National University of Science & Technology, Islamabad, Pakistan
School of Mechanical & Manufacturing Engineering

ME 825 Design Optimization
Instructor: M. Asghar Bhatti

Tentative Course Outline

Goal: To introduce fundamental concepts and applications of design optimization.

1	Fri 28 Dec 2012	Optimization Problem Formulation, Time Value of Money
2	Mon 31 Dec 2012	Graphical Solution
3	Tue 1 Jan 2013	Optimality Conditions
4	Wed 2 Jan 2013	Linear programming
5	Thu 3 Jan 2013	LP Examples
6	Fri 4 Jan 2013	Numerical methods for unconstrained problems
7	Mon 7 Jan 2013	Numerical methods for constrained problems
8	Tue 8 Jan 2013	NLP Examples
9	Wed 9 Jan 2013	Genetic Algorithm
10	Thu 10 Jan 2013	Stress analysis using solid elements
11	Fri 11 Jan 2013	Stress analysis using frame and plate elements
12	Mon 14 Jan 2013	Heat transfer and fluid flow modeling
13	Tue 15 Jan 2013	Design optimization using FEA
14	Wed 16 Jan 2013	Design optimization using FEA

Course conduct

Lecture with in-class problem solving using *Mathematica*/Matlab and Ansys/Abaqus

Each class period will be for 3 hours 5 PM - 8 PM.

Active learning with in-class group problem solving. Lectures will typically consist of segments of about 15 minutes of explanation and then a hands-on activity.

Lectures will be posted on-line at the Piazza site (www.piazza.com). Please go through the material before the lecture. The lectures will not cover all details of a topic and will focus on problem solving or explaining difficult concepts.

Required background

Computer software

It'll be useful if you are familiar with either *Mathematica* or MATLAB. You will be expected to use one of these tools in all class activities.

Finite element analysis based optimization will be introduced. Familiarity of basic finite element concepts and a working knowledge of Ansys will be useful.

Mathematics

Calculus of single and multiple variables. Key concepts used: Taylor series and differentiation

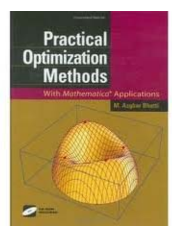
Matrix algebra. Must be familiar with the matrix notation. Matrix multiplication, transpose, inverse, eigenvalues, system of equations

Deformable bodies

Elementary notions of stresses and deformations of bars and beams. Some familiarity with two and three dimensional stress state useful.

Reference

M.A.Bhatti
Practical Optimization Methods : With *Mathematica* Applications
Springer Verlag, New York, 2000



Lecture 1: Optimization Problem Formulation & Time Value of Money

Read before the lecture

Chapter 1—Problem Formulation

1.1 Optimization Problem Formulation

1.2 Standard Form of an Optimization Problem

1.3 Solution of Optimization Problems

1.4 Time value of money

1.5 Concluding remarks

After this lecture you should

Understand terms such as design variables, objective function, and constraints.

Be able to formulate typical design situations as optimization problem.

Be able to classify a given optimization problem into an appropriate standard form.

Be able to properly account for economic conditions on long term engineering projects

Introduction, Optimization Problem Formulation

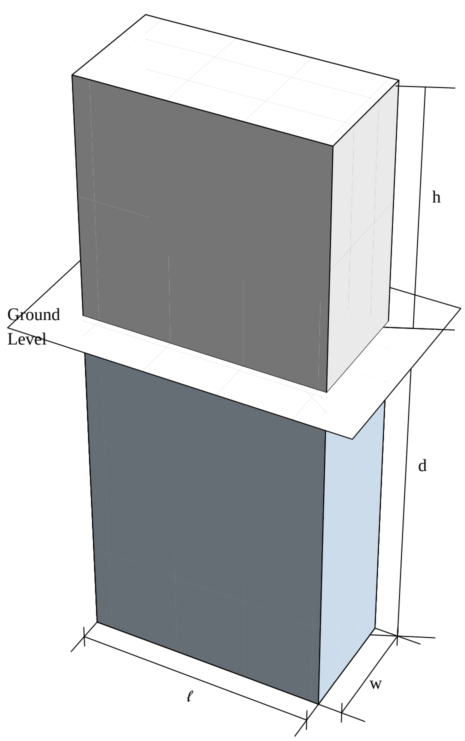
Formulation of an optimization problem involves taking statements defining general goals and requirements of a given activity and transcribing them into a series of well defined mathematical statements. More precisely, the formulation of an optimization problem involves:

