Math 140

Lecture 2

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with modifications by T. Milev

University of Massachusetts Boston

January 31

Outline

1. (1.2) A Catalog of Essential Functions
   - Power Functions
   - Rational Functions
   - Algebraic Functions
   - Transcendental Functions

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   - Power Functions
   - Rational Functions
   - Algebraic Functions
   - Transcendental Functions

2. (1.3) New Functions from Old Functions
   - Transformations of Functions
   - Combinations of Functions

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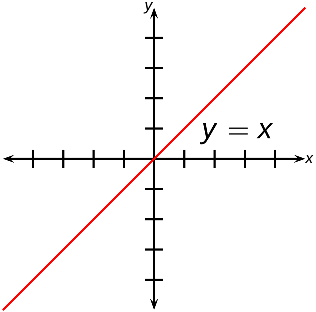

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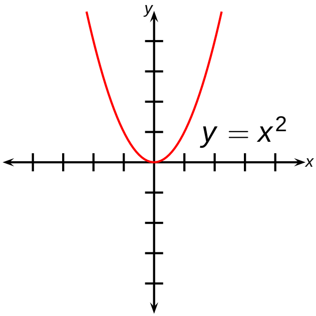

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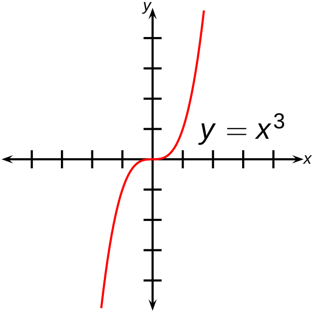

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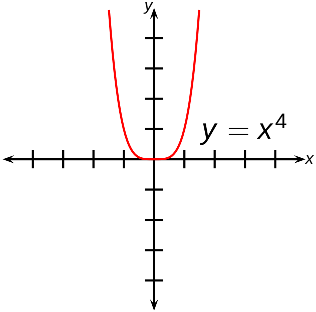

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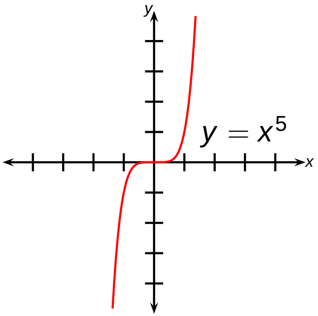

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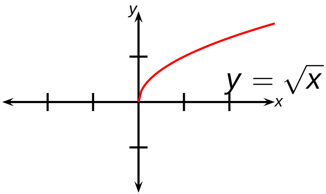

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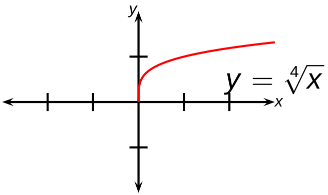

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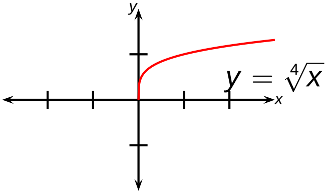

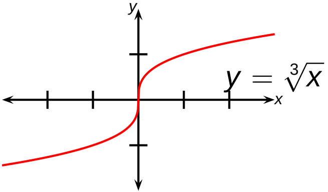

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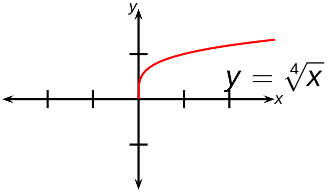
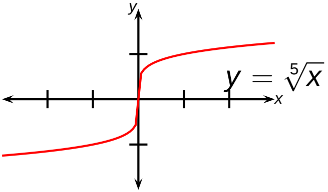
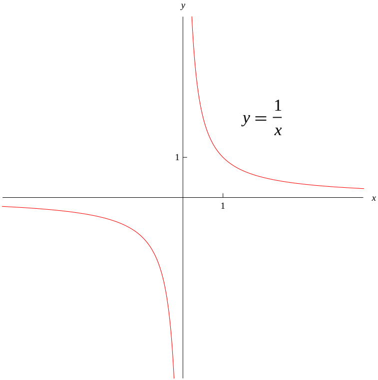




$f(x) = x^{-1} = \frac{1}{x}$ is called the reciprocal function. Its graph has equation $y = \frac{1}{x}$, or $xy = 1$, and is an hyperbola with the coordinate axes as its asymptotes.



