STAT 310 - Homework 4

Due Date - 2/7/2013 @ 1pm

Problem One

Let X be a geometric random variable, show that

$$P(X > n + k - 1 \mid X > n - 1) = P(X > k)$$

In light of the construction of a geometric distribution from a sequence of independent Bernoulli trials, how can this be interpreted so that it is "obvious"?

Problem Two

In LOTTO 49, Michigan's lottery game, a player selects 6 integers out of the first 49 positive integers. The state then randomly selects 6 out of the first 49 positive integers. Cash prizes are given to a player who matches 4, 5, or 6 integers. Let X equal the number of of integers selected by a player that match integers selected by the state.

(a) State the pmf of X

(b) Calculate the mean, variance, and standard deviation of X

(c) What value of X is most likely to occur?

(d) On February 25, 1995, the jackpot was worth \$45 million. When the prize is this large, many bets are placed. Out of the 25 million bets that were placed, 3 people matched all six numbers, with each winning \$15 million (most of which was paid by losers during the preceding games); 390 matched fives numbers, to win \$2500 each and; 22,187 matched four numbers to win \$100 each. Are these numbers of winners consistent with the probability model?

(e) A mathematics professor convinced some colleagues to pool their LOTTO bets, so that they were able to purchase 138 tickets together. They let the state computer randomly select their numbers on which they placed their bets. Among their 138 bets, 65 matched none of the winning LOTTO numbers, (3,20,33,34,43,46); 55 matched one, 16 matched two, and 2 matched three numbers. How do these results compare with what they could have expected?

Problem Three

(i) Give the name of the distribution of X (if it has a name), (ii) find the values of μ and σ^2, and (iii) calculate $P(1 \leq X \leq 2)$ when the moment generating function of X is given by

1. $M(t) = (0.3 + 0.7e^t)^5$
2. $M(t) = \frac{0.3e^t}{1-0.7e^t}$, with $t < \ln(0.7)$
3. $M(t) = 0.45 + 0.55e^t$
4. $M(t) = 0.3e^t + 0.4e^{2t} + 0.2e^{3t} + 0.1e^{4t}$
5. $M(t) = (0.6e^t)^2(1 - 0.4e^t)^{-2}$, with $t < -\ln(0.4)$
6. $M(t) = \sum_{x=1}^{10}(0.1)e^{tx}$

Problem Four

An airline always overbooks if possible. A particular plane has 95 seats on a flight in which a ticket sells for $300. The airline sells 100 such tickets for this flight.

1. If the probability of an individual not showing up is 0.05, assuming independence, what is the probability that the airline can accommodate all the passengers who do show up?
2. If the airline must return the $300 price plus a penalty of $400 to each passenger that cannot get on the flight, what is the expected payout (penalty plus ticket refund) that the airline will pay?

Problem Five

Let X be Bernoulli random variable.

1. Calculate $E(X)$
2. Calculate $E(X^2)$
3. Calculate $Var(X)$
4. Derive the mgf.
5. Calculate $E(X)$ using the mgf
6. Calculate $Var(X)$ using the mgf

Problem Six

A particular type of tennis racket comes in a midsize version and an oversize version. Sixty percent of all customers at a certain store want the oversize version.

1. Among ten randomly selected customers who want this type of racket, what is the probability that at least six want the oversize version?

2. Among ten randomly selected customers, what is the probability that the number who want the oversize version is within 1 standard deviation of the mean value?

3. The store currently has seven rackets of each version. What is the probability that all of the next ten customers who want this racket can get the version they want from current stock?

Problem Seven

Three brothers and their wives decide to have children until each family has two female children. What is the pmf of X = the total number of male children born to the brothers? What is $E(X)$, and how does it compare to the expected number of male children born to each brother?