1. Selection of one or more optimization variables
2. Choosing an objective function
3. Identifying a set of constraints

The objective function and the constraints must all be functions of one or more optimization variables. The following examples illustrate the process.

Building design

To save energy costs for heating and cooling, an architect is considering designing a partially buried rectangular building.



The total floor space needed is 20, 000 m^2. Lot size limits the building plan dimension to 50 m. It has already been decided that the ratio between the plan dimensions must be equal to the *golden ratio* (1.618) and that each story must be 3.5 m high. The heating and cooling costs are estimated at \$100 per $m^2$of the exposed surface area of the building. The owner has specified that the annual energy costs should not exceed \$225, 000. Formulate the problem of determining building dimensions to minimize cost of excavation.

Optimization variables

n = Number of stories
d = Depth of building below ground
h = Height of building above ground
ℓ = Length of building in plan
w = Width of building in plan

Objective function

The stated design objective is to minimize excavation cost. Assuming the cost of excavation to be proportional to the volume of excavation, the objective function can be stated as follows.

Minimize $d \ell w$

Constraints

Since the height of each story is given the number of stories and the total height are related to each other as follows.

$$\frac{d+h}{n} = 3.5$$

Also the requirement that the ratio between the plan dimensions must be equal to the golden ratio makes the two plan dimensions dependent on each other as follows.

$$\ell = 1.618\, w$$

The total floor space is equal to the area per floor multiplied by the number of stories. Thus the floor space requirement can be expressed as follows.

$$n \ell w \geq 20,000$$

The lot size places the following limits on the plan dimensions.

$$\ell \leq 50 \quad w \leq 50$$

The energy cost is proportional to the exposed building area which includes the areas of the exposed sides and the roof. Thus the energy budget places the following restriction on the design.

$$100\,(2 h \ell + 2 h w + \ell w) \leq 225,000$$

To make the problem mathematically precise, it is also necessary to explicitly state that the design variables cannot be negative.

$\ell, w, h, d \geq 0 \quad n \geq 1$, must be an integer

Formal statement of optimization problem

Find (n, ℓ, w, h, d) in order to

Minimize $d \ell w$

$$\text{Subject to} \left(\begin{array}{c} \frac{d+h}{n} = 3.5 \\ \ell = 1.618\, w \\ n\, \ell\, w \geq 20,000 \\ \ell \leq 50 \\ w \leq 50 \\ 100\,(2\,h\,\ell + 2\,h\,w + \ell\,w) \leq 225,000 \\ n \geq 1 \\ \ell, w, h, d \geq 0 \end{array} \right)$$

Class Activity: Concrete blocks production

A company can produce three different types of concrete blocks, identified as A, B, and C. The production process is constrained by facilities available for mixing, vibration and inspection/drying. Using the data given in the following table formulate the production problem in order to maximize the profit.

