Introduction to Social Statistics

Day 10

Instructor Rob Kemp

Wednesday, February 13, 2013

Announcements

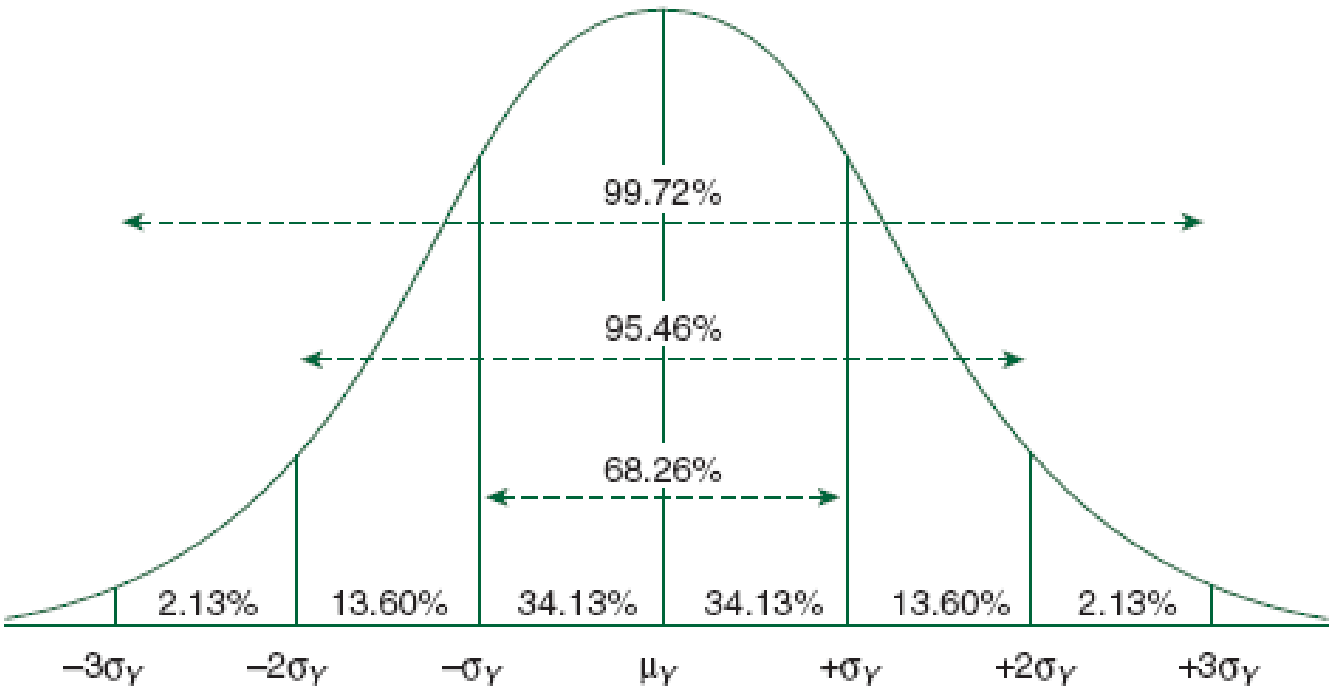
- HW #2 posted.
- Exam #2 is on March 11th

Agenda
Z Scores in multiple ways…

- Review from Friday
- Transforming Proportions into Z Scores
- Finding the Percentile Rank of a Raw Score
- Finding the Raw Score for a Percentile