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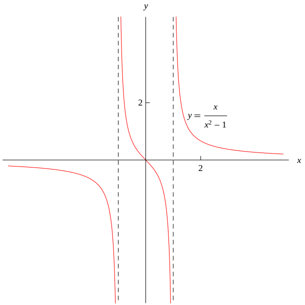

Example ($x/(x^2 - 1)$)

The function

$$f(x) = \frac{x}{x^2 - 1}$$

is a rational function.

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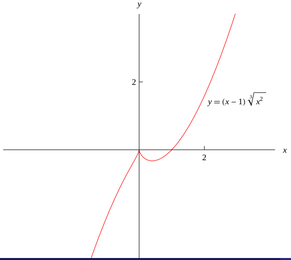
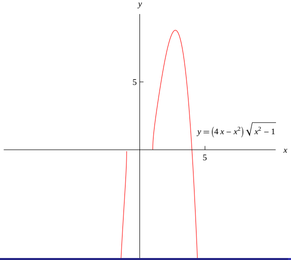
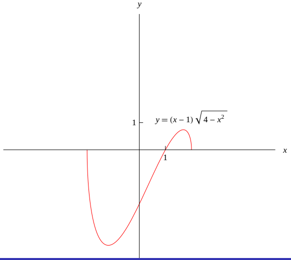
Examples.







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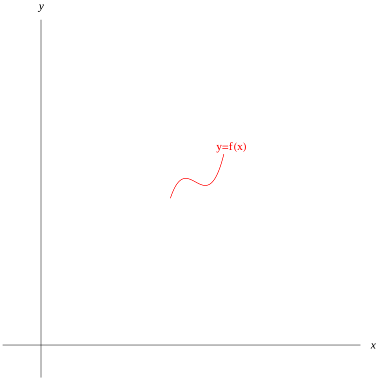
- Outside of Calculus I: by definition, a function is transcendental if it is not algebraic, i.e., if it satisfies no polynomial equation with polynomial coefficients.

Transformations of Functions



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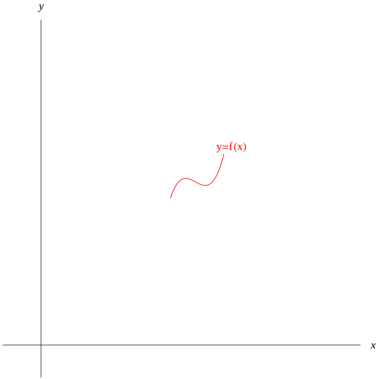

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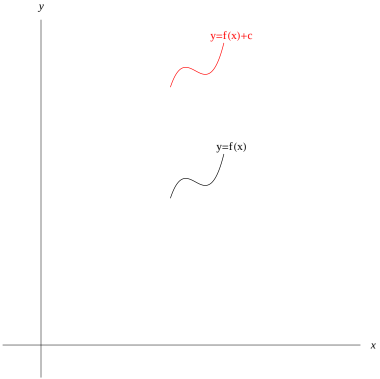

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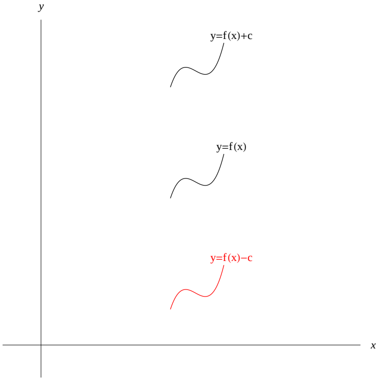

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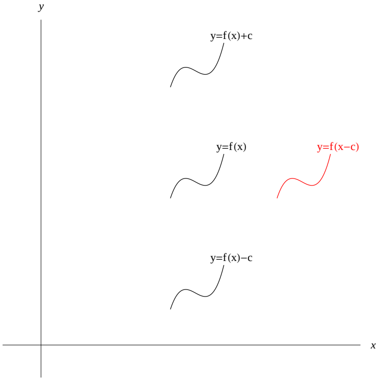

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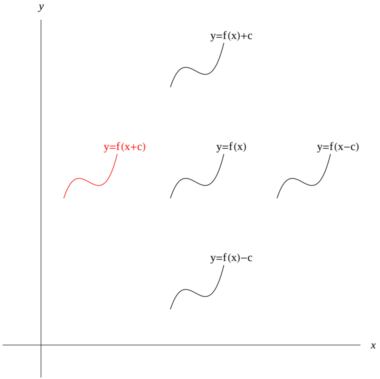