	—	**Blocks**	—	
	A	***B***	***C***	**Available**
Mixing (hours/batch)	1	3	9	900
Vibration (hours/batch)	2	3	6	1200
Inspection/Drying (hours/batch)	0.7	0.8	1	400
Profit : ($/batch)	7	17	30	

Optimization variables

$x_1 = \#$ of batches of blocks A produced

$x_2 = \#$ of batches of blocks B produced

$x_3 = \#$ of batches of blocks C produced

Objective function

Profit to be maximized

$7\,x_1 + 17\,x_2 + 30\,x_3$

Constraints

Available manpower and equipment for various operations is limited

$$\begin{array}{l|l} \text{Mixing} & x_1 + 3\,x_2 + 9\,x_3 \leq 900 \\ \text{Vibration} & 2\,x_1 + 3\,x_2 + 6\,x_3 \leq 1200 \\ \text{Inspection} & 0.7\,x_1 + 0.8\,x_2 + x_3 \leq 400 \end{array}$$

The company does not want to operate in a loss. Thus all design variables must be positive.

Formal statement of optimization problem

Find (x_1, x_2, x_3) in order to

Maximize $7x_1 + 17x_2 + 30x_3$

$$\text{Subject to} \begin{pmatrix} x_1 + 3x_2 + 9x_3 \le 900 \\ 2x_1 + 3x_2 + 6x_3 \le 1200 \\ 0.7x_1 + 0.8x_2 + x_3 \le 400 \\ x_1, x_2, x_3 \ge 0 \end{pmatrix}$$

See additional examples in the textbook

1.1.2 Plant operation

1.1.3 Financial portfolio management

1.1.4 Data fitting

Standard Form of an Optimization Problem

Find a vector of optimization variables, $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)^T$ in order to

Minimize an objective (or cost) function, $f(\boldsymbol{x})$

Subject to

$g_i(\boldsymbol{x}) \le 0$	$i = 1, 2, \ldots, m$	Less than type inequality constraints (LE)
$h_i(\boldsymbol{x}) = 0$	$i = 1, 2, \ldots, p$	Equality constraints (EQ)
$x_{iL} \le x_i \le x_{iU}$	$i = 1, 2, \ldots, n$	Bounds on optimization variables

The bounds constraints are in fact inequality constraints. They are sometimes kept separate from the others because of their simple form. Certain numerical optimization algorithms take advantage of their special form in making the computational procedure efficient.

Equality constraints can be used to eliminate design variables.

Consider the building design example.

Statement of building design problem as given previously

Find (n, ℓ, w, h, d) in order to

Minimize $d\,\ell\,w$

$$\text{Subject to} \begin{pmatrix} \frac{d+h}{n} = 3.5 \\ \ell = 1.618\,w \\ n\,\ell\,w \ge 20,000 \\ \ell \le 50 \\ w \le 50 \\ 100\,(2\,h\,\ell + 2\,h\,w + \ell\,w) \le 225,000 \\ n \ge 1 \\ \ell, w, h, d \ge 0 \end{pmatrix}$$

The formulation includes two simple relationships between the optimization variables (equality constraints). Using these we can eliminate two design variables. Substituting $n = (d+h)/3.5$ and $\ell = 1.618\,w$ we get the following formulation in terms of three variables.

Find $(d,\ h$ and $w)$ in order to

Minimize $f = 1.618\, d\, w^2$

$$\text{Subject to} \quad \begin{pmatrix} 100\left(5.236\, h\, w + 1.618\, w^2\right) \le 225\,000 \\ 1.618\, w \le 50 \\ w \le 50 \\ 0.462286\, (d + h)\, w^2 \ge 20\,000 \\ d \ge 0,\;\; h \ge 0, \text{ and } w \ge 0 \end{pmatrix}$$

Number of equality constraints must be less than the number of design variables.