Areas Under the Normal Curve by Measuring Standard Deviations

Figure 6.3 Percentages Under the Normal Curve



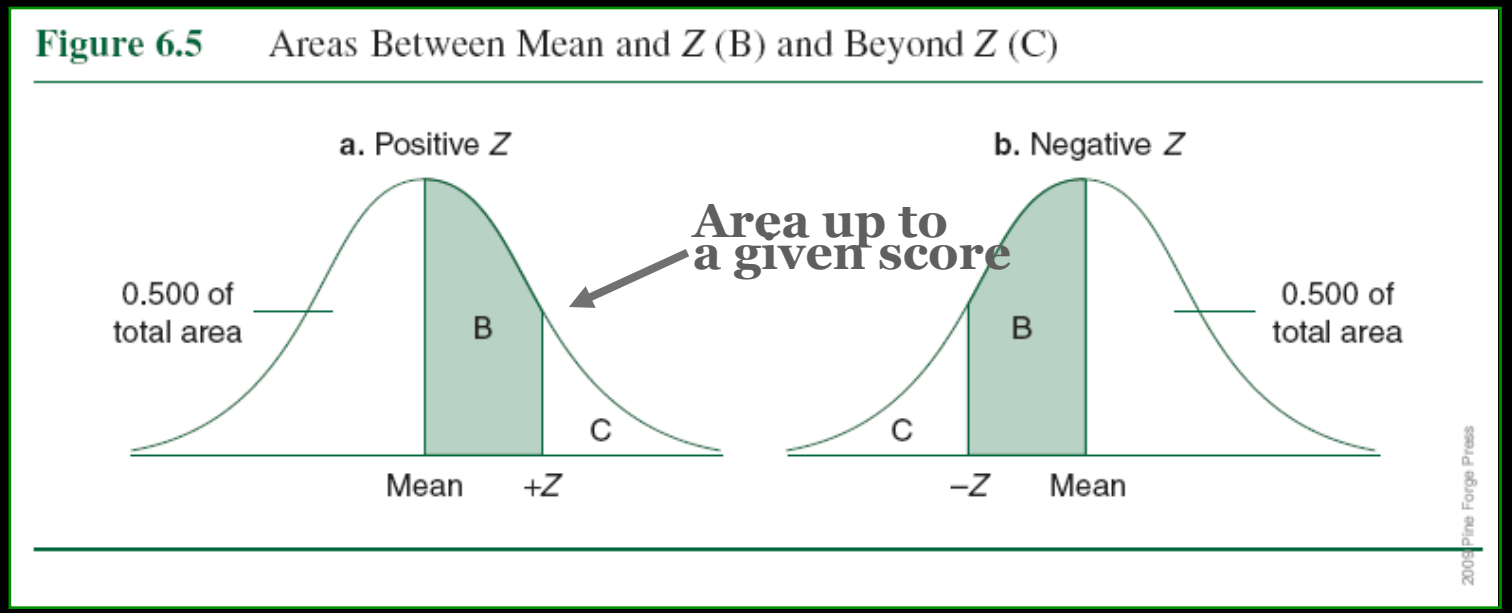
Standard (Z) Scores

- A ***standard score*** (also called ***Z score***) is the number of standard deviations that a given raw score is **above** or **below** the mean.

$$Z = \frac{Y - \bar{Y}}{S_y}$$

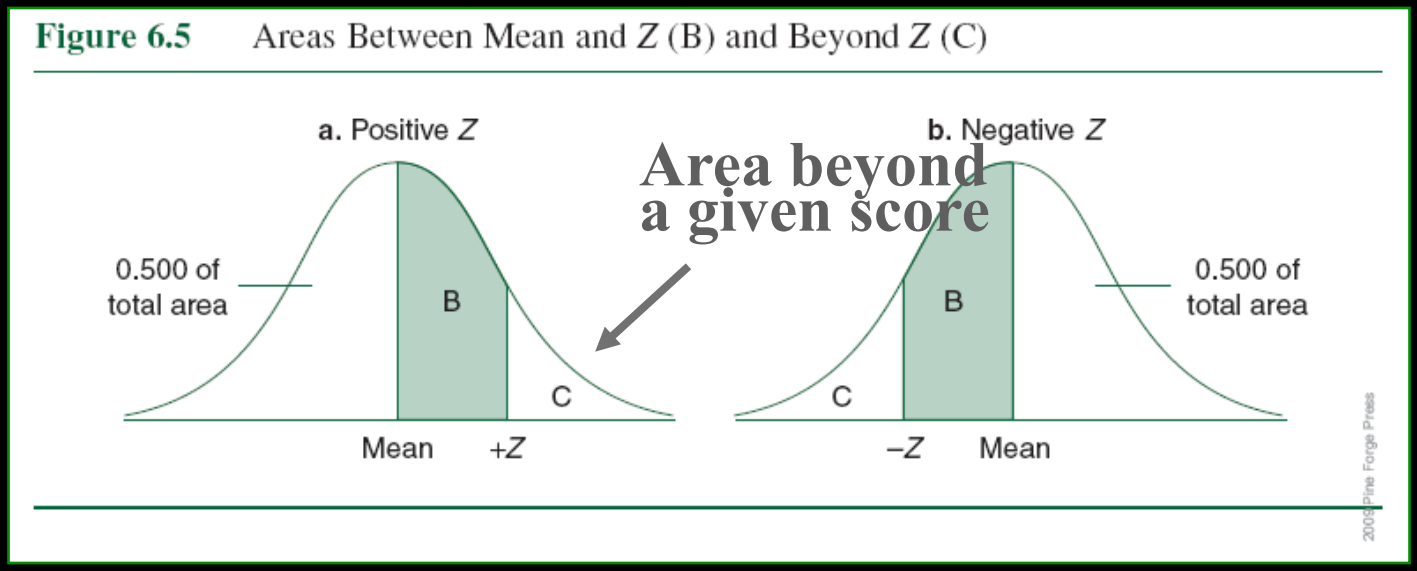
The Standard Normal Table

- A table showing the **area** (as a proportion, which can be translated into a percentage) under the standard normal curve **corresponding** to any Z score or its fraction



The Standard Normal Table

- A table showing the **area** (as a proportion, which can be translated into a percentage) under the standard normal curve **corresponding** to any Z score or its fraction