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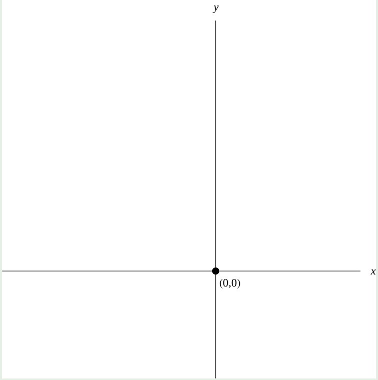
Example (Example 2, p. 39)

Draw a graph of the function $f(x) = x^2 + 6x + 10$.



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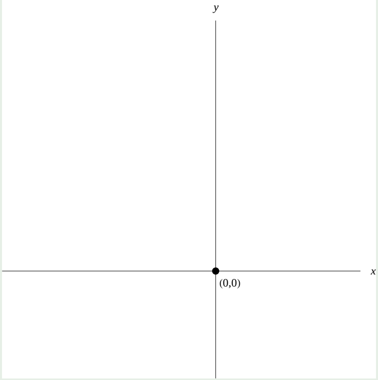

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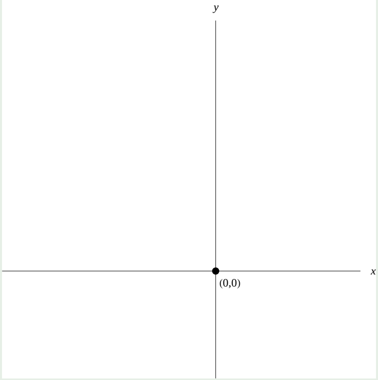

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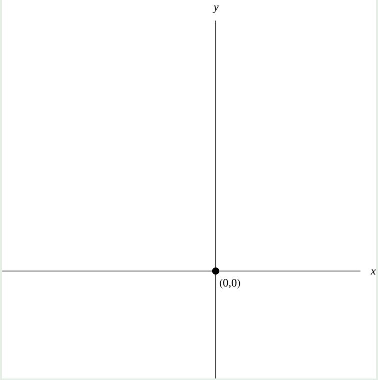

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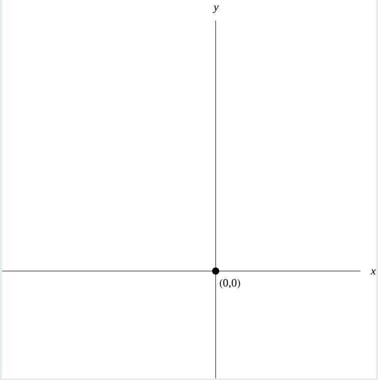

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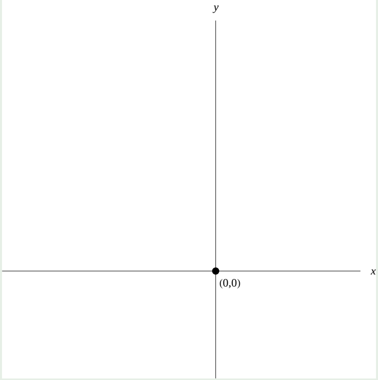

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$$\begin{aligned} f(x) &= x^2 + 6x + 10 \\ &= (x^2 + 6x + 9) + 10 - 9 \\ &= (x + 3)^2 + 1 \end{aligned}$$

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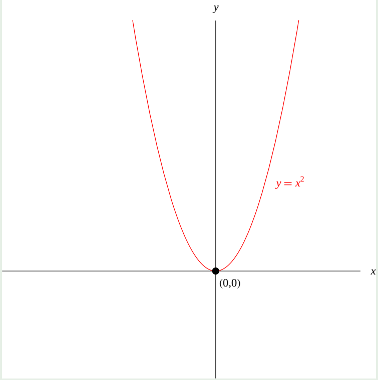
Draw a graph of the function $f(x) = x^2 + 6x + 10$.



