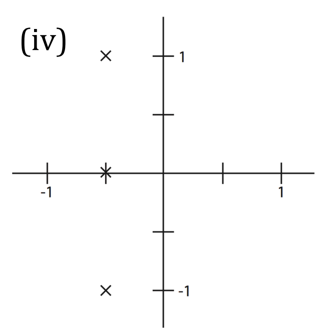
(ii)



(iii)

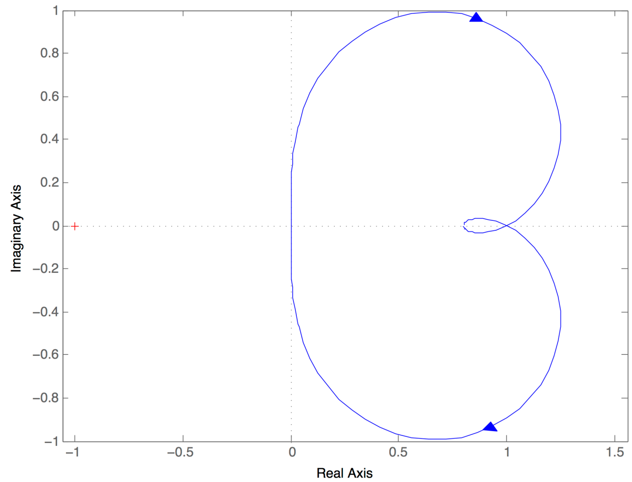


(iv)

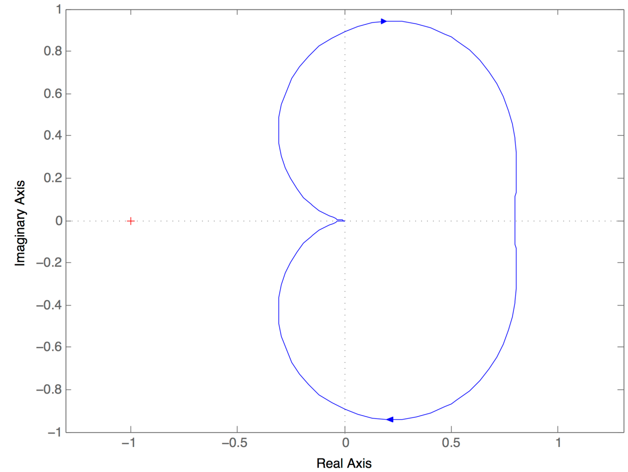


Nyquist Plots

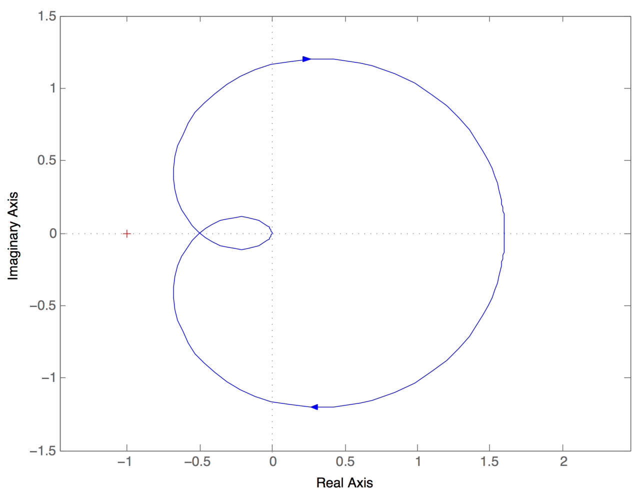
(i)



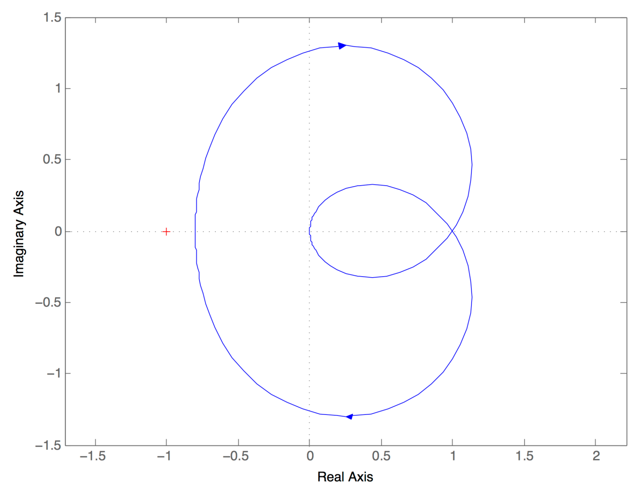
(ii)



(iii)



(iv)



Note: Each correct entry of a plot number received 1.5 points. Each correct s-plane/Nyquist pairing (even if not listed under the correct step response) received 2 points.

a. For step response (a), the corresponding s-plane is _(ii)_ and the Nyquist plot is _(iv)_.

