Math 140
Lecture 1

Greg Maloney

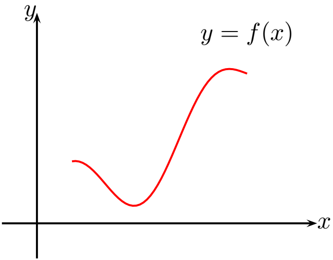
with modifications by T. Milev

University of Massachusetts Boston

January 29, 2013

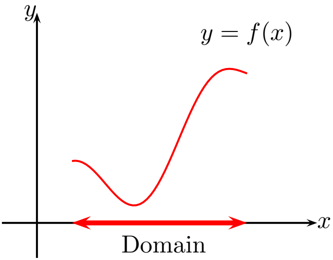
Outline

Outline



Definition (Function)

A function f is a rule that assigns to each element x in a set D exactly one element, called $f(x)$, in a set E.



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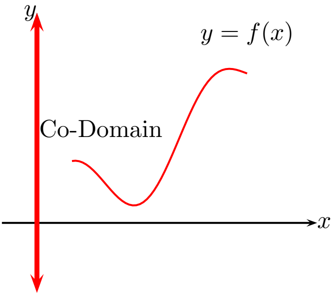
Definition (Domain)

The set D is called the domain.



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Definition (Co-domain)

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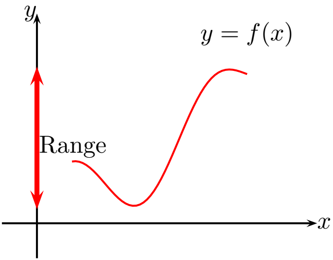

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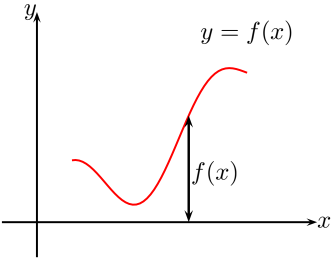
Definition (Range)

The set of all possible values taken by $f(x)$ as the element x runs over elements of D is called the range of f.



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Definition (Value of f at x)

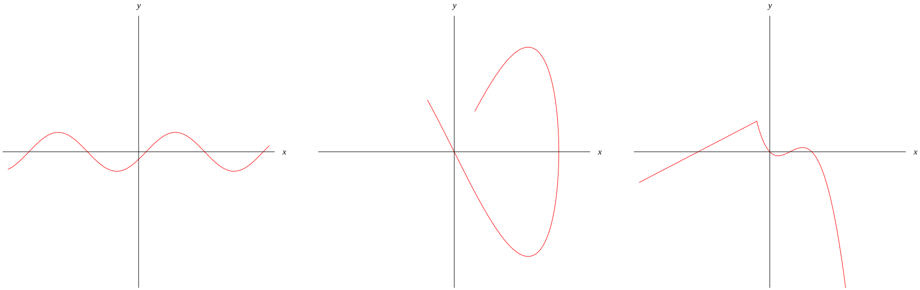
The number $f(x)$ is called the value of f at x, and is read "f of x."

The Vertical Line Test

Question: How can we tell if a curve is the graph of a function or not?
Answer: Use the vertical line test.

The Vertical Line Test.

A curve is the graph of a function if and only if no vertical line intersects it more than once.



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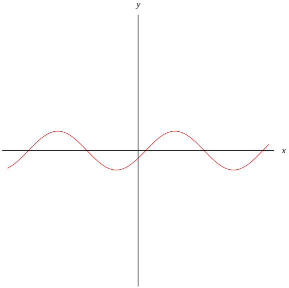

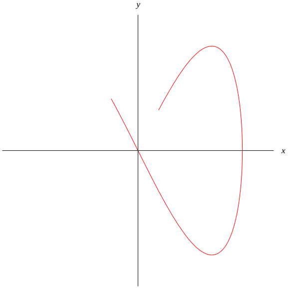

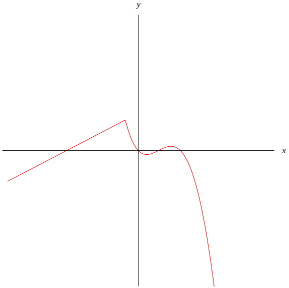

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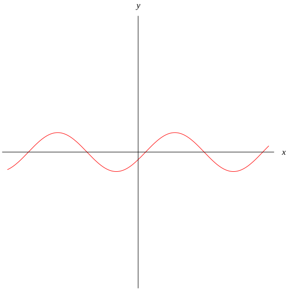