Complete the square:

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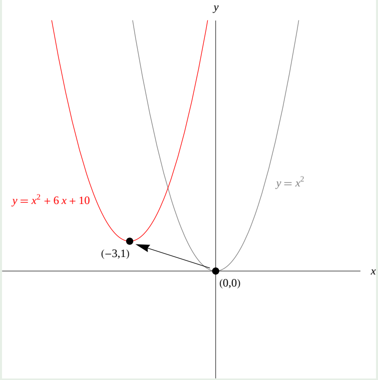

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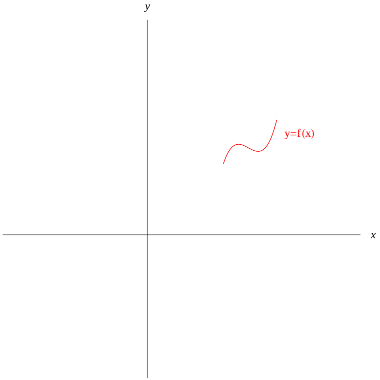
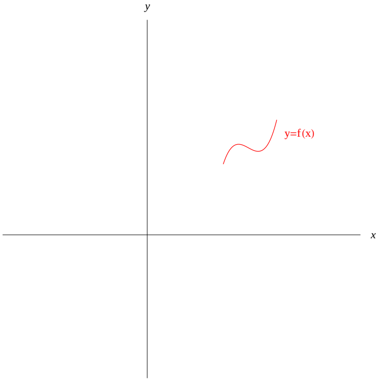


What happens if we multiply or divide by a constant $c > 1$ in the equation of a function f? What happens if we multiply f by -1? What happens if we multiply x by -1 before applying f?



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$cf(x)$	
$(1/c)f(x)$	
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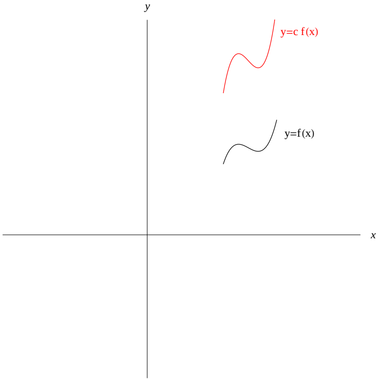
$f(-x)$	



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$cf(x)$	Stretch the graph of $f(x)$ vertically by a factor of c.
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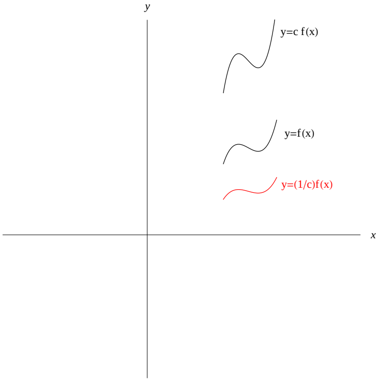

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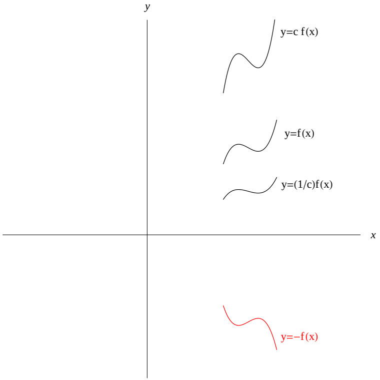

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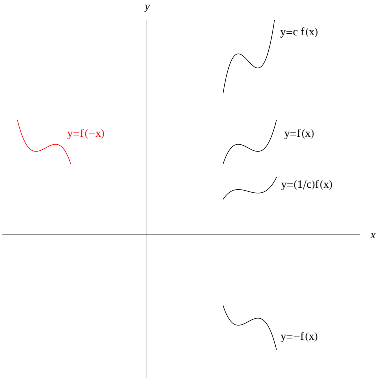

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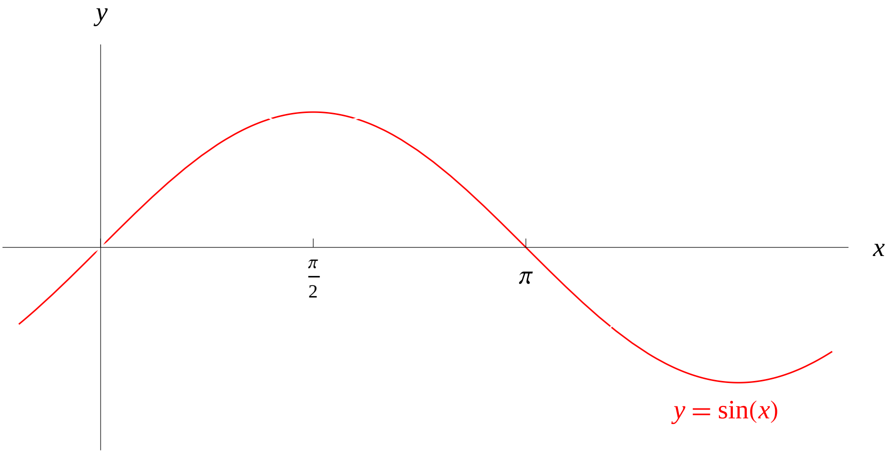