No restriction on the number of inequality constraints

Maximization of a function is equivalent to minimization of its negative

Statement of production problem as given previously

Find (x_1, x_2, x_3) in order to

Maximize $7\, x_1 + 17\, x_2 + 30\, x_3$

$$\text{Subject to} \begin{pmatrix} x_1 + 3\, x_2 + 9\, x_3 \le 900 \\ 2\, x_1 + 3\, x_2 + 6\, x_3 \le 1200 \\ 0.7\, x_1 + 0.8\, x_2 + x_3 \le 400 \\ x_1,\, x_2,\, x_3 \ge 0 \end{pmatrix}$$

Statement of production problem in standard form

Find (x_1, x_2, x_3) in order to

Minimize $-(7\, x_1 + 17\, x_2 + 30\, x_3)$

$$\text{Subject to} \begin{pmatrix} x_1 + 3\, x_2 + 9\, x_3 \le 900 \\ 2\, x_1 + 3\, x_2 + 6\, x_3 \le 1200 \\ 0.7\, x_1 + 0.8\, x_2 + x_3 \le 400 \\ x_1,\, x_2,\, x_3 \ge 0 \end{pmatrix}$$

GE (≥) constraints can be converted to LE (≤) to form my multiplying both sides by a negative sign

Statement of building design problem as given previously

Find (n, ℓ, w, h, d) in order to

Minimize $d\, \ell\, w$

$$\text{Subject to} \begin{pmatrix} \frac{d+h}{n} = 3.5 \\ \ell = 1.618\, w \\ n\, \ell\, w \ge 20,\, 000 \\ \ell \le 50 \\ w \le 50 \\ 100\, (2\, h\, \ell \; + \; 2\, h\, w \; + \; \ell\, w) \le 225,\, 000 \\ n \ge 1 \\ \ell,\, w,\, h,\, d \ge 0 \end{pmatrix}$$

Statement of building design problem in standard form

Find (n, ℓ, w, h, d) in order to

Minimize $d\,\ell\,w$

$$\text{Subject to}\quad \left(\begin{array}{c} \frac{d+h}{n} = 3.5 \\ \ell = 1.618\,w \\ -n\,\ell\,w \le -20,000 \\ \ell \le 50 \\ w \le 50 \\ 100\,(2\,h\,\ell + 2\,h\,w + \ell\,w) \le 225,000 \\ -n \le -1 \\ -\ell, -w, -h, -d \le 0 \end{array}\right)$$

Multiple objective functions

Standard optimization problem formulation considers only a single objective function.

Real life problems typically have more than one goals.

Strategy 1

Select the most important goal as the single objective function and treat others as constraints with reasonable limiting values.

Strategy 2

Define a composite objective function as a weighted sum of all functions expressing individual goals

$$f(\boldsymbol{x}) = w_1\, f_1(\boldsymbol{x}) + w_2\, f_2(\boldsymbol{x}) + \ldots$$

where $w_1,\ w_2,\ \ldots$ are suitable weighting factors. The success of the method clearly depends on a clever choice of these weighting factors.

Classification of optimization problems

Unconstrained problems

These problems have an objective function but no constraints.

The data fitting problem formulated in the textbook is an example.

Linear programming (LP) problems

If the objective function and all constraints are linear functions of optimization variables. Production problem is an LP problem.

Quadratic programming (QP) problems

If the objective function is a quadratic function and all constraint functions are linear functions of optimization variables.

Nonlinear programming (NLP) problems

The general constrained optimization problem, in which one or more functions are nonlinear, are called nonlinear programming problems. The building design problem is an NLP problem.

Class Activity

Express the following optimization problem in the standard NLP form

$$\text{Find} \quad (x_1, x_2)$$

$$\text{Maximize} \quad f(x_1, x_2) = (x_1 - 2)^2 + (x_2 - 10)^2$$

$$\text{Subject to} \quad \begin{pmatrix} x_1^2 + x_2^2 \le 50 \\ x_1^2 + x_2^2 + 2\,x_1\,x_2 - x_1 - x_2 + 20 \ge 0 \\ x_1,\ x_2 \ge 0 \end{pmatrix}$$

Solution

$$\text{Find} \quad (x_1, x_2)$$

$$\text{Minimize} \quad -f(x_1, x_2) = -(x_1 - 2)^2 - (x_2 - 10)^2$$

$$\text{Subject to} \quad \begin{pmatrix} x_1^2 + x_2^2 \le 50 \\ -\left(x_1^2 + x_2^2 + 2\,x_1\,x_2 - x_1 - x_2 + 20\right) \le 0 \\ x_1,\ x_2 \ge 0 \end{pmatrix}$$

Time Value of Money

Basic design principle: "Best" possible design. Best might mean least-cost design, the safest design, etc.

