Process Dynamics and Control
CHEN 461, Spring 2013

Homework #6
SHOW YOUR WORK!

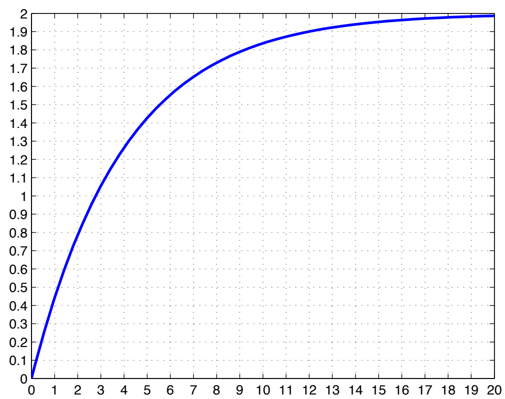
1. Identify the transfer functions that produced the following step responses shown in the figures on the following page, where

    (a) $U(s) = \frac{1}{s}$ (a unit step input)

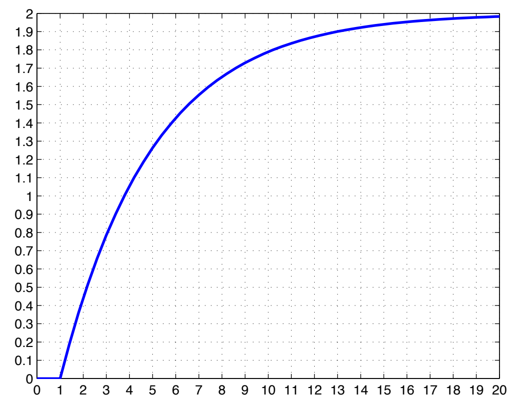
    (b) $U(s) = \frac{1}{s}$ (a unit step input)

    (c) $U(s) = \frac{1}{s}e^{-s}$ (a unit step input delayed by one time unit)

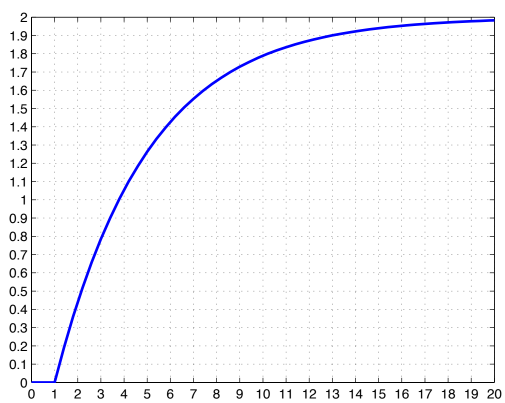
    (d) $U(s) = \frac{2}{s}e^{-s}$ (a step input of value 2 delayed by one time unit)



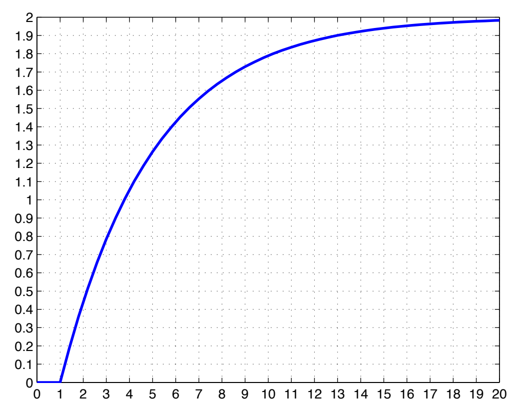
(a)



(b)

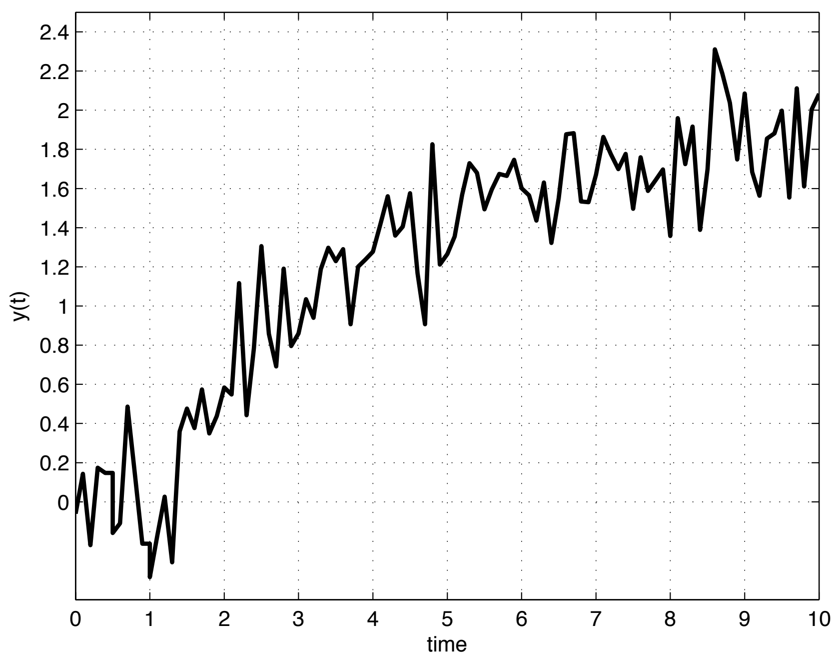


(c)