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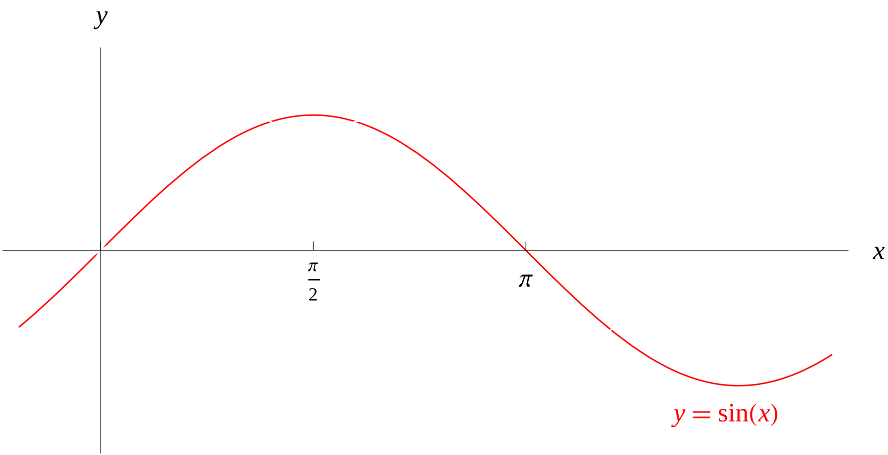

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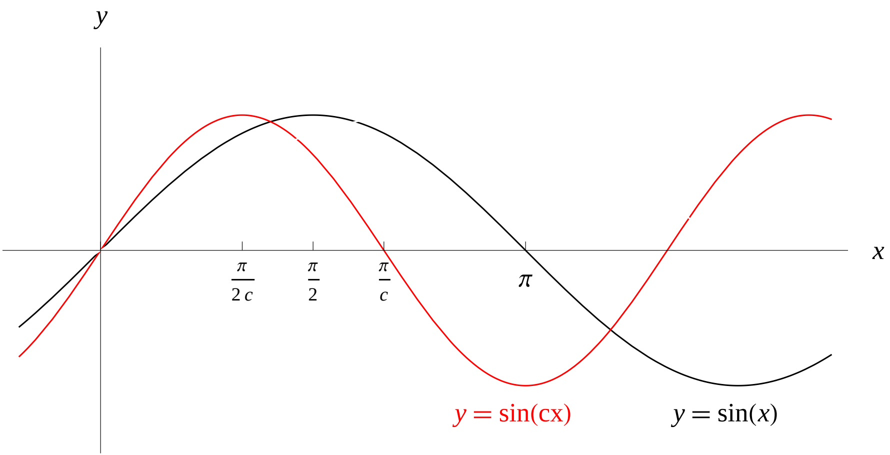

What happens if we multiply or divide x by a constant $c > 1$ before applying f?

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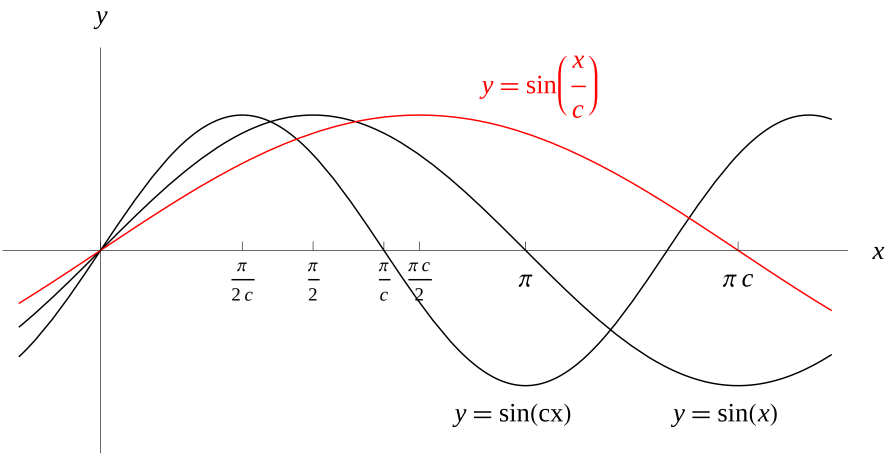
$f((1/c)x)$	



What happens if we multiply or divide x by a constant $c > 1$ before applying f?

