# Problem 1

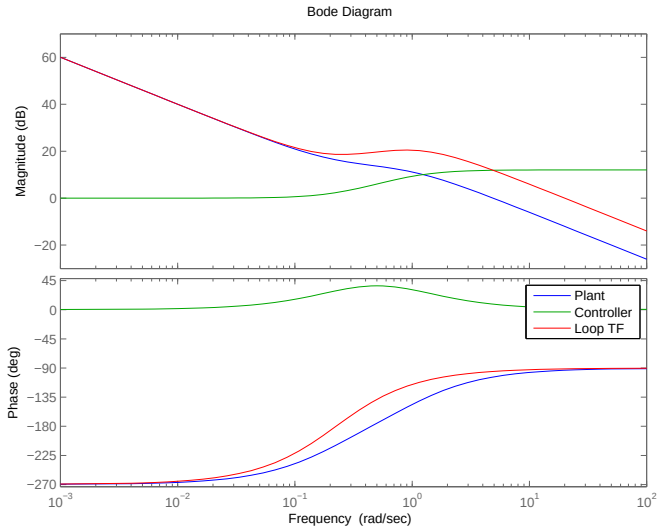
Consider the plant

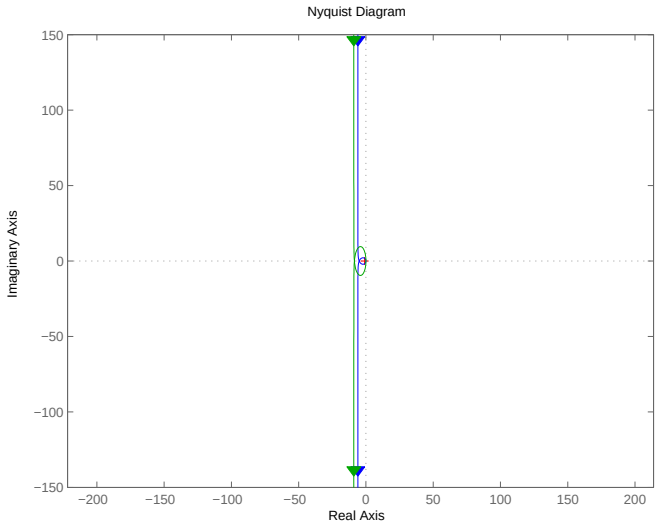
$$G(s) = \frac{5s+1}{s(s-1)}$$

## part a

Use the Nyquist Criterion to determine whether this system is stable in closed-loop feedback.

*There is one open loop pole, so we require exactly one encirclement of the origin.*



