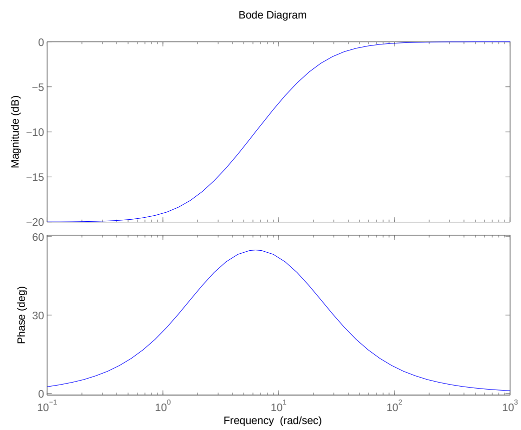
Problem 1

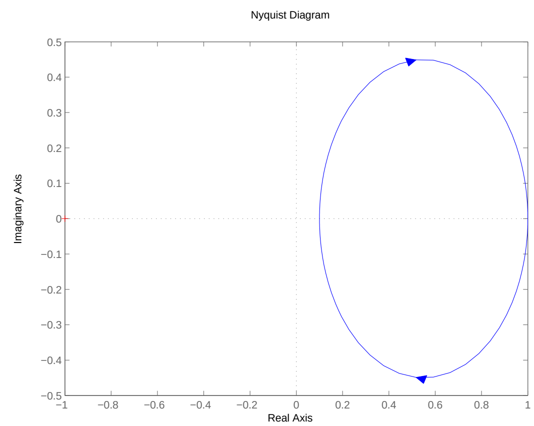
part a

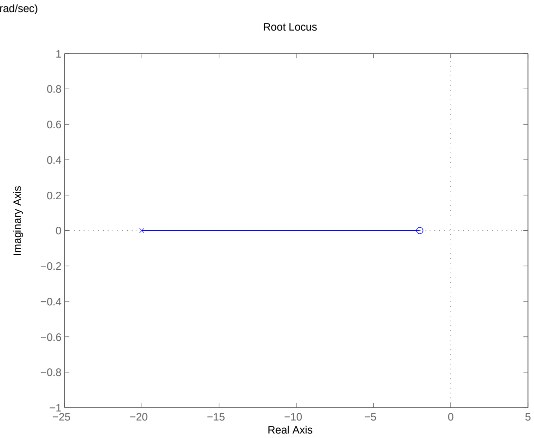
This loop transfer function is

$$L(s) = (1/10)\frac{s/2+1}{s/20+1}$$

The plots are shown below







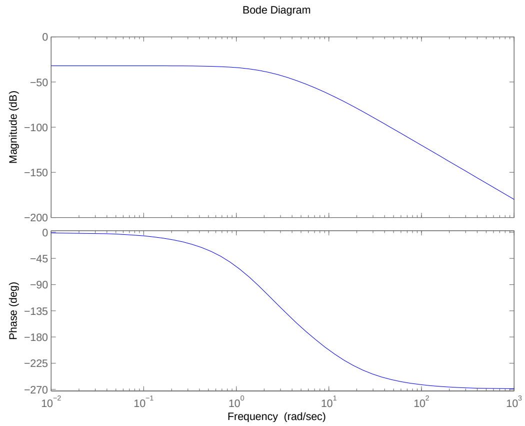
The Nyquist plot does not approach -1 for any positive K.