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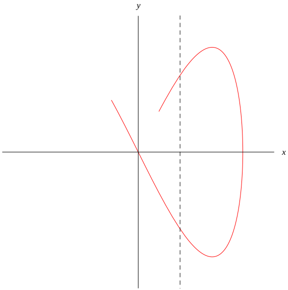
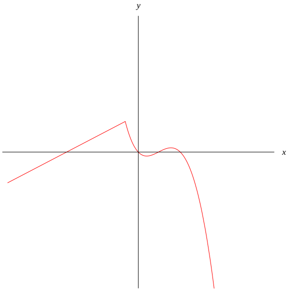


Not a function



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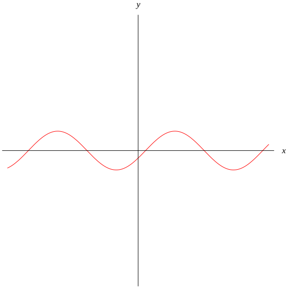

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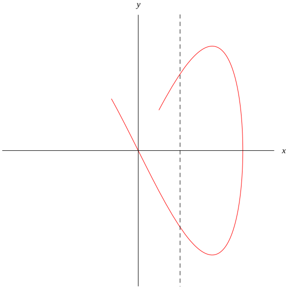

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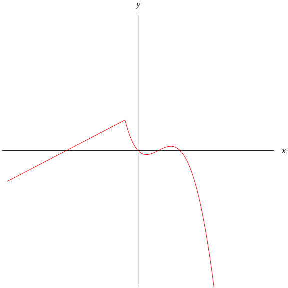

Function

Piecewise Defined Functions

Definition (Piecewise Defined Function)

A piecewise defined function is a function that is defined by possibly different formulas on different subsets of its domain.

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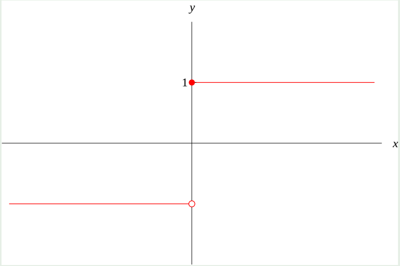
Example



$$f(x) = \begin{cases} 1 & \text{if} \quad x \geq 0 \\ -1 & \text{if} \quad x < 0 \end{cases}$$

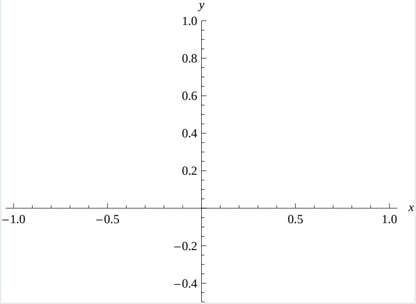
The filled red circle means $(0, 1)$ is on the curve.
The open circle means $(0, -1)$ is not on the curve.

Example (Example 8, p. 18)

The absolute value $|a|$ of a number a is defined to be

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0. \end{cases}$$

Sketch a graph of the function $f(x) = |x|$.



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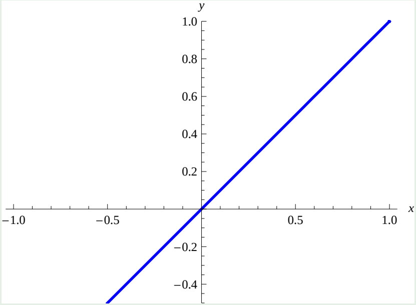

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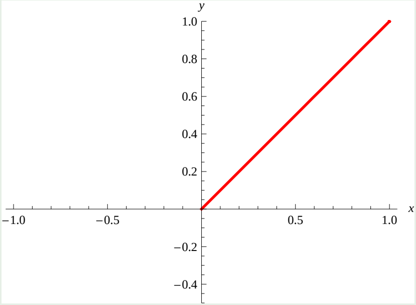

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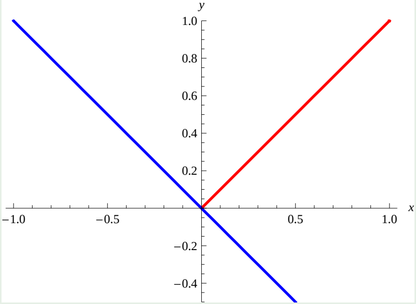

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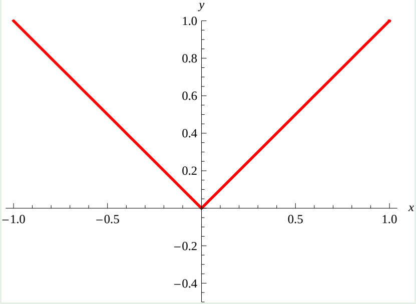

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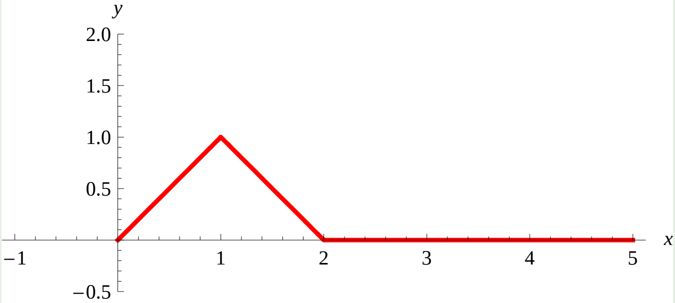
Find a formula for the function f in the graph.



