ENGR 105: Feedback Control Design
Winter 2013

Lecture 24 - The Negative (0°) Root Locus

Wednesday, March 13, 2013

Today's Objectives

1. root locus for negative K
2. application to non-minimum phase systems

Reading: FPE Section 5.6.1

1 The 0° root locus

Most our discussion up to now has examined positive values of the gain K. What if we want to look at negative values of K? Let's look at this from the perspective of the root locus.

$$1 + K\frac{b(s)}{a(s)} = 0$$

$$K\frac{b(s)}{a(s)} = -1$$

$$\text{If } K < 0 \text{ then } |K|\frac{b(s)}{a(s)} = 1$$

$$\Rightarrow \text{Magnitude of 1 and phase of } 0^\circ$$

We can look at this on a 0° root locus. To do this, we rewrite the root locus rules for a 0° angle condition in place of the 180° condition.

What changes?

- Points to the left of an even number of real poles and zeros are now on the locus.

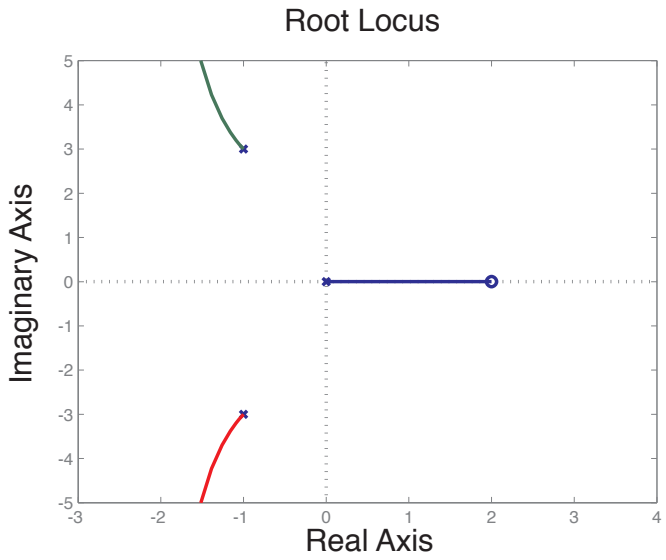
- Asymptotes are now found from

$$\phi_l = \frac{360^\circ(l-1)}{n-m} \quad \text{for} \quad l = 1, 2, \ldots, n-m$$

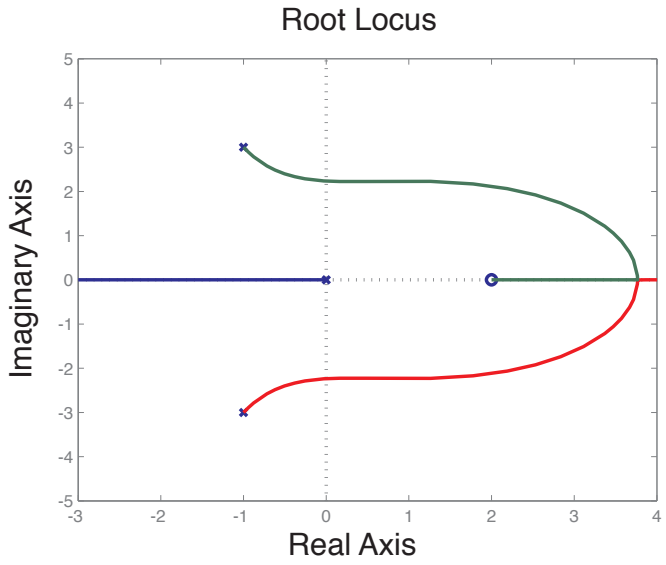
- Departure angle calculations now use the 0° angle condition.

2 Use for examining non-minimum phase systems

This can be particularly useful when examining non-minimum phase systems. (One example of a non-minimum phase system is attitude control of an airplane.)



If $K > 0$, no gain will stabilize the system.



What happens if $K < 0$?

Draw the 0° root locus

Note that the right half plane zero still means that poles move into the right half plane as $K \to \infty$. The negative gain doesn't change that, but may buy a period of stability.