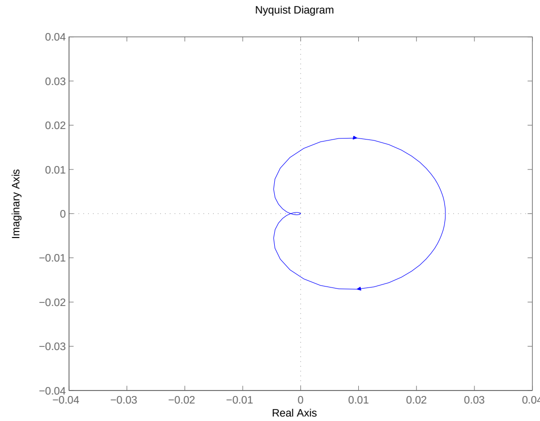
part b

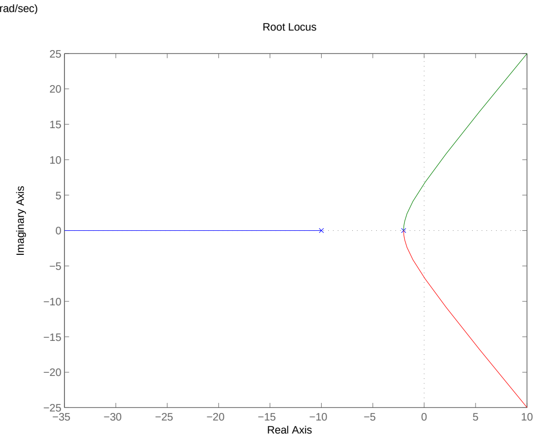
This loop transfer function is

$$L(s) = (1/10)\frac{1}{(s/10+1)(s+1)^2}$$

The plots are shown below

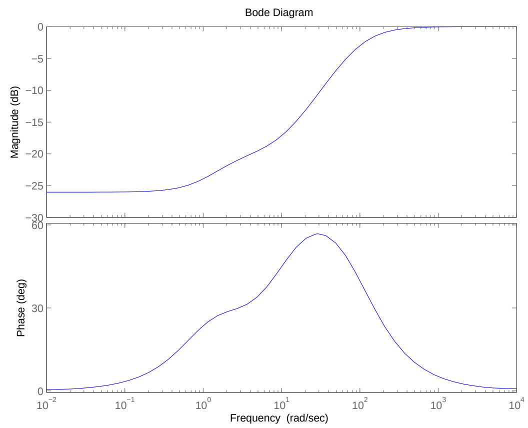


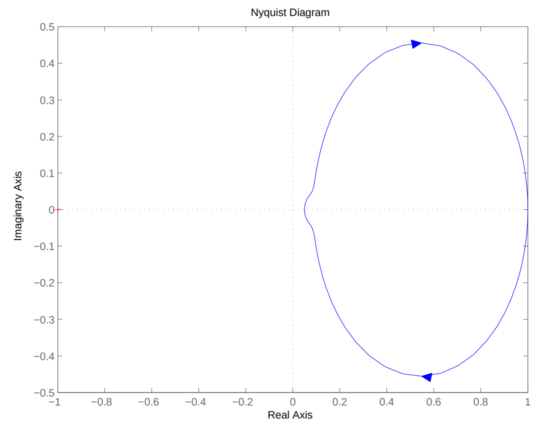


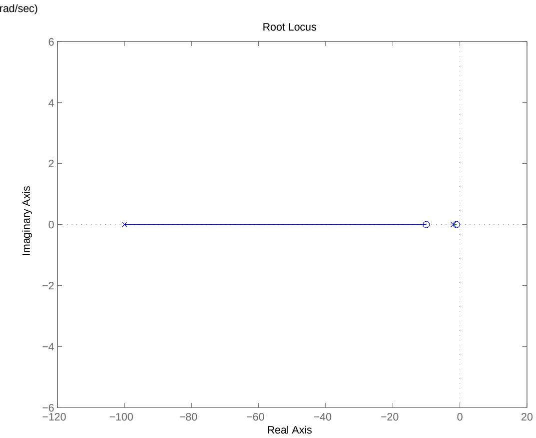


There are no open loop RHP poles, so we want no encirclements of -1. When the phase is $-180°$, the magnitude is about 0.0017, so the gain would have to be around $K = 1/0.0017 \approx 576$ for any encirclements of -1.