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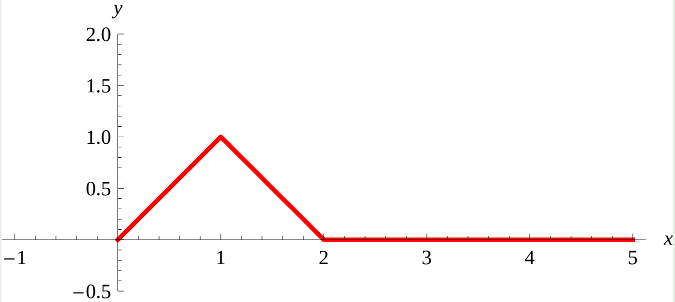

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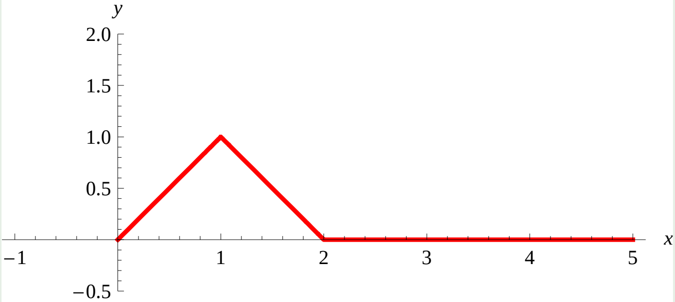

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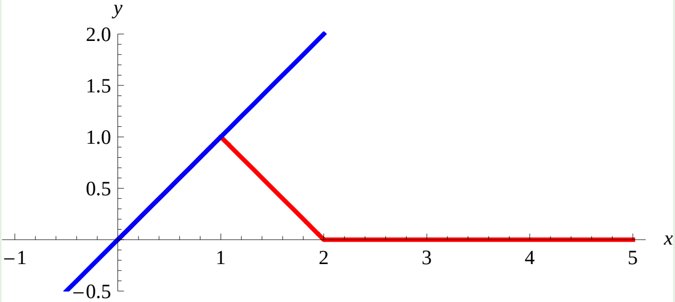

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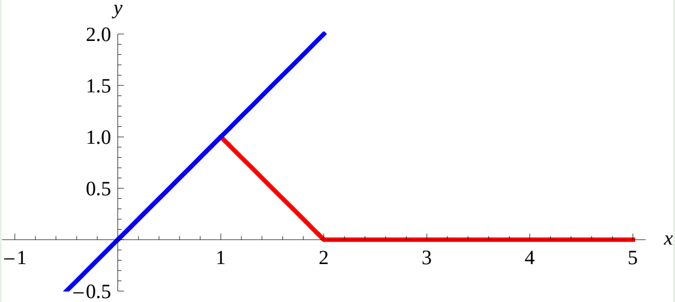

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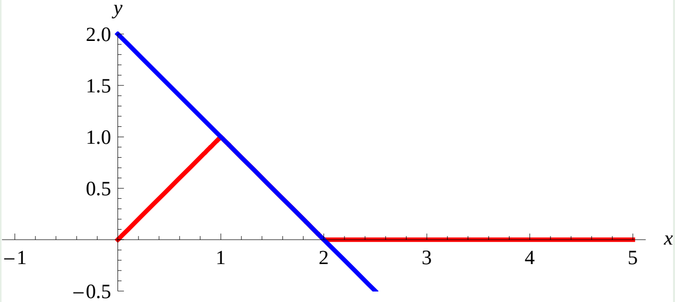

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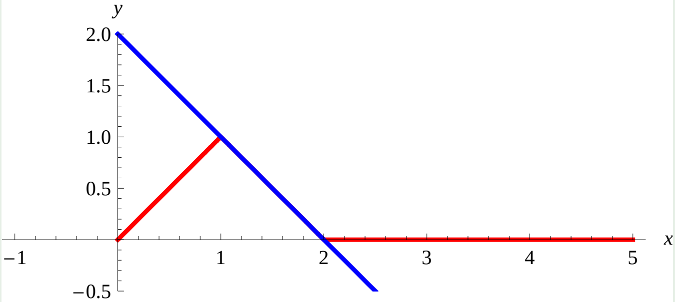