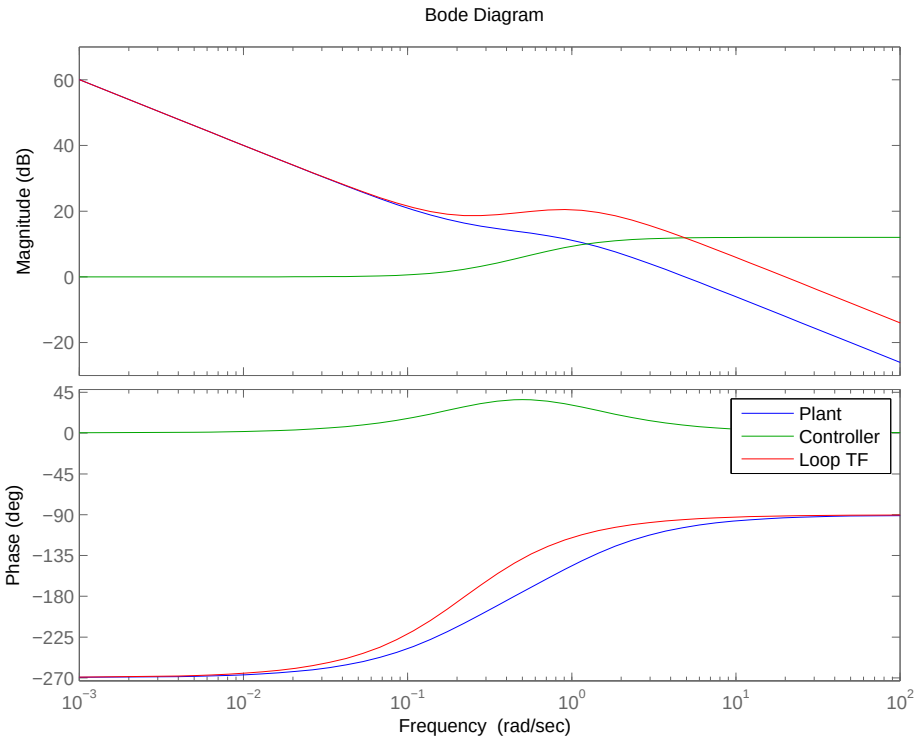


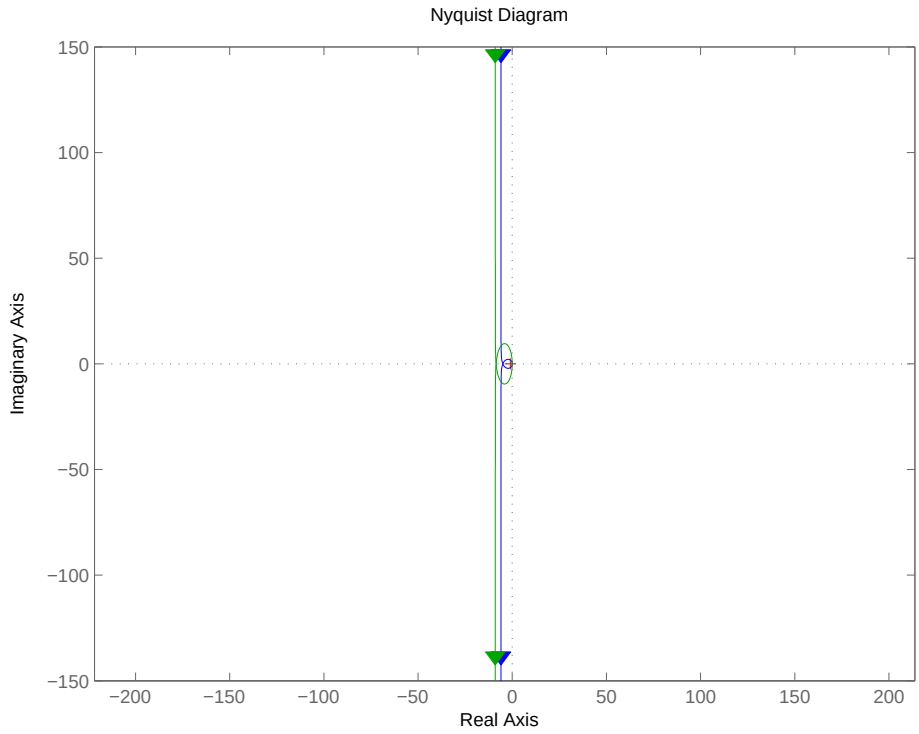

## part b

Design a controller that increases the robustness margins of the closed-loop system.

*We can try "boosting" the phase with a controller like*

$$K(s) = \frac{\frac{s}{5/3} + 1}{\frac{s}{5 \cdot 3} + 1}$$


Bode Diagram
Magnitude (dB)
60
40
20
0
−20
Phase (deg)
45
0
−45
−90
−135
−180
−225
−270
$10^{-3}$
$10^{-2}$
$10^{-1}$
$10^{0}$
$10^{1}$
$10^{2}$
Frequency (rad/sec)
Plant
Controller
Loop TF




## part c

For what frequencies is your controller good for reference tracking? For disturbance rejection? For noise insensitivity? How are these goals related? Can you think of controllers which achieve good robustness yet perform differently for these goals?

*For the standard feedback configuration, and for $L(s) = K(s)G(s)$, note that*

$$|L(j\omega)| \gg 1 \implies \frac{Y(j\omega)}{R(j\omega)} = \frac{L(j\omega)}{1 + L(j\omega)} \approx 1$$

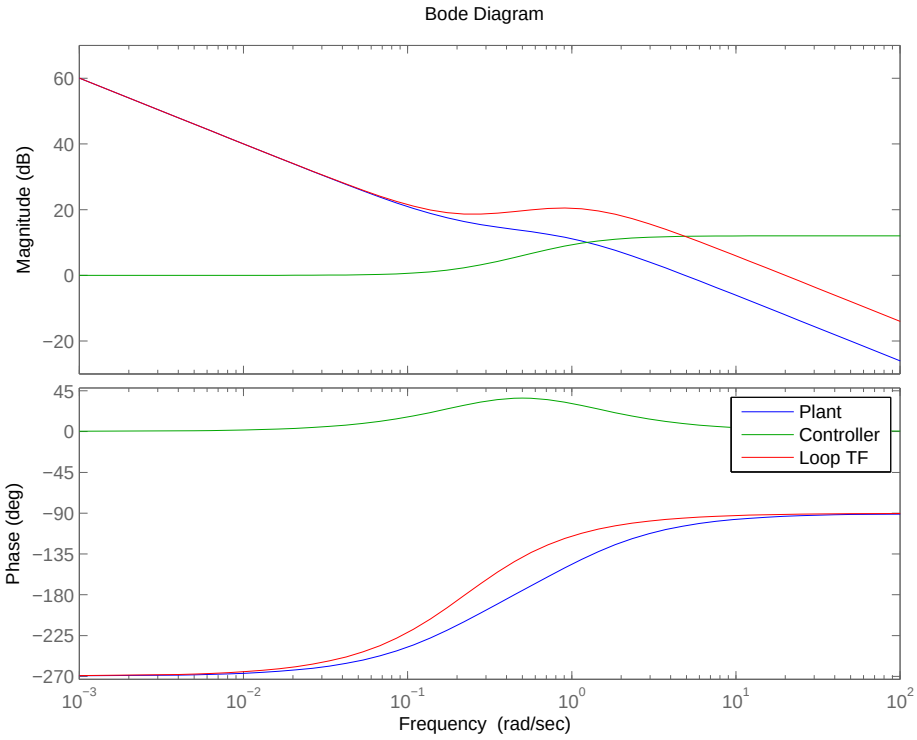
*so reference sinusoids are mapped to output sinusoids pretty accurately when the frequency is less than around 10 rad/s (using the Bode plot), so we have "good reference tracking" when the signal changes "slower" than 10 rad/s. Note that also*