part c





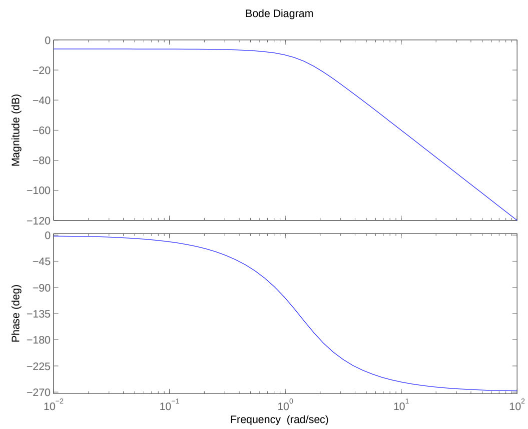


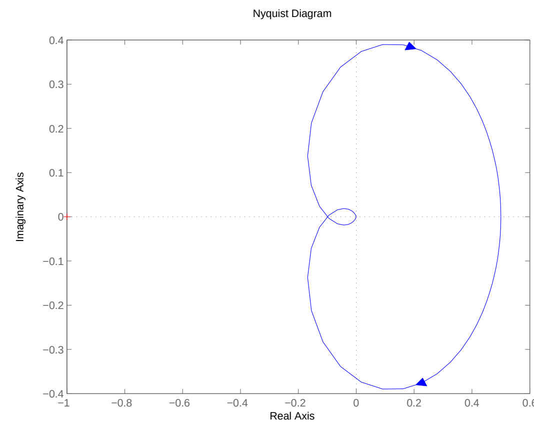
The Nyquist plot does not approach -1 for any positive K.

```
clc
clear
s=tf('s');

%% part a
G=(s+2)/(s+20);
bode(G);
print -dpdf ~/Dropbox/E105-Win2013/Assignments/assignment8-nick/soln/p1ai
nyquist(G);
print -dpdf ~/Dropbox/E105-Win2013/Assignments/assignment8-nick/soln/p1ai
rlocus(G);
print -dpdf ~/Dropbox/E105-Win2013/Assignments/assignment8-nick/soln/p1av

%% part b
G=1/((s+10)*(s+2)^2);
bode(G);
print -dpdf ~/Dropbox/E105-Win2013/Assignments/assignment8-nick/soln/p1bi
```

```
nyquist(G);
axis([-.04,.04,-.04,.04])
print -dpdf ~/Dropbox/E105-Win2013/Assignments/assignment8-nick/soln/p1bi
rlocus(G);
print -dpdf ~/Dropbox/E105-Win2013/Assignments/assignment8-nick/soln/p1bv
```

Problem 2





We want to analyze the stability of the CL system

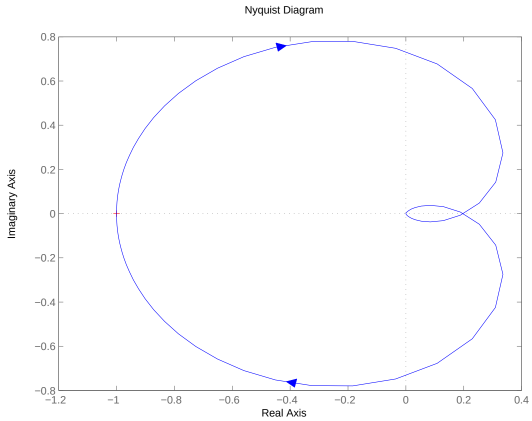
$$T(s) = \frac{K\frac{1}{s^2+2s+2}}{1 + K\frac{1}{s^2+2s+2}\frac{1}{s+1}}$$