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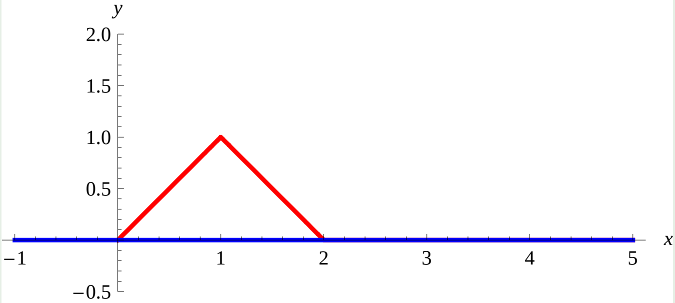

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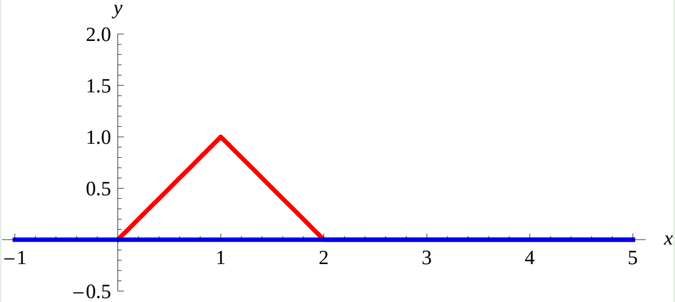

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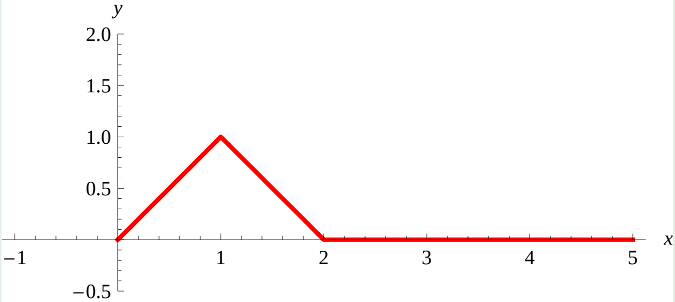

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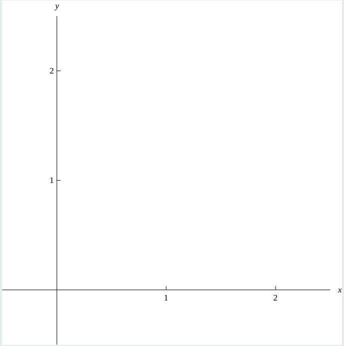

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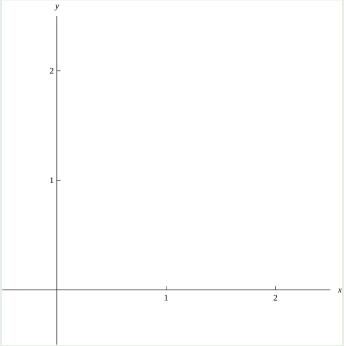

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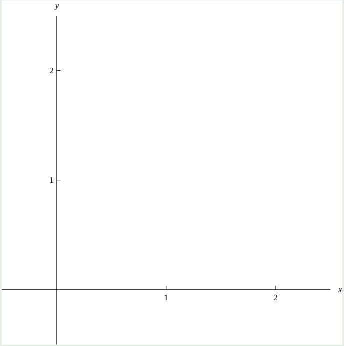

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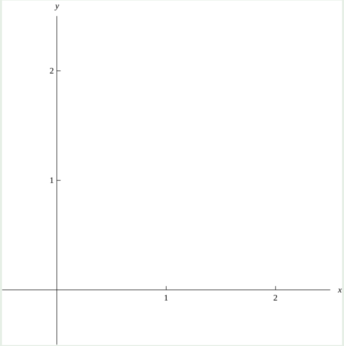

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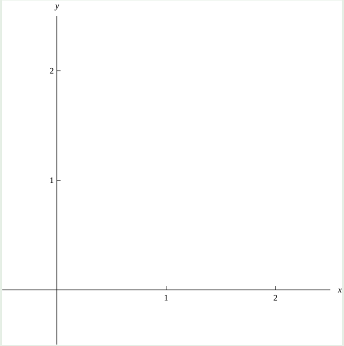