Cost plays an important role in most engineering designs, hence we start by looking at how value of money changes over time because of interest, inflation, taxes, etc.

Interest functions

Single payment compound amount factor — spcaf

Consider the simplest case of determining future value of a fixed amount of money invested.

P = Amount invested $\qquad$ i = Interest rate per period

The appropriate period depends upon the investment type. Most commercial banks base their computations on a daily basis. Some of the large capital projects may be based on monthly or annual compounding.

Total investment at the end of first period $= P + i\,P = (1 + i)\,P$

Total investment at the end of second period $= (1 + i)\,P + i(1 + i)\,P = (1 + i)^2\,P$

Continuing this process for n periods, it is easy to see that

Total investment after n periods, $S_n = (1 + i)^n\,P \equiv \text{spcaf}[i, n]\,P$

Class Activity 0.1

A father deposits \$2000 in his daughter's account on her 6th birthday. If the bank pays an annual interest rate of 8%, compounded monthly, how much money will be there when his daughter reaches her 16th birthday.

Answer: \$4439 .28

Solution

$i = 0.08/12 \quad n = 120 \quad \text{spcaf}[0.08/12,\ 120] = (1 + 0.08/12)^{120} = 2.21964$

$S_{120} = 2000 \times 2.21964 = \$4439.28$

Single payment present worth factor — sppwf

The inverse of the spcaf will give the present value of a future sum of money. That is

$$P = (1 + i)^{-n}\, S_n \equiv \text{sppwf}[i,\ n]\, S_n$$

Class Activity 0.2

A person will need $10,000 in exactly 5 years from today. How much money should he deposit in a bank account that pays an annual interest rate of 9%, compounded monthly.

Answer: $6387

Solution

$i = 0.09/12 \quad n = 60 \quad \text{sppwf}[0.09/12,\ 60] = (1 + 0.09/12)^{-60} = 0.6387$

$P = 10,000 \times 0.6387 = \$6387$

Uniform series compound amount factor — uscaf

Now consider a more complicated situation in which a series of payments (or investments) are made at regular intervals.

R = Uniform series of amounts invested per period $\quad i$ = Interest rate per period

The first payment earns interest over $n - 1$ periods and thus after n periods is equal to $= (1 + i)^{n-1} R$

The second payment earns interest over $n - 2$ periods and thus after n periods is equal to $= (1 + i)^{n-2} R$

Continuing this process, the total investment after n periods is as follows.

$$S_n = (1 + i)^{n-1} R + (1 + i)^{n-2} R + \ldots + (1 + i) R + R$$

This represents a geometric series whose sum, derived in most calculus textbooks, is as follows.

$$S_n = \frac{(1+i)^n - 1}{i} R \equiv \text{uscaf}[i,\ n]\, R$$

Class Activity 0.3

A mother deposits $200 every month in her daughter's account starting on her 6th birthday. If the bank pays an annual interest rate of 8%, compounded monthly, how much money will be there when her daughter reaches her 16th birthday.

Answer: $36589.20

Solution

$i = 0.08/12 \quad n = 120 \quad \text{uscaf}[0.08/12,\ 120] = \frac{(1 + 0.08/12)^{120} - 1}{0.08/12} = 182.946$

$S_{120} = 200 \times 182.946 = \$36589.20$

Sinking fund deposit factor — sfdf

The inverse of the uscaf can be used to compute uniform series of payments from a given future amount.

$$R = \frac{i}{(1+i)^n - 1} S_n \equiv \text{sfdf}[i,\ n]\, S_n$$

Class Activity 0.4

A person will need $10,000 in exactly 5 years from today. How much money should he save every month in a bank account that pays an annual interest rate of 9%, compounded monthly.