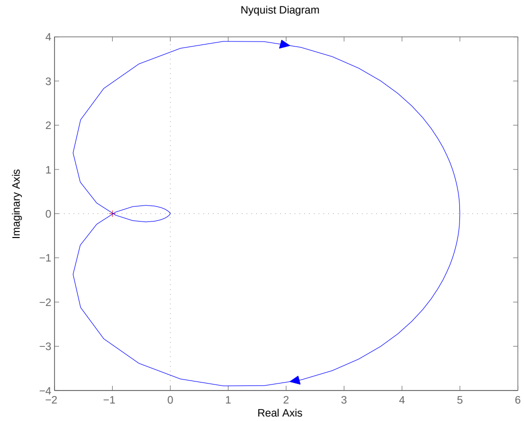
So the first task is to get this into an amenable form. We can easily analyze the stability of

$$\widetilde{T}(s) = \frac{K\frac{1}{s^2+2s+2}\frac{1}{s+1}}{1 + K\frac{1}{s^2+2s+2}\frac{1}{s+1}}$$

Note that stability analysis for $\widetilde{T}(s)$ is exactly equivalent to that of $T(s)$ (to see this, note that if $T(s)$ is stable, then $\widetilde{T}(s) = \frac{1}{s+1}T(s)$ is stable, because $\frac{1}{s+1}$ does not introduce any new RHP poles, and if $\widetilde{T}(s)$ is stable, then $T(s) = (s+1)\widetilde{T}(s)$ is stable, because the $s+1$ term does not add any new RHP poles to $\widetilde{T}(s)$). You are not expected to go into this level of detail.

There are no open-loop poles, so we want no encirclements of -1. The Bode plot crosses $180°$ at around 2 rad/sec, and the amplitude is $1/10$, so if we "blow up" the Nyquist plot by adding a gain $K \geq 10$, then we get an extra encirclement (two, actually), and therefore instability. If we add a negative gain, the Nyquist plot rotates by $180°$ (note: make sure you can justify this!). Therefore, we are interested in when the of $L(s)$ is $0°$. On the Nyquist contour Γ, this happens for $s = 0$, $G(s) = .5$. So we would have to add a gain of -2 to get the extra encirclement, so the system is stable for $K \in (-2, 10)$. We can confirm this using Nyquist plots of $KG(s)$:



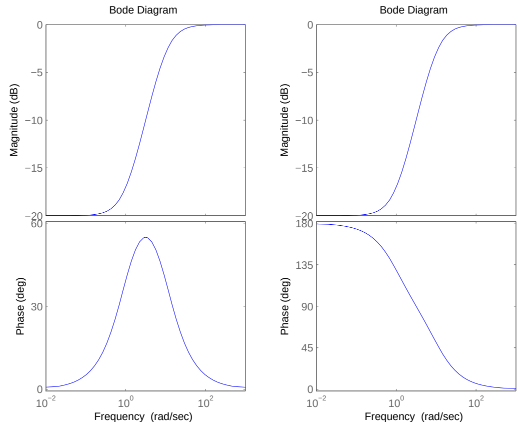



```
clc
clear
s=tf('s');

G=1/((s^2+2*s+2)*(s+1));
nyquist(G)
print -dpdf ~/Dropbox/E105-Win2013/Assignments/assignment8-nick/soln/p2_n
bode(G)
print -dpdf ~/Dropbox/E105-Win2013/Assignments/assignment8-nick/soln/p2_b

nyquist(10*G)
print -dpdf ~/Dropbox/E105-Win2013/Assignments/assignment8-nick/soln/p2_1
nyquist(-2*G)
print -dpdf ~/Dropbox/E105-Win2013/Assignments/assignment8-nick/soln/p2_2
```