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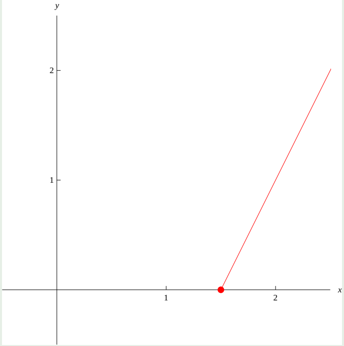

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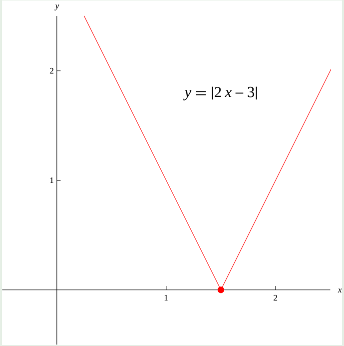

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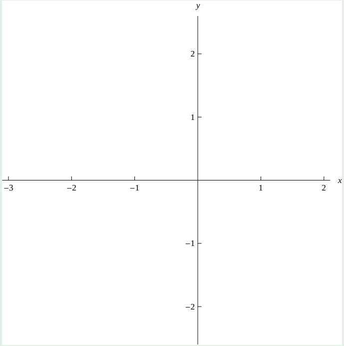

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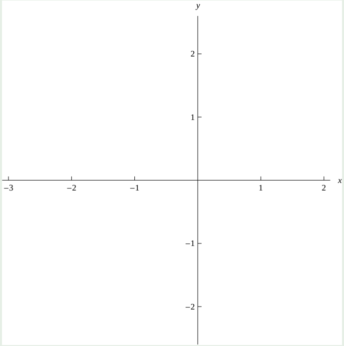

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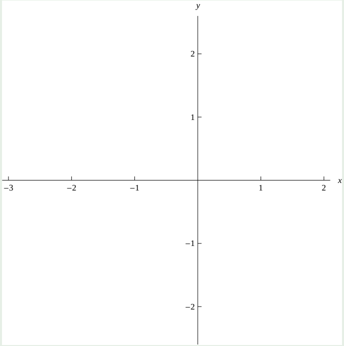

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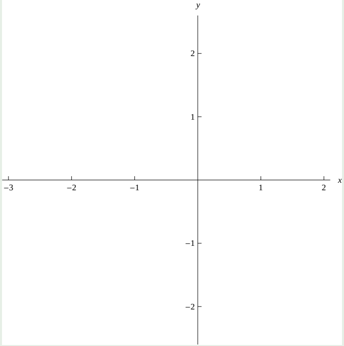

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$$\frac{|4x+2|}{2x+1} = \begin{cases} \frac{\color{red}{4x+2}}{2x+1} & \text{if } 4x+2 > 0 \\ \frac{-(4x+2)}{2x+1} & \text{if } 4x+2 < 0 \end{cases}$$

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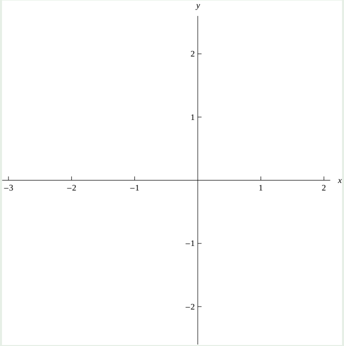

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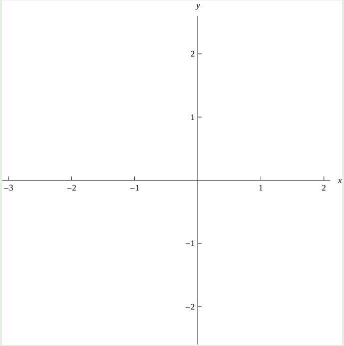
Sketch the function $f(x) = \dfrac{|4x+2|}{2x+1}$.



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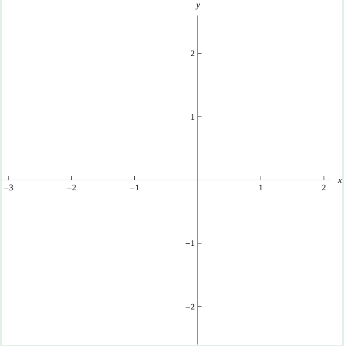

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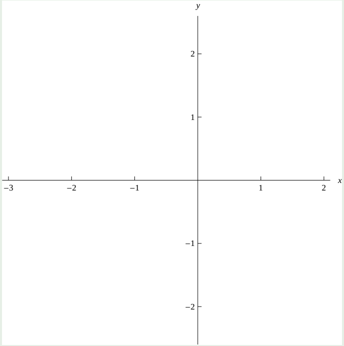