Answer: $136 .10

Solution

$$i = 0.09/12 \quad n = 60 \quad \text{sfdf}[0.09/12,\ 60] = \frac{0.08/12}{(1 + 0.08/12)^{60} - 1} = 0.0136097$$

$$R = 10,\ 000 \times 0.0136097 = \$136\ .10$$

Capital recovery factor — crf

The above formulas can be combined to give a simple formula to convert a given present amount to a series of uniform payments.

$$R = \frac{i}{(1+i)^n - 1} S_n = \frac{i}{(1+i)^n - 1}(1+i)^n P = \frac{i}{1-(1+i)^{-n}} P$$

or $$R = \frac{i}{1-(1+i)^{-n}} P \equiv \text{crf}[i,\ n]\ P$$

Class Activity 0.5

A car costs $15,000. You put $5,000 down and finance the rest for 3 years at 10% annual interest rate compounded monthly. What are your monthly car payments?

Answer: $322 .67

Solution

Cost = $15,000 Down payment = $5,000 Amount financed = $10,000

$$i = 0.1/12 \quad n = 36 \quad \text{crf}[0.1/12,\ 36] = \frac{0.1/12}{(1 + 0.1/12)^{36} - 1} = 0.0322672$$

$$R = 10,\ 000 \times 0.0322672 = \$322\ .67$$

Uniform series present worth factor — uspwf

The inverse of the crf can be used to compute present worth of a uniform series of payments.

$$P = \frac{1-(1+i)^{-n}}{i} R = \ \equiv \text{uspwf}[i,\ n]\ R$$

Note that when n is large $(1 + i)^{-n}$ goes to zero. Thus

$$\text{uspwf}[i,\ \infty] = \frac{1}{i} \qquad \text{and} \qquad \text{crf}[i,\ \infty] = i$$

Example

A car costs $15,000. You put $5,000 down and finance the rest for 3 years at 10% annual interest rate compounded monthly. Your monthly payments are $322.67. Compute the amount needed to pay off the loan after making 12 payments?

Convert the remaining 24 payments to an equivalent single amount using uspwf as follows.

$$i = 0.1/12 \quad n = 24 \quad \text{uspwf}[0.1/12,\ 24] = \frac{1-(1+0.1/12)^{-24}}{0.1/12} = 21.6709$$

Remaining loan after 12 payments = $P = 322.67 \times 21.6709 = \$6992\ .53$

Summary

P = Single amount at present

S_n = Single amount after n periods

R = Uniform series of amounts at each period

Given	To find	Multiply given value by
P	S_n	$\text{spcaf}[i, n] = (1+i)^n$
S_n	P	$\text{sppwf}[i, n] = (1+i)^{-n}$
R	S_n	$\text{uscaf}[i, n] = \frac{(1+i)^n - 1}{i}$
S_n	R	$\text{sfdf}[i, n] = \frac{i}{(1+i)^n - 1}$
P	R	$\text{crf}[i, n] = \frac{i}{1-(1+i)^{-n}}$, $\text{crf}[i, \infty] = i$
R	P	$\text{uspwf}[i, n] = \frac{1-(1+i)^{-n}}{i}$, $\text{uspwf}[i, \infty] = \frac{1}{i}$

Comparison of alternatives based on economic factors

Annual-cost comparisons—for each alternative all costs/revenues are expressed in terms of equivalent annual amounts.

Present-worth comparisons—for each alternative all costs/revenues are expressed in terms of equivalent present amounts.

It becomes easy to make these comparisons if all financial transactions related to a particular alternative are represented on a time line showing the amount of the transaction and the time when the transaction takes place. Such diagrams are called *Cash flow diagrams* and are used in the following examples. The following function, included in the OptimizationToolbox package, is useful in drawing these cash flow diagrams.

Example: Water treatment facilities

A city is considering following two proposals for its water supply.

Proposal 1:

First dam and treatment plant — Construction cost = \$500,000, annual operations cost = \$36,000. The capacity of this facility will be enough for the community for 12 years.

Second dam and treatment plant — Construction cost = \$480,000, additional annual operations cost = \$28,000. The capacity of the expanded facility will be enough for the foreseeable future

Proposal 2:

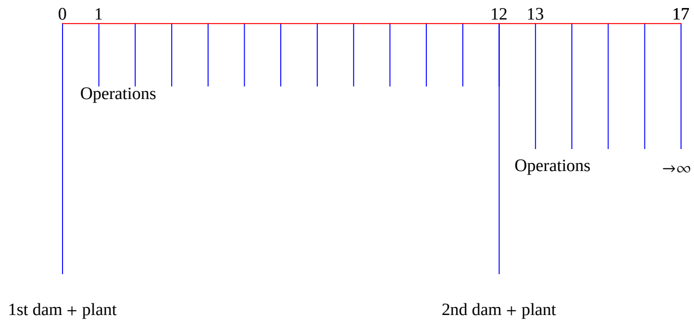
Large dam and first treatment plant — Construction cost = \$700,000, annual operations cost = \$37,000. The capacity of this facility will be enough for the community for 15 years.

Second treatment plant — Construction cost = \$100,000, additional annual operations cost = \$26,000. The capacity of the expanded facility will be enough for the foreseeable future

Use a present worth analysis to compare the alternatives. Assume annual interest rate of 10% and annual compounding.

Solution Present worth of Proposal 1

The cash flow diagram for this option is as follows.



The cost of the first dam and the treatment plant is already in terms of present dollars. Annual operations costs for 12 years can be brought to present by multiplying by uspwf[i,12]. The cost of the second dam and the treatment plant can be brought to present by multiplying by sppwf[i,12]. Starting from year 13, the operations costs increase and stay the same for the foreseeable future. If we multiply these costs by uspwf[i,∞], this will bring all these costs to a single amount at the year 12. To bring it to present this single amount needs to be multiplied by sppwf[i, 12]. Thus the complete calculations for the present worth are as follows.

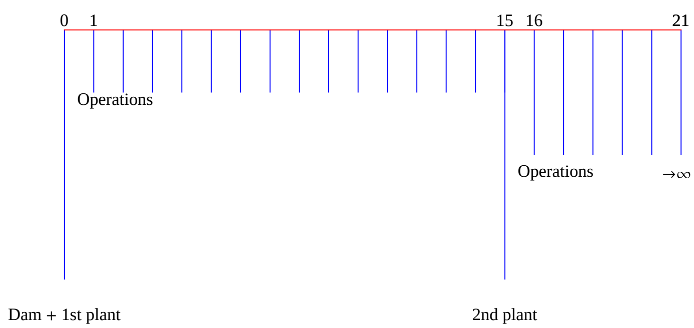
```
c1 = 500000; a1 = 36000; a2 = a1+28000; c2 = 480000;
i = 0.1;
```

```
PW1 = c1 + a1*uspwf[i,12] + c2*sppwf[i,12] + a2*uspwf[i,∞]*sppwf[i,12]
```

1.10216×10^{6}

Solution Class Activity - Present worth of Proposal 2

The cash flow diagram for this option is as follows.




```
c1 = 700000; a1 = 37000; a2 = a1+26000; c2 = 100000;
i = 0.1;
```

```
PW2 = c1 + a1*uspwf[i,15] + c2*sppwf[i,15] + a2*uspwf[i,∞]*sppwf[i,15]
```

1.15618×10^{6}

(c) Conclusion

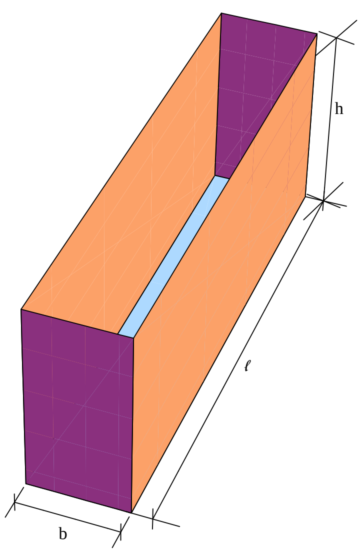
In terms of present dollars, the cost of the first proposal is 1.10216×10^{6} and that of the second proposal is 1.15618×10^{6}. The first proposal is slightly cheaper for the city and should be adopted.

Optimization problem with economic factors

Example: Open-top rectangular containers to transport material

A company requires open-top rectangular containers to transport material. Using the following data, formulate an optimum design problem to determine the container dimensions for minimum annual cost.

Construction costs	Sides $= \$65/m^2$ Ends $= \$80/m^2$ Bottom $= \$120/m^2$
Useful life	10 years
Salvage value	20 % of the initial construction cost
Yearly maintenance cost	$\$12/m^2$ of the outside surface area
Minimum required volume of the container	$1200\, m^3$
Nominal interest rate	10 % (Annual compounding)



Design variables

b = Width of container ℓ = Length of container h = Height of container

Objective function

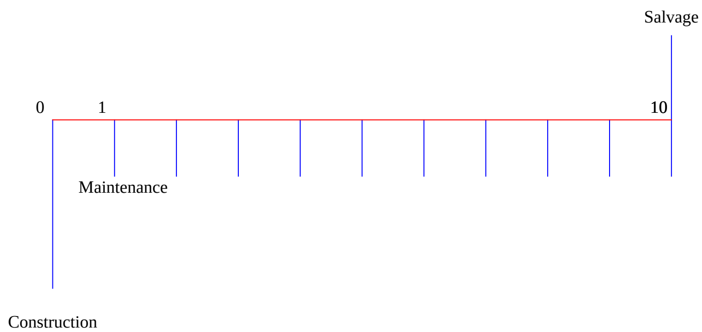
Minimum annual cost

Construction cost $= 120\,\ell\, b + 2\times 65\,\ell\, h + 2\times 80\, b\, h = 120\,\ell\, b + 130\,\ell\, h + 160\, b\, h$

Yearly maintenance cost $= 12\,(\ell\, b + 2\,\ell\, h + 2\, b\, h)$

Salvage value $= 0.2\,(120\,\ell\, b + 130\,\ell\, h + 160\, b\, h)$

To determine the annual cost, it is useful to draw the cash flow diagram.



Using this cash flow diagram, it ia easy to see that to convert all transactions to annual cost, we need to multiply the construction cost by crf[i,10] and the salvage value by sfdf[i,10]. Thus the annual cost of the project is as expressed as follows.

$$\text{Annual cost } = \text{crf}[0.1,\ 10]\,(120\,\ell\,b + 130\,\ell\,h + 160\,b\,h) + 12\,(\ell\,b + 2\,\ell\,h + 2\,b\,h) \\ -\text{sfdf}[0.1,\ 10]\times 0.2\,(120\,\ell\,b + 130\,\ell\,h + 160\,b\,h)$$

$$\text{Annual cost } = 48.0314\,b\,h + 30.0236\,b\,\ell + 43.5255\,h\,\ell$$

Constraints

$$\text{Container volume } = b\,h\,\ell \geq 1200$$

Complete optimization problem statement

Find $b,\ h,$ and ℓ to

Minimize annual cost $= 48.0314\,b\,h + 30.0236\,b\,\ell + 43.5255\,h\,\ell$

Subject to $b\,h\,\ell \geq 1200$ and $b,\ h,$ and $\ell\ \geq 0$

Extra