Problem 3

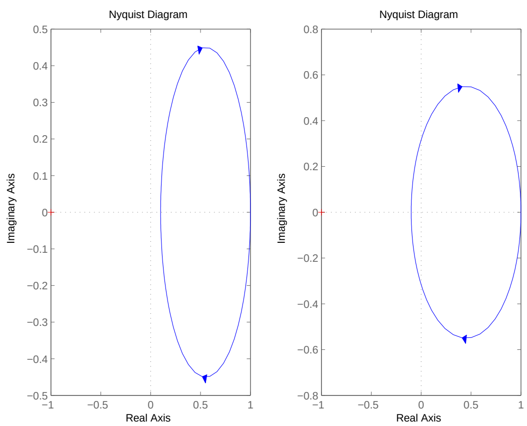
part a

The Bode plots for G_1 and G_2, respectively, are



part b

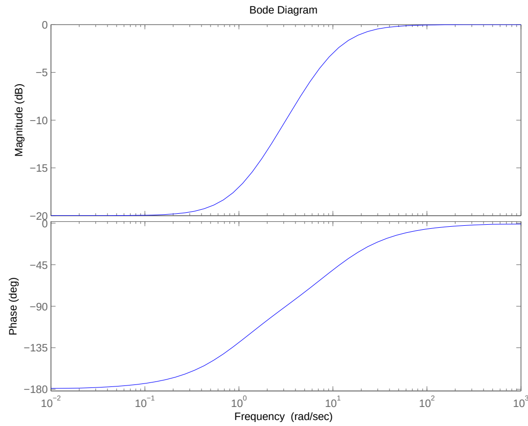
The Nyquist plots for G_1 and G_2, respectively, are



A RHP zero does not affect the number of encirclements required for stability. In particular, for these plants, we need 0 encirclements of -1 for stability. (Of course, the Nyquist plots may be different for these two plants, so this condition may be met for different values of the gain K.)