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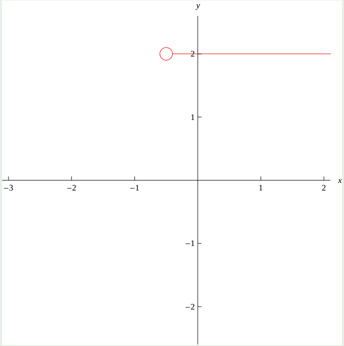

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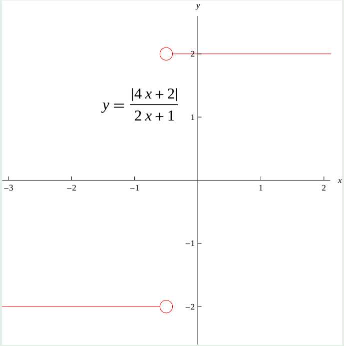

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Therefore f is odd.

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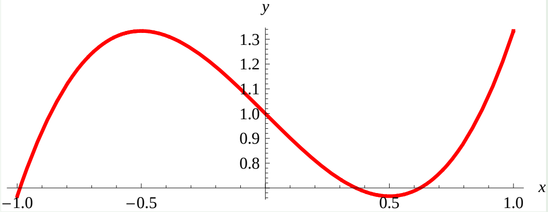

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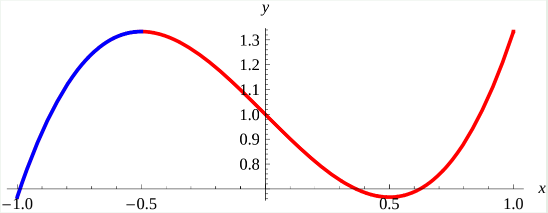

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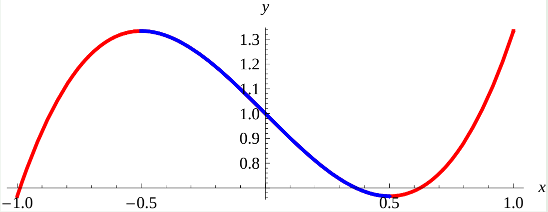

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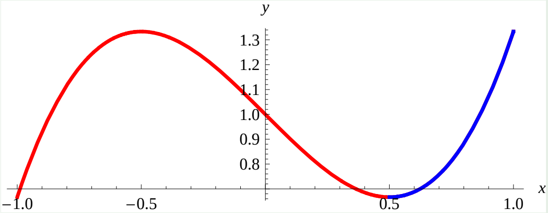

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$$g(x) = \frac{x^2 - 9}{x^2 - x - 6}$$

- Any risk of dividing by 0?

Example (Two Functions and Their Domains)

Find the implied domains of the following two functions:

$$f(x) = \sqrt[4]{x-2} + \sqrt[3]{6-x}$$

- Any risk of dividing by 0? No.
- Any risk of taking the even root of a negative number? Yes.
- $x - 2$ must not be negative.

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$$x^2 - x - 6 \neq 0$$

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$$\begin{aligned} x^2-x-6 &\neq 0 \\ (x-3)(x+2) &\neq 0 \\ x &\neq 3 \text{ or } -2 \end{aligned}$$

Domain is all real numbers except 3 and -2; that is, $(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$.

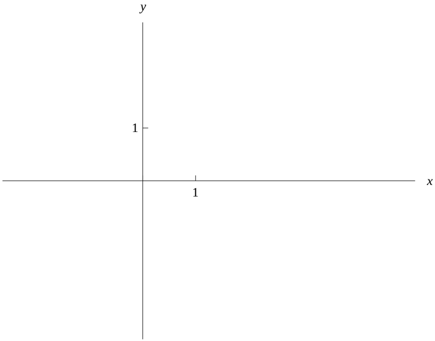
Linear Functions

Definition (Linear Function)

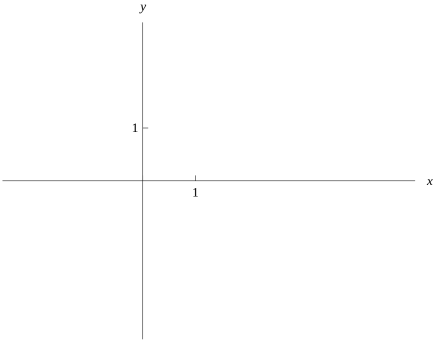
A linear function is a function the graph of which is a line. We can write any linear function in slope-intercept form:

$$f(x) = mx + b.$$

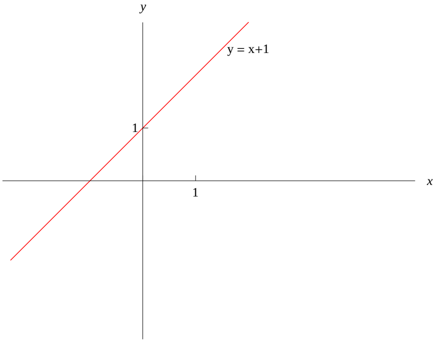
m is called the slope, and b is called the y-intercept.



