

Review problems

To be tested March 7

Problem 1 Find an expression for the function $(f \circ g)(x)$ and $(g \circ f)(x)$. Simplify your answer to a single fraction.

1. $f(x) = \frac{3x-5}{x-2}$, $g(y) = \frac{y-2}{y-4}$. Answer. $(f \circ g)(x) = \frac{-2x+14}{-x+6}$, $(g \circ f)(x) = \frac{x-1}{-x+3}$.
2. $f(x) = \frac{x-3}{x+2}$, $g(y) = \frac{y+3}{y-4}$. Answer. $(f \circ g)(x) = \frac{-2(x)+15}{3x-5}$, $(g \circ f)(x) = \frac{4x+3}{-3x-11}$.

Problem 2 Find all solutions in the interval $[0, 2\pi]$ of the equation.

1. $\sqrt{2} \cos x = \sin(2x)$. Answer $x = \frac{\pi}{4}, \frac{3\pi}{4}, \pi, 2\pi$.
2. $\sqrt{3} \sin x = \sin(2x)$. Answer. $x = \frac{\pi}{6}, \frac{11\pi}{6}, 0, \pi, 2\pi$.