Final Grade Data…

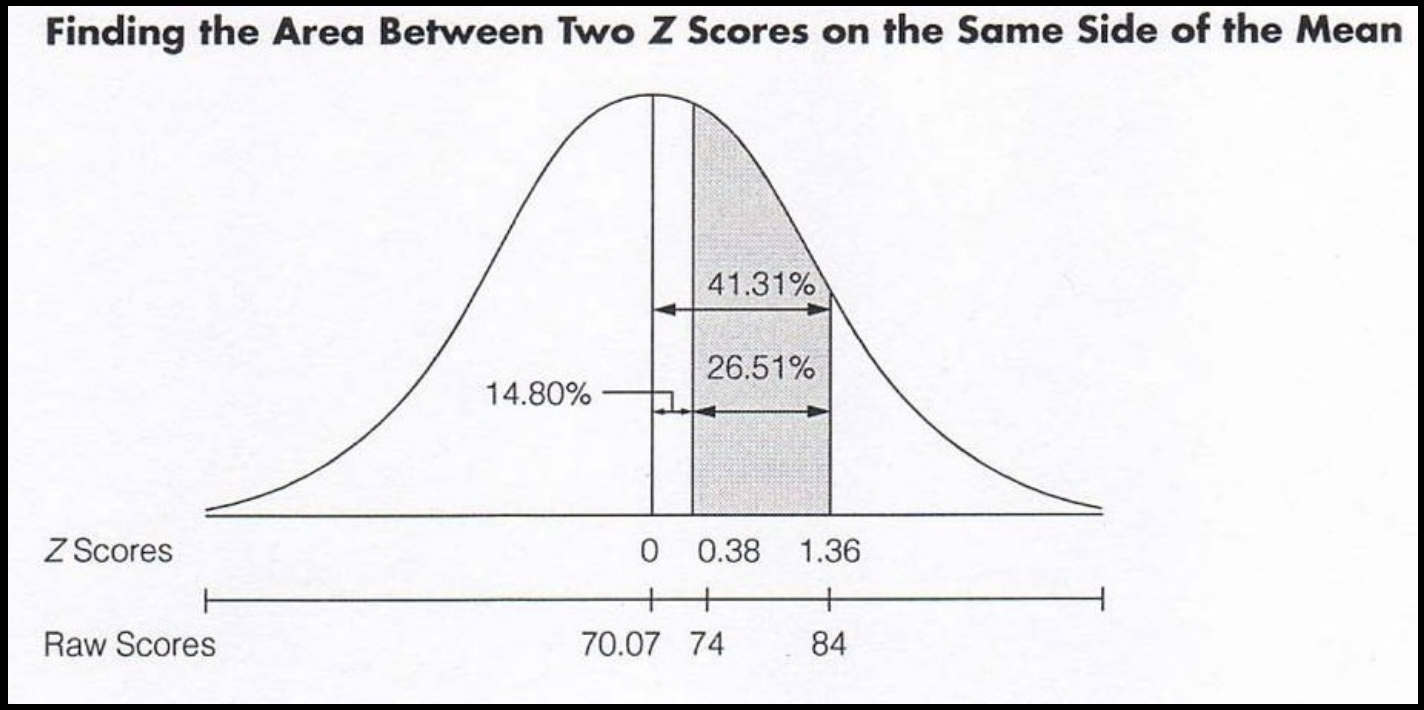
Table 14.1 Final Grades in Social Statistics of 1,200 Students					
Midpoint Score	Frequency Bar Chart	Freq.	Cum. Freq. (below)	%	Cum % (below)
40	*	4	4	0.33	0.33
50	*******	78	82	6.5	6.83
60	**********	275	357	22.92	29.75
70	**********	483	840	40.25	70
80	**********	274	1114	22.83	92.83
90	*******	81	1195	6.75	99.58
100	*	5	1200	0.42	100

Finding the Area Between 2 Z Scores on the Same Side of the Mean

- Using the same data presented in Table 14.1, find the percentage of students scoring between 74 and 84.
- (1) Find the Z scores for 74 and 84:

  $Z = 0.38$ and $Z = 1.36$
- (2) Look up the corresponding areas for those Z scores: **0.1480 and 0.4131**

Finding the Area Between 2 Z Scores on the Same Side of the Mean



(3) To find the highlighted area above, **subtract** the smaller area from the larger area (0.4131-0.1480 = **0.2651**)
Now, we have the percentage of students scoring between 74 and 84.

Finding the Area Between 2 Z Scores on Opposite Sides of the Mean

- Using the same data, find the percentage of students scoring between 62 and 72.
- (1) Find the Z scores for 62 and 72:

  Z = (72-70.07)/10.27 = **0.19**

  Z = (62-70.07)/10.27 = **-0.79**

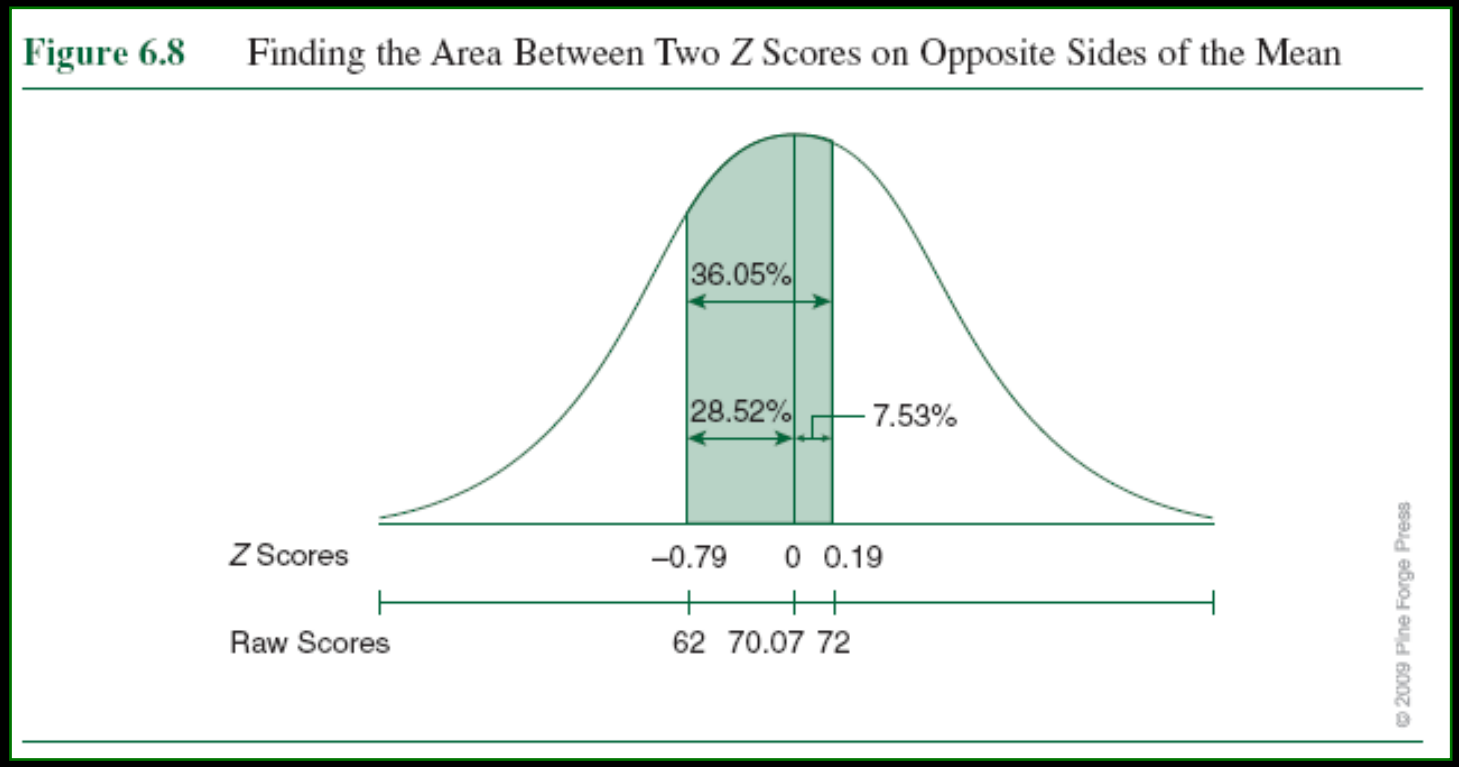
  (2) Look up the areas **between** these Z scores and the mean, like in the previous 2 examples:

  Z = 0.19 is 0.0753 and Z = -0.79 is 0.2852

  (3) **Add** the two areas together: 0.0753 + 0.2852

  = 0.3605

Finding the Area Between 2 Z Scores on Opposite Sides of the Mean



(4) Convert the proportion (.3605) to a percentage (36.05%); this is the percentage of students scoring **between** 62 and 72.

Finding Area Above a Positive Z Score or Below a Negative Z Score

- Find the percentage of students who did (a) very well, scoring above 85, and (b) those students who did poorly, scoring below 50.
- (a) Convert 85 to a Z score, then look up the value in **Column C** of the Standard Normal Table:

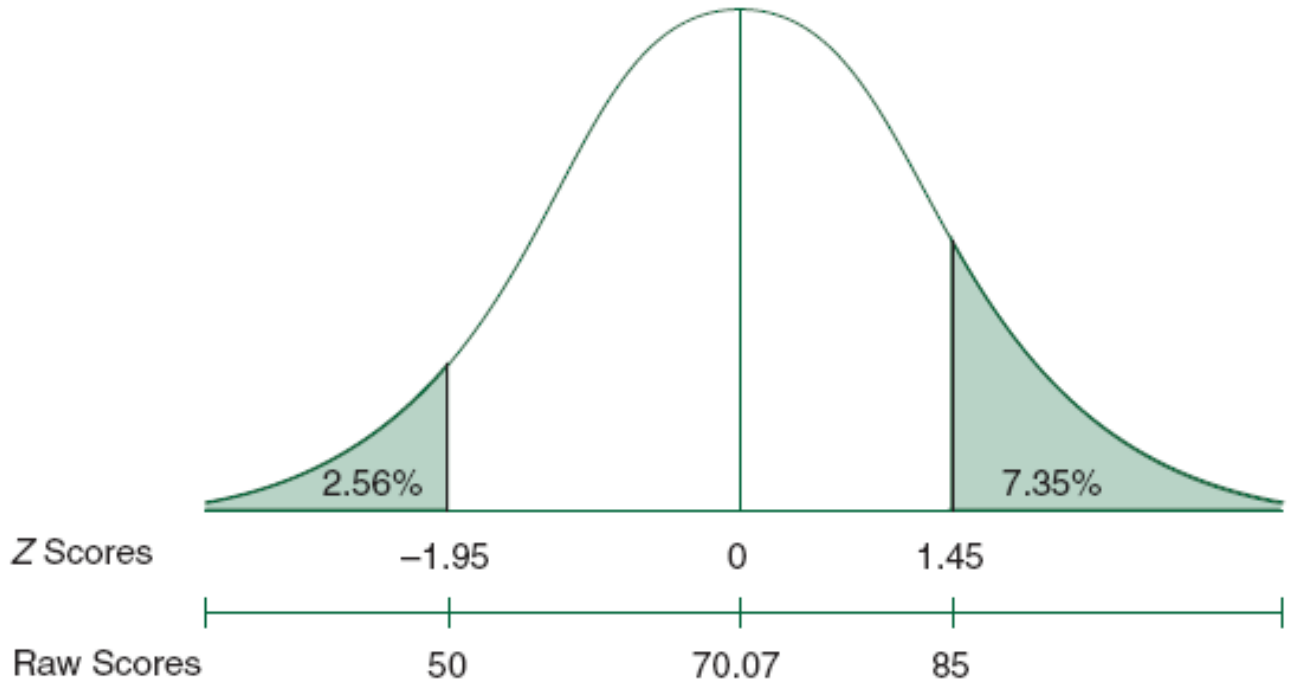
  Z = (85-70.07)/10.27 = **1.45 → 7.35%**

  (b) Convert 50 to a Z score, then look up the value (look for a **positive** Z score!) in **Column C**:

  Z = (50-70.07)/10.27 = **-1.95 → 2.56%**

Finding Area Above a Positive Z Score or Below a Negative Z Score

Figure 6.9 Finding the Area Above a Positive *Z* Score or Below a Negative *Z* Score



Finding a Z Score Bounding an Area Above It

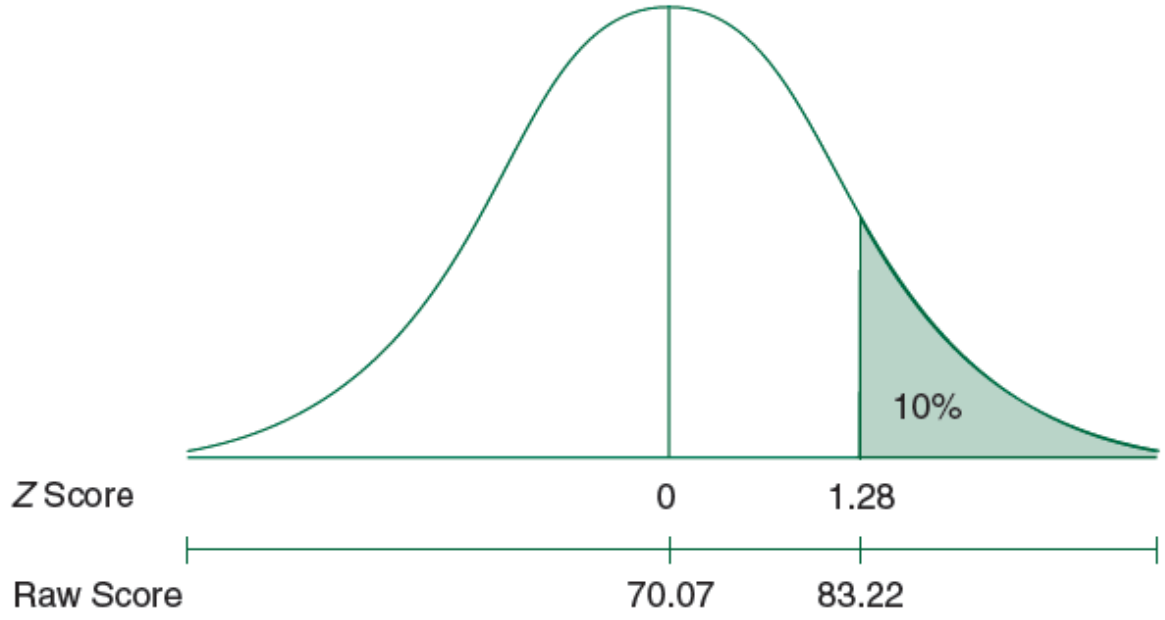
- Find the raw score that bounds the top 10 percent of the distribution (Table 14.1)
- (1) 10% = a proportion of 0.10
- (2) Using the Standard Normal Table, look in **Column C** for .1000, then take the value in **Column A**; this is the Z score **(1.28)**

(3) Finally convert the Z score to a raw score:

$$Y=70.07 + 1.28\ (10.27) = \quad \mathbf{83.22}$$

Finding a Z Score Bounding an Area Above It

Figure 6.10 Finding a Z Score Bounding an Area Above It



(4) 83.22 is the raw score that bounds the upper 10% of the distribution. The Z score associated with 83.22 in this distribution is 1.28