$f(x)$	Direction	y-intercept

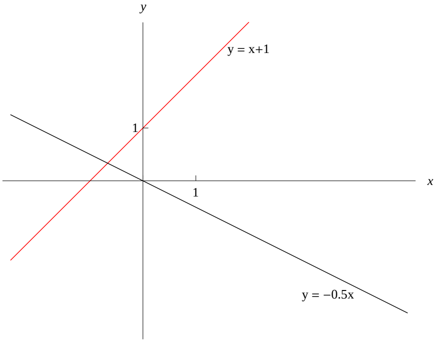


$f(x)$	Direction	y-intercept



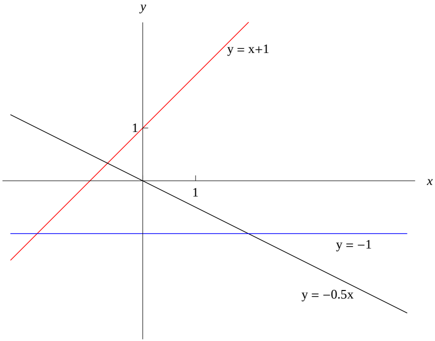
$f(x)$	Direction	y-intercept
$x+1$	$\nearrow$	

- $m > 0$ means the graph of f points up ($\nearrow$).



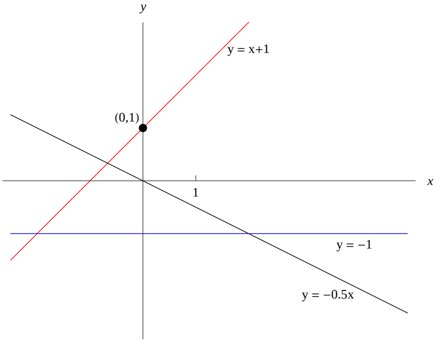
$f(x)$	Direction	y-intercept
$x+1$	$\nearrow$	
$-0.5x$	$\searrow$	

- $m > 0$ means the graph of f points up ($\nearrow$).
- $m < 0$ means the graph of f points down ($\searrow$).



$f(x)$	Direction	y-intercept
$x+1$	$\nearrow$	
$-0.5x$	$\searrow$	
-1	$\rightarrow$	

- $m > 0$ means the graph of f points up ($\nearrow$).
- $m < 0$ means the graph of f points down ($\searrow$).
- $m = 0$ means the graph of f is horizontal ($\rightarrow$).



$f(x)$	Direction	y-intercept
$x + 1$	$\nearrow$	1
$-0.5x$	$\searrow$	
-1	$\rightarrow$	

- $m > 0$ means the graph of f points up ($\nearrow$).
- $m < 0$ means the graph of f points down ($\searrow$).
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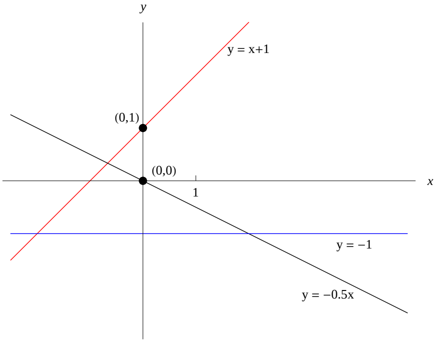
- b tells us the height of the point where the graph hits the y-axis.



$f(x)$	Direction	y-intercept
$x+1$	$\nearrow$	1
$-0.5x + 0$	$\searrow$	0
-1	$\rightarrow$	

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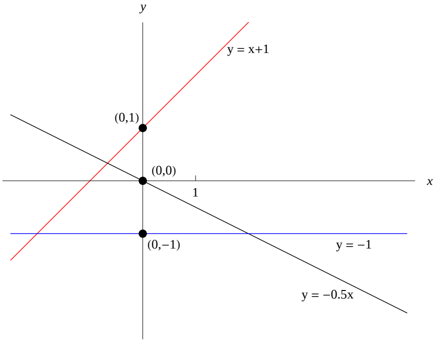

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Polynomials

Definition (Polynomial Function)

A polynomial function is a function f of the form

$$f(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_{n-1} x^{n-1} + a_n x^n,$$

where n is a non-negative integer and $a_0, \ldots, a_n$ are real numbers, called the coefficients. If $a_n \neq 0$ the integer n is called the degree of f.