$f(cx)$	Compress the graph of $f(x)$ horizontally by a factor of c.
$f((1/c)x)$	Stretch the graph of $f(x)$ horizontally by a factor of c.

What happens when we take the absolute value of a function?

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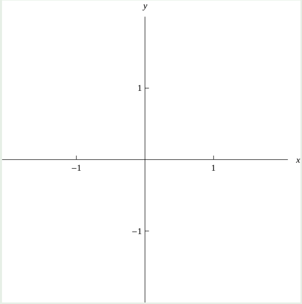
Example (Example 5, p. 41)

Draw the graph of the function $f(x) = |x^2 - 1|$.



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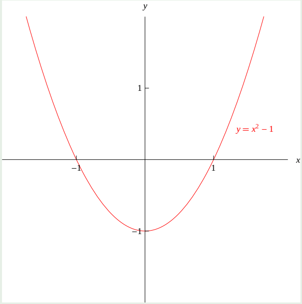

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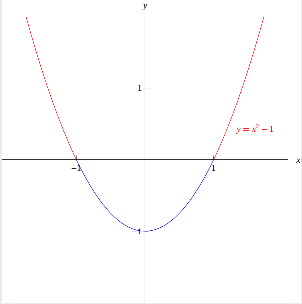

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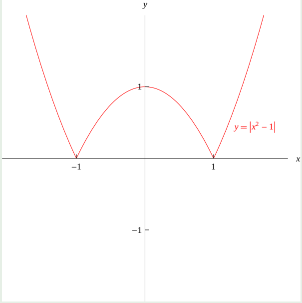


- Draw the graph of $f(x) = x^2 - 1$.
- Identify the part(s) below the x-axis.
- Flip those parts over the x-axis.

Combinations of Functions

Two functions f and g can be combined to form new functions $f+g$, $f-g$, fg, and f/g. The sum and difference functions are defined by the formulas

$$(f+g)(x) = f(x) + g(x), \qquad (f-g)(x) = f(x) - g(x).$$

If A is the domain of f and B is the domain of g, then the domain of $f+g$ and $f-g$ is $A \cap B$, the intersection of A and B.
The product and quotient functions are defined by the formulas

$$(fg)(x) = f(x)g(x), \qquad \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}.$$

These functions also have the domain $A \cap B$, with one exception: in the quotient function, we aren't allowed to divide by 0, so we must exclude those values of x that make $g(x) = 0$. We write this domain as

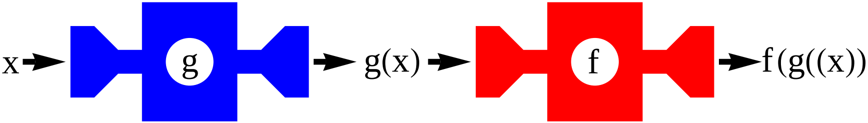
$$\{x \in A \cap B |\ g(x) \neq 0\}.$$

Definition (Composition of f and g)

If f and g are two functions, then the composition of f and g is written $f \circ g$ and is defined by the formula

$$(f \circ g)(x) = f(g(x)).$$

Imagine f and g as machines taking some input and producing some output. Then $f \circ g$ corresponds to attaching both machines end-to-end so that the output of g becomes the input of f.



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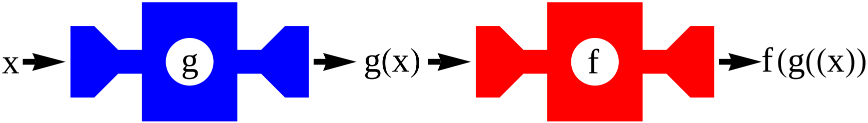

The domain of $f \circ g$ is the set of all numbers x in the domain of g such that $g(x)$ is in the domain of f. If the domain of f is A and the domain of g is B, we write this as

$$\{x \in B |\ g(x) \in A\}.$$

Example (Example 7, p. 42)

If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{2 - x}$, find each function and its domain.

$f \circ g$ $\qquad$ $g \circ f$ $\qquad$ $g \circ g$

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