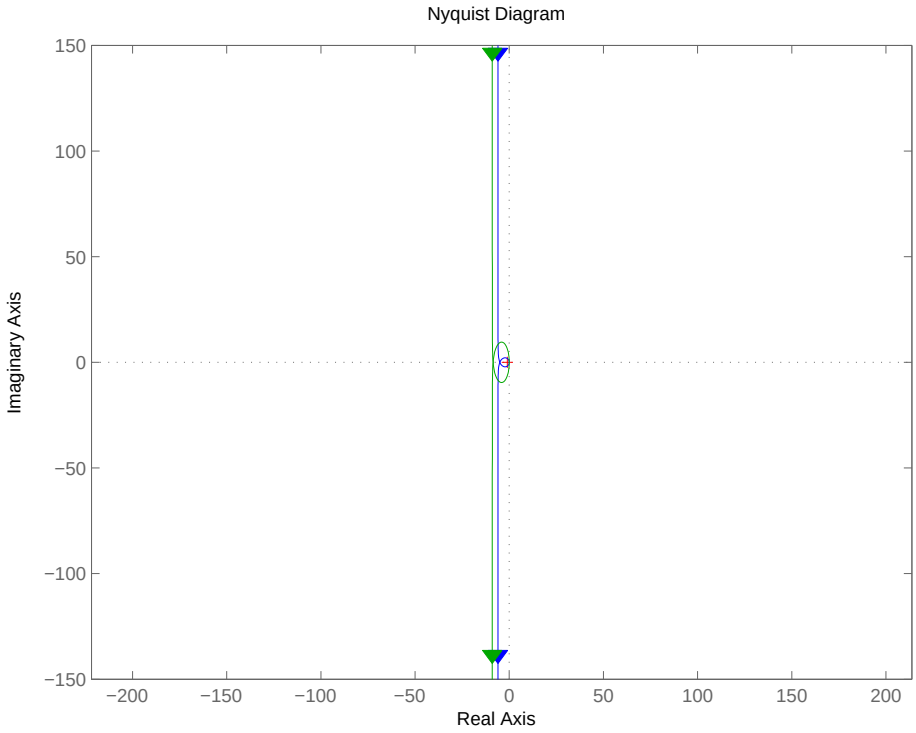
$$|L(j\omega)| \gg 1 \implies \left|\frac{Y(j\omega)}{D(j\omega)}\right| = \left|\frac{1}{1 + L(j\omega)}\right| \approx 0$$

*so the disturbance does not affect the output for less than around 10 rad/s (disturbances are attenuated if they are "slow" enough). Also,*

$$|L(j\omega)| \ll 1 \implies \left|\frac{Y(j\omega)}{N(j\omega)}\right| = \left|\frac{-L(j\omega)}{1 + L(j\omega)}\right| \approx 0$$

*so the noise signal (injected in the feedback loop) does not affect the output for less than around 10 rad/s (low frequency noise is suppressed, but high frequency noise is passed through, where high/low is relative to 10 rad/s). Note that for a given frequency, the first and second goals are complementary, and the third directly opposes the first two. In other words, you must choose between either "effective" control and noise insensitivity for each frequency in the spectrum.*


Bode Diagram
Magnitude (dB)
60
40
20
0
−20
Phase (deg)
45
0
−45
−90
−135
−180
−225
−270
Plant
Controller
Loop TF
10^−3
10^−2
10^−1
10^0
10^1
10^2
Frequency (rad/sec)


Nyquist Diagram
Imaginary Axis
Real Axis
150
100
50
0
−50
−100
−150
−200
−150
−100
−50
0
50
100
150
200