If we interpret x as an indeterminate formal expression, rather than a number, we say that $f(x)$ is a polynomial (rather than a polynomial function).

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$f(x)$	Polynomial?	Degree	a_0	a_1	a_2
$x^4 - x + 1$					
6					
$3x^2 - \frac{1}{2}x + \sqrt{x}$					
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$f(x)$	Polynomial?	Degree	a_0	a_1	a_2
$x^4 - x + 1$	Yes	4			
6					
$3x^2 - \frac{1}{2}x + \sqrt{x}$					
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Polynomials

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6	Yes	0	6	0	
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$3x^2 - \frac{1}{2}x + \sqrt{2}$	Yes	2	$\sqrt{2}$		
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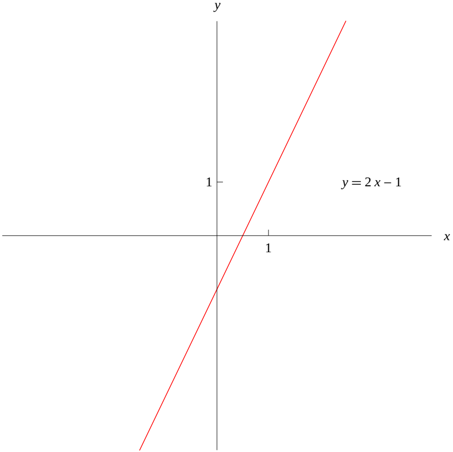
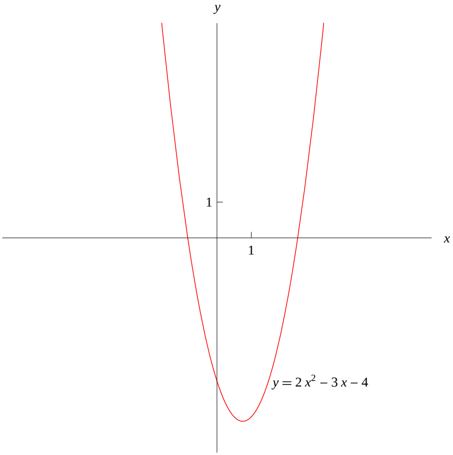
$3x^2 - \frac{1}{2x} + \sqrt{2}$	No				

- Linear functions are polynomial (functions).



Linear

- Linear functions are polynomial (functions).
- So are quadratic functions. Their graphs are parabolas.



Quadratic

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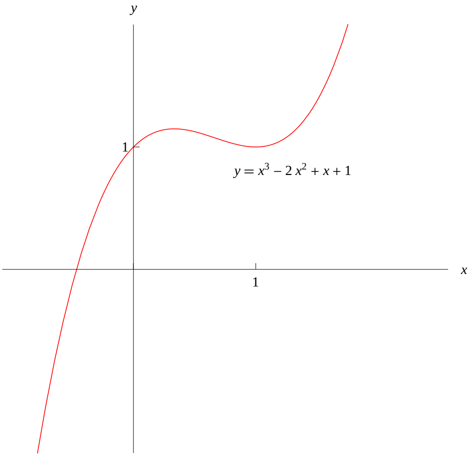
- So are quadratic functions. Their graphs are parabolas.
- And there are many more.



Cubic

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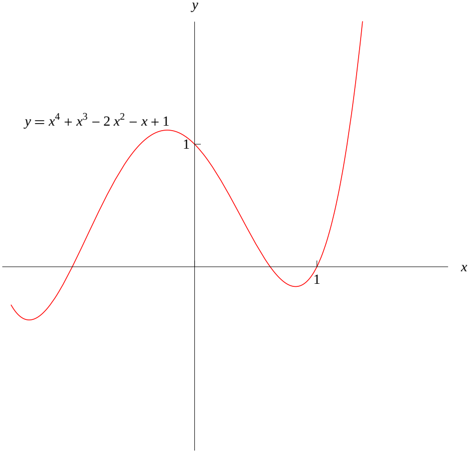

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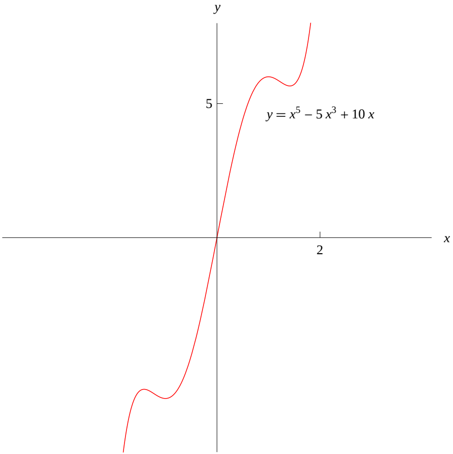

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