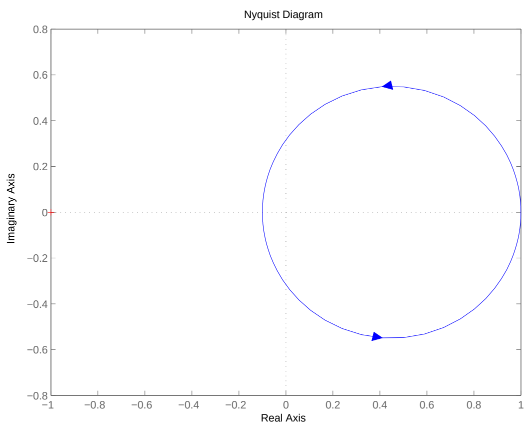
part c

The Bode plots for G_3 is



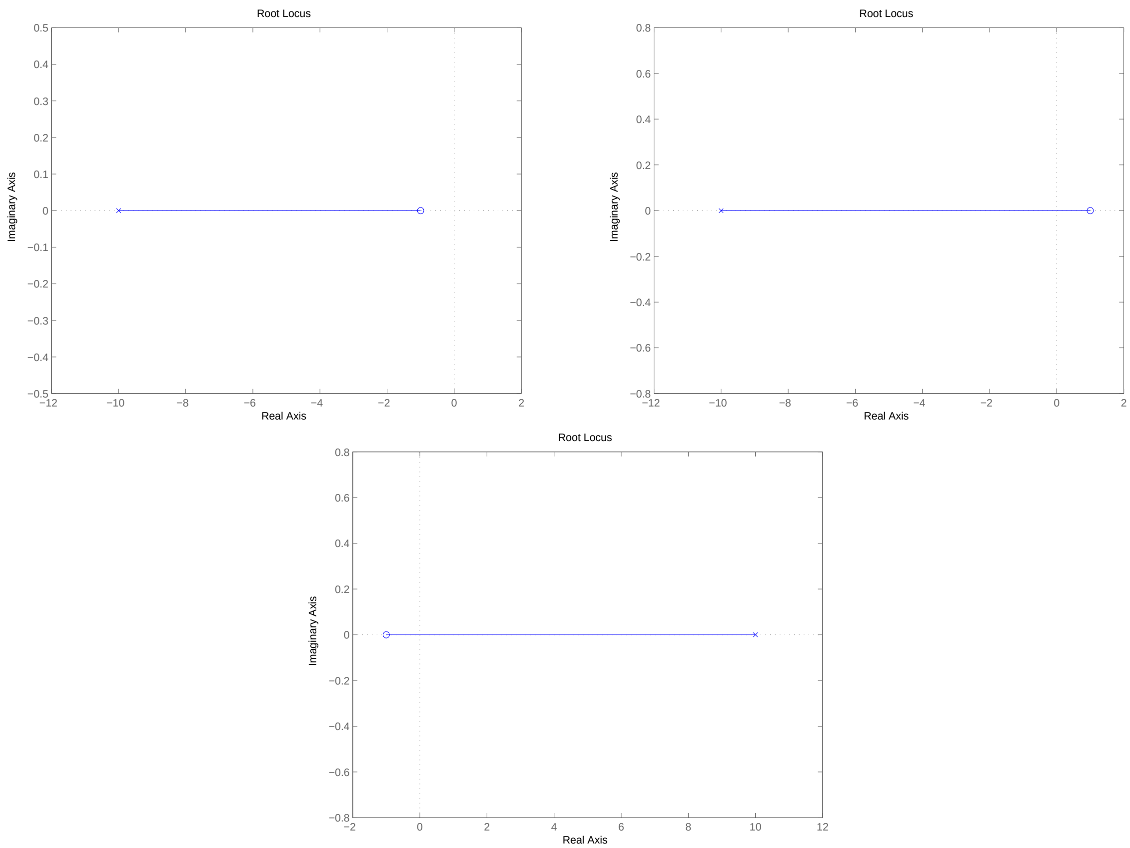
part d

The Nyquist plot for G_3 is



Unlike the first two plants, this plant has a RHP pole, so we require 1 encirclement of -1 for stability. Therefore, the first plant will not be unstable for any K. The second plant will be unstable for $K \geq 10$, and the third plant will be unstable for $K \geq 10$.

part e



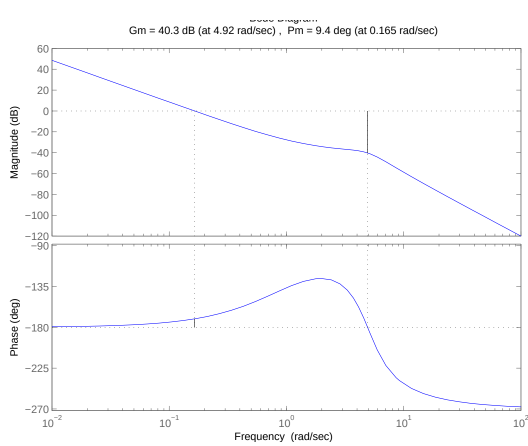

```
clc
clear
s=tf('s');

%% part a
G1=(s+1)/(s+10);
G2=(s-1)/(s+10);
subplot(121); bode(G1);
subplot(122); bode(G2);
print -dpdf ~/Dropbox/E105-Win2013/Assignments/assignment8-nick/soln/p3a.

%% part b
subplot(121); nyquist(G1);
subplot(122); nyquist(G2);
print -dpdf ~/Dropbox/E105-Win2013/Assignments/assignment8-nick/soln/p3b.

%% part c
clf
```

```
G3=(s+1)/(s-10);
bode(G3);
print -dpdf ~/Dropbox/E105-Win2013/Assignments/assignment8-nick/soln/p3c.

%% part d
nyquist(G3);
print -dpdf ~/Dropbox/E105-Win2013/Assignments/assignment8-nick/soln/p3d.

%% part e
rlocus(G1);
print -dpdf ~/Dropbox/E105-Win2013/Assignments/assignment8-nick/soln/p3ei
rlocus(G2);
print -dpdf ~/Dropbox/E105-Win2013/Assignments/assignment8-nick/soln/p3ei
rlocus(G3);
print -dpdf ~/Dropbox/E105-Win2013/Assignments/assignment8-nick/soln/p3ei
```

Problem 4

part a

The Bode plot of the OL system is:



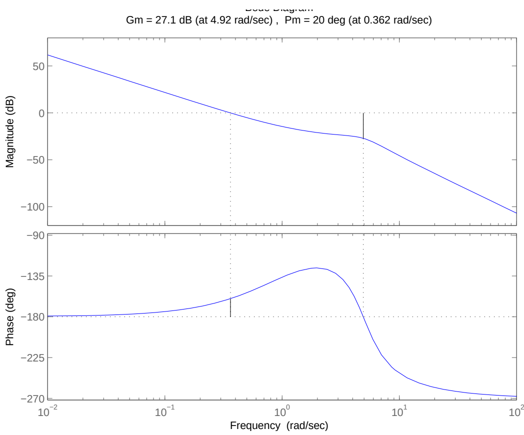
Note that we can use the Bode plot to tell stability; the magnitude crosses 1 only once, and increasing gain leads to instability (the asymptotics for the root locus will be $\pm 60°, 180°$).