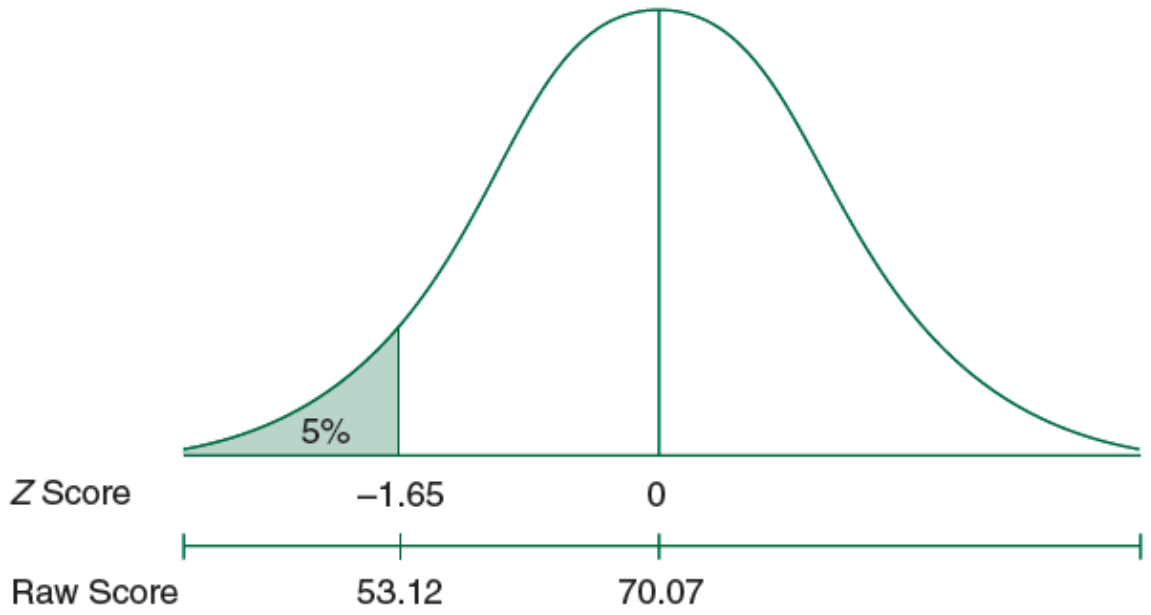
Finding a Z Score Bounding an Area Below It

- Find the raw score that bounds the lowest 5 percent of the distribution (Table 14.1)
- (1) 5% = a proportion of 0.05
- (2) Using the Standard Normal Table, look in **Column C** for 0.05, then take the value in **Column A**; this is the Z score (-1.65); **negative**, since it is on the **left** side of the distribution
- (3) Finally convert the Z score to a raw score:

  Y=70.07 + -1.65 (10.27) = **53.12**

Finding a Z Score Bounding an Area Below It

Figure 6.11 Finding a *Z* Score Bounding an Area Below It



(4) 53.12 is the raw score that bounds the lower 5% of the distribution. The Z score associated with 53.12 in this distribution is -1.65

Finding the Percentile Rank of a Score Higher than the Mean

- Suppose your raw score was 85. You want to calculate the percentile (to see where in the class you rank.)
- (1) Convert the raw score to a Z score:

  Z = (85-70.07)/10.27 = **1.45**

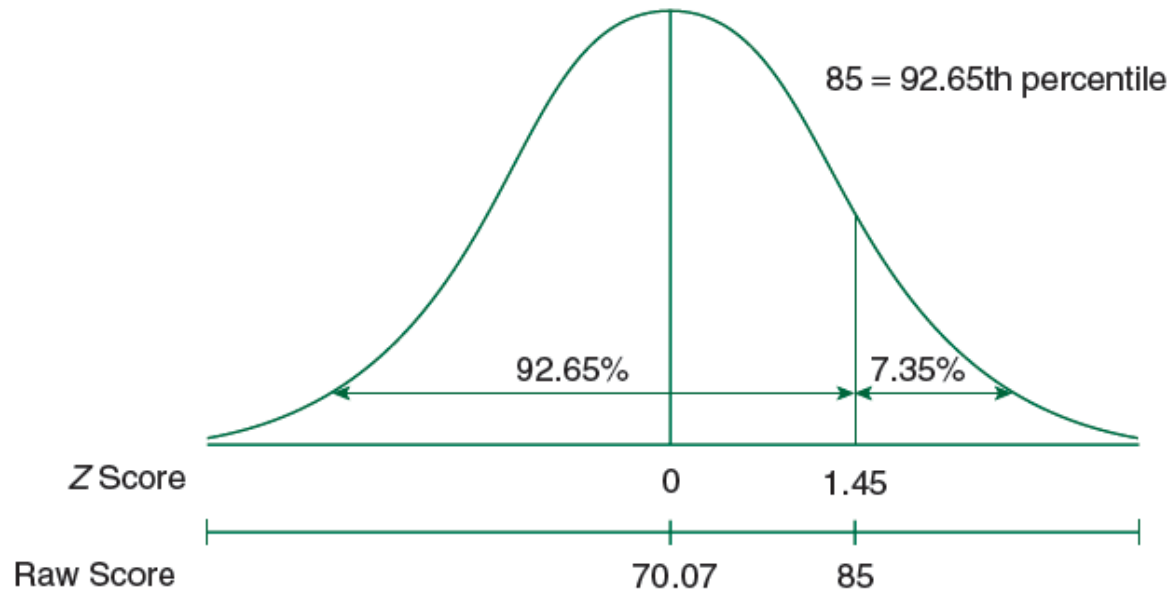
(2) Find the area beyond Z in the Standard Normal Table (**Column C**): **0.0735**