Rationale:

This system in non-minimum phase, so there is a RHP zero.

zero at 1
poles at -0.5 +/- j

(See next page for corresponding Bode Plots that you can get from the Nyquist.)

b. For step response (b), the corresponding s-plane is _(i)_ and the Nyquist plot is _(i)_.

Rationale:

Has a more overshoot than (d), this is due to a LHP zero.

zero at -1
poles at -0.5 +/- j

c. For step response (c), the corresponding s-plane is _(iv)_ and the Nyquist plot is _(iii)_.

Rationale:

Less oscillation due to extra real pole.

poles at -0.5 and -0.5 +/- j

d. For step response (d), the corresponding s-plane is _(iii)_ and the Nyquist plot is _(ii)_.

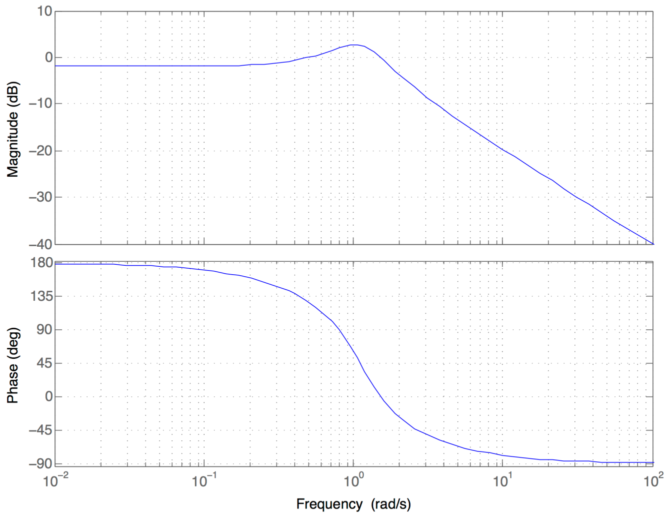
Rationale:

Regular second-order response

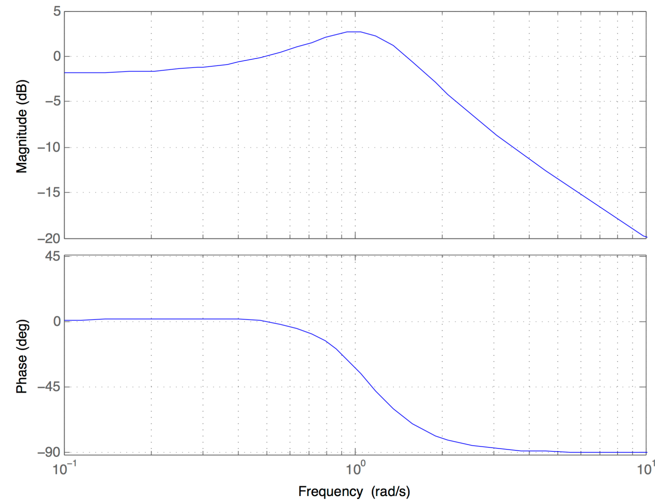
poles at -0.5 +/- j

Bode plots:

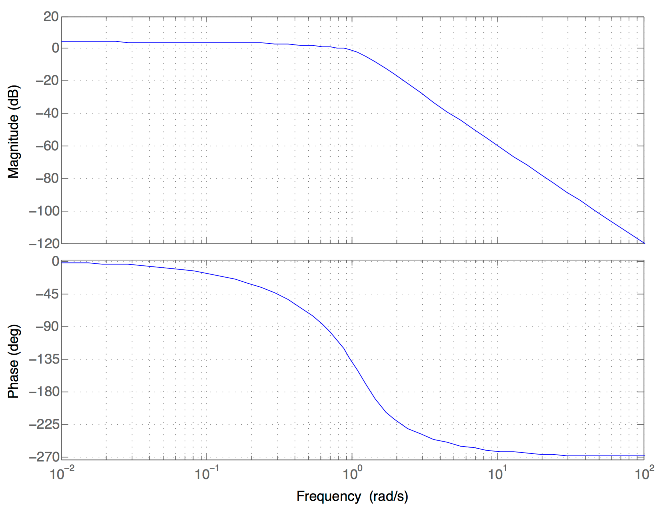
(a)



(b)



(c)



(d)