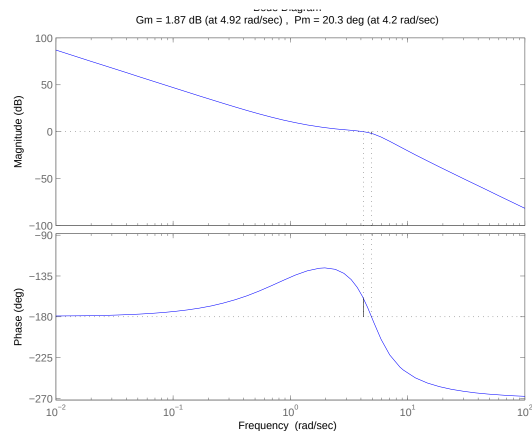
part b

Looking at the Bode plot, We cross 180° at $\omega = 4.92$ rad/s, with magnitude 0.0097, so our margin is around 104. (Note: For the original problem (before we modified it) there was a small hump, and for the critical gain, there were actually *two* frequencies unity magnitude, so we actually couldn't use the Bode plot to analyze stability for the critical gain. But we would know that if we use a tiny bit larger gain, we could use the rule again to get instability, and if we use a tiny bit smaller gain, we could use the rule again to get stability. If you followed reasoning like this for the original problem, you got credit.)

part c

We add gain ("shifting" the magnitude plot up and down) to ensure that the phase plot is not within $20°$ of $180°$ for all crossover frequencies (in this case, the magnitude plot is monotonic, so there will only be one such crossover frequency). The phase will be at least $-160°$ for any $K \in [4.5709, 83.1764]$, approximately. (Note that we could make the phase lower than $200°$ for higher gains, but the system would be unstable, so phase margin wouldn't make much sense!) If you took the problem as originally stated, you got full credit if you said something about why you can't analyze the phase margin when there are two crossover frequencies without using Nyquist.