(3) **Subtract** the area from 1.00 for the percentile, since .0735 is **only** the area **not** below the score:

1.00 - .0735 = **0.9265** (proportion of scores below 85)

Finding the Percentile Rank of a Score Higher than the Mean

Figure 6.12 Finding the Percentile Rank of a Score Higher Than the Mean



(4) .9265 represents the proportion of scores less than 85 corresponding to a percentile rank of 92.65%

Finding the Percentile Rank of a Score Lower than the Mean

- Now, suppose your raw score was 65.
- (1) Convert the raw score to a Z score

  Z = (65-70.07)/10.27 = **-0.49**

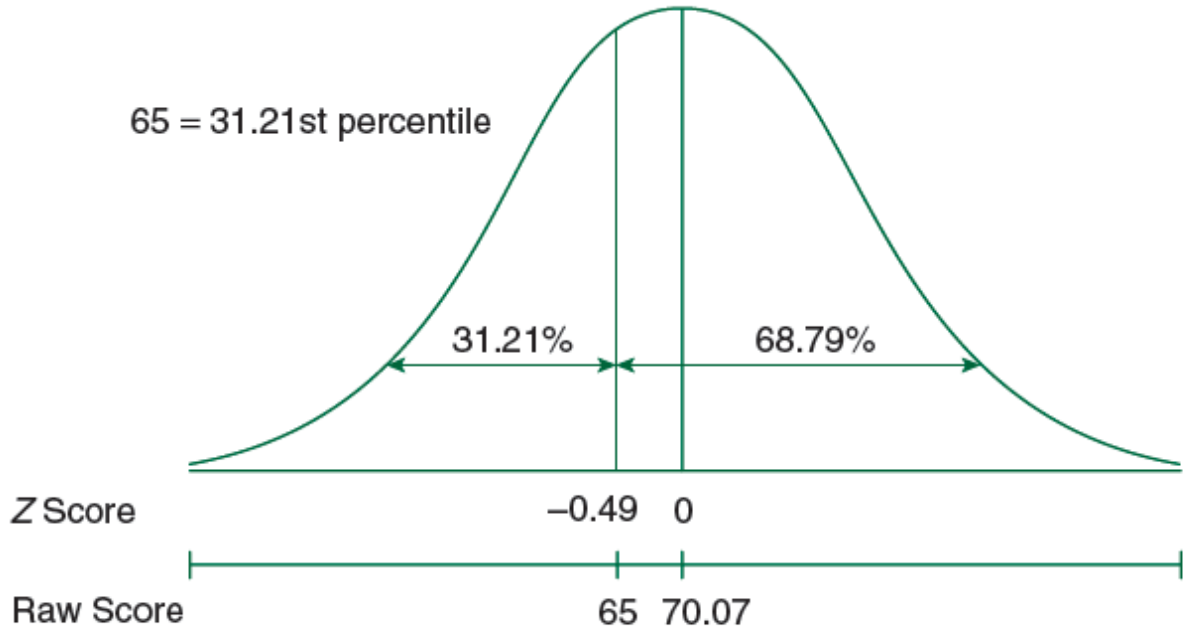
(2) Find the are **beyond** Z in the Standard Normal Table, **Column C**: **0.3121**

(3) **Multiply by 100** to obtain the percentile rank:

0.3121 x 100 = **31.21**%

Finding the Percentile Rank of a Score Lower than the Mean

Figure 6.13 Finding the Percentile Rank of a Score Lower Than the Mean



Finding the Raw Score of a Percentile Higher than 50

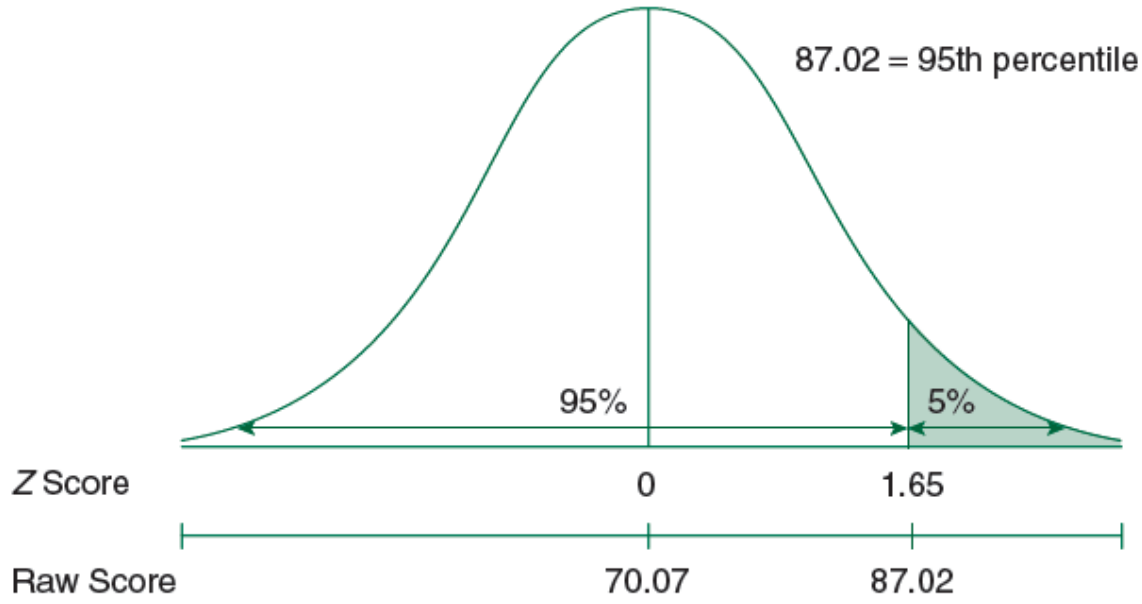
- Say you need to score in the 95th percentile to be accepted to a particular grad school program. What's the cutoff for the 95th percentile?
- (1) Find the area associated with the percentile:

  95/100 = 0.9500
- (2) **Subtract** the area from **1.00** to find the area above & beyond the percentile rank:

  1.00 - 0.9500 = 0.0500
- (3) Find the Z Score by looking in **Column C** of the Standard Normal Table for 0.0500: Z = 1.65

Finding the Raw Score of a Percentile Higher than 50

Figure 6.14 Finding the Raw Score Associated With a Percentile Higher Than 50



(4) Convert the Z score to a raw score.

Y= 70.07 + 1.65(10.27) = **87.02**

Finding the Raw Score of a Percentile Lower than 50

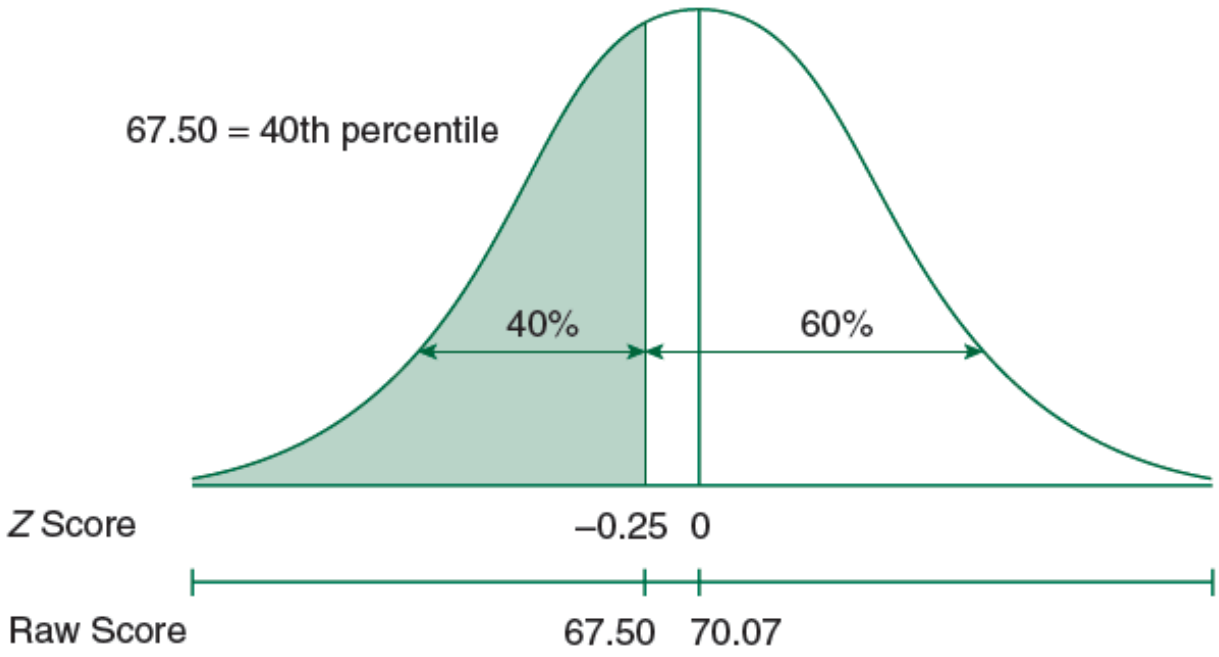
- What score is associated with the 40^{th} percentile?
- (1) Find the area **below** the percentile:

  40/100 = 0.4000
- (2) Find the Z score associated with this area. Use **Column C**, but remember that **this is a negative Z score** since it is **less than** the mean; so, S_y = -0.25
- (3) Convert the Z score to a raw score:

  Y = 70.07 + -0.25(10.27) = **67.5**

Finding the Raw Score of a Percentile Lower than 50

Figure 6.15 Finding the Raw Score Associated With a Percentile Lower Than 50



See You Wednesday!