(d)

2. Facilities that process natural gas often perform onsite separation of heavier components from the natural gas. You facility involves a series of columns. The first column separates the light methane/ethane from the heavier hydrocarbons. The natural gas (methane/ethane) stream is sent to compression units and on to the pipeline. The heavier hydrocarbons are further separated. The second column (called the depropanizer) separates propane from the heavy stream, the third column (called the debutanizer) separates butane from the heavy stream, etc. In an effort to improve control of the debutanizer, one of your colleagues performs a step test on the column. She makes a unit step change in the reboiler duty and measures the change in the boiling point temperature of the product stream from the top of the column (as a proxy for the butane concentration). Her data is shown in the following figure. Note: The raw data is available in matlab form on piazza (`hw5_data1.m`).



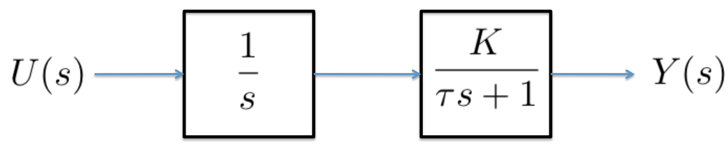
(a) What is the steady state gain for the process?

(b) Determine a first-order plus time delay model for this system. Specifically, give k_p, τ_p, and t_w. Use the data and simulations to tune your model.

(c) Determine a second-order model for this system. Specifically, identify k_p, τ_1, and τ_2. Use the data and simulations to tune your model.

(d) Which model do you think fits the data better? Justify your answer.

3. Consider a process described by the following block diagram, containing both an integrating component and a first order component. Assume that you are given the



step response for a step change of Q. Describe a process for obtaining the gain K and time-constant τ. Show all your work and draw the expected response.

4. The dynamic behavior of a liquid level in each leg of a manometer tube, responding to a change in pressure is given by:

$$\frac{d^2h'}{dt^2} + \frac{6\mu}{R^2\rho}\frac{dh'}{dt} + \frac{3}{2}\frac{g}{L}h' = \frac{3}{4\rho L}p'(t)$$

where $h'(t)$ is the level of fluid measured with respect to the initial steady-state value, $p'(t)$ is the pressure chance, and R, L, g, ρ, and μ are constants.

(a) Rearrange this equation into standard gain-time constant form and find expression for K, τ, and ξ in terms of the physical constants.

(b) For what values of physical constants does the manometer response oscillate?

(c) Would changing the manometer fluid so that ρ is larger make its response more oscillatory, or less? Repeat the analysis for an increase in μ.

5. For the equation:

$$\frac{d^2y}{dt^2} + K\frac{dy}{dt} + 4y = u$$

(a) Find the transfer function and put it in standard gain/time constant form.

(b) Discuss the qualitative form of the response of this system (independent of the input forcing) over the range -10 $\leq K \leq$ 10.
Specify values of K where the response will converge and where it will not. Write the form of the response without evaluating any coefficients.

6. Given the following transfer function,

$$G(s) = \frac{0.7(s^2 + 2s + 2)}{s^5 + 5s^4 + 2s^3 - 4s^2 + 6},$$

complete the following.

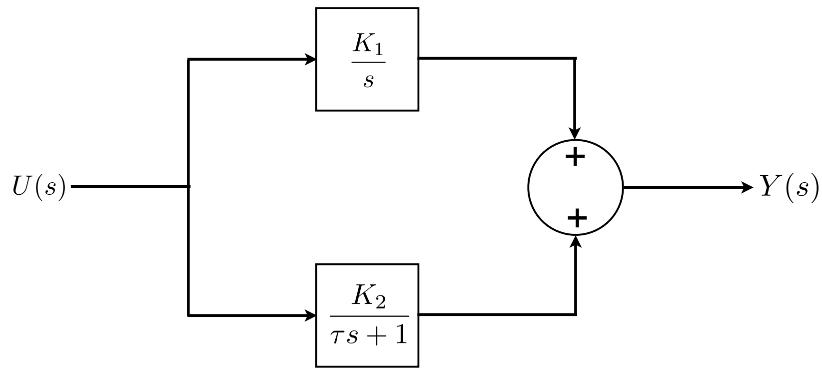
(a) Find the poles and zeros for the transfer function and sketch them in the complex plane.

(b) Describe the response characteristics you expect from a step change.

(c) Simulate a unit step response for the system. Show the plots for your response.

(d) Does the simulated response agree with your analysis from part (b)? If not, why?

7. The process shown below consists of an integrating element operating in parallel with a first-order element:



(a) What is the order of the overall transfer function, $G(s) = Y(s)/U(s)$?

(b) What is the gain of $G(s)$?

(c) What are the poles of $G(s)$? Where are they located in the complex s-plane?

(d) What are the zeros of $G(s)$? Where are they located? Under what condition(s) will one or more of the zeros be located in the right-half s-plane?

(e) Under what conditions, will the process exhibit a right-half plane zero?

(f) For any input change, what function of time (response modes) will be included in response, $y(t)$?

(g) Is the output bounded for any bounded input change, for example, $u(t) = M$?