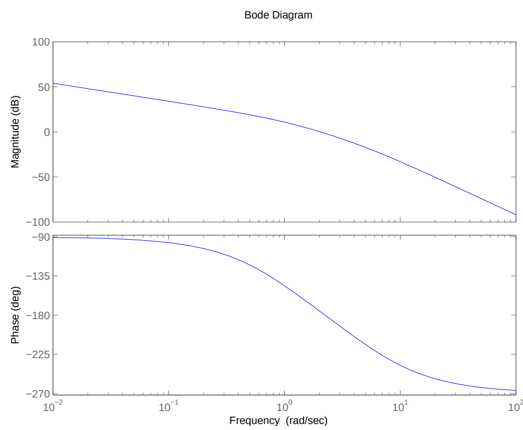




Problem 5

part a

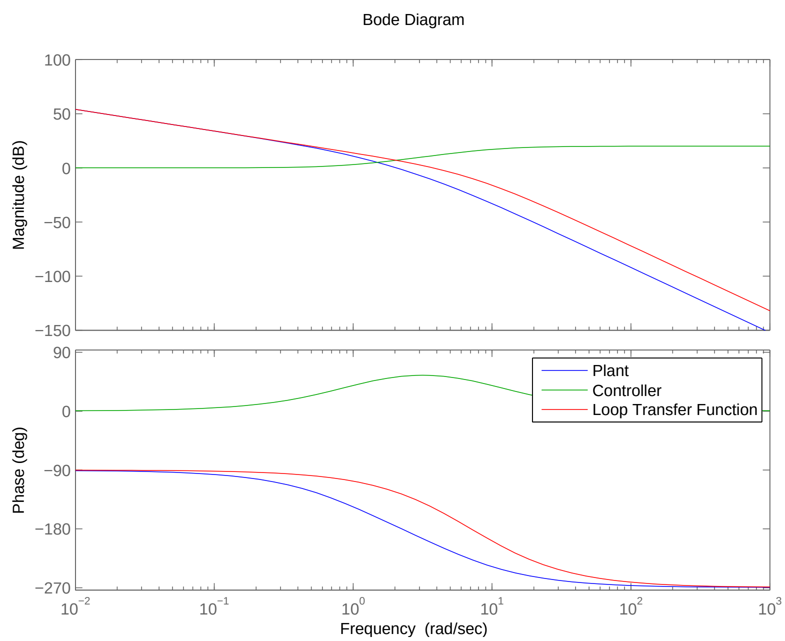
The Bode plot of the OL system is:



When the magnitude is about unity, the phase is around -180, so the phase margin is virtually non existent. We know a lead compensator has phase up to $90°$ in the center, so we can put the lead compensator near the crossover frequency to boost the phase margin up to $90°$. So a possible controller is

$$D(s) = \frac{s+1}{s/10+1}$$

part b

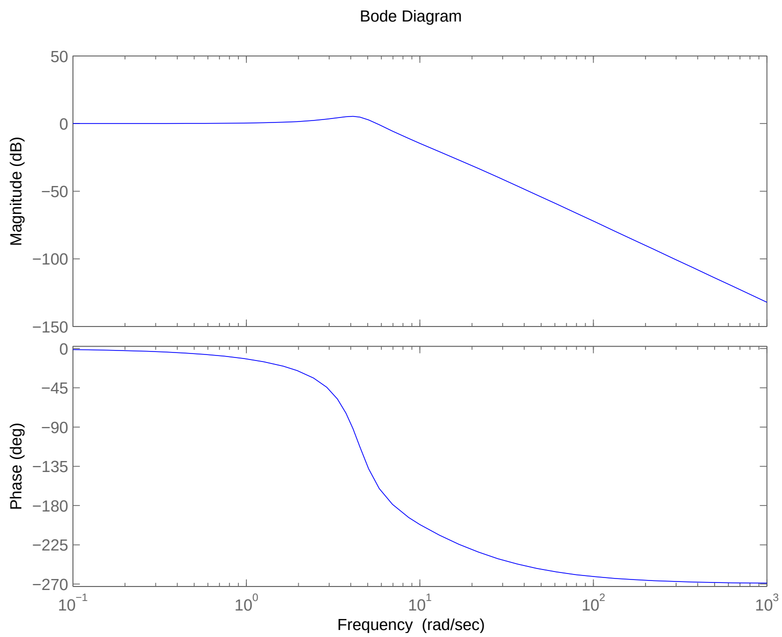


part c

We want to calculate the transfer function from R to Y, which is

$$\frac{R(s)}{Y(s)} = \frac{D(s)G(s)}{1 + D(s)G(s)}$$

The Bode plot of this transfer function shows that the frequency rolls off around $\omega \approx 6$, which means our system can't follow references "faster" than this frequency.




```
clc
clear
s=tf('s');

%% part a
G=(5)/(s*(s+1)*(s/5+1));
bode(G)
print -dpdf ~/Dropbox/E105-Win2013/Assignments/assignment8-nick/soln/p5a.

%% part b
D=(s/1+1)/(s/10+1);
bode(G,D,G*D)
legend('Plant','Controller','Loop␣Transfer␣Function')
print -dpdf ~/Dropbox/E105-Win2013/Assignments/assignment8-nick/soln/p5b.

%% part c
```

```
T=D*G/(1+D*G);
bode(T)
print -dpdf ~/Dropbox/E105-Win2013/Assignments/assignment8-nick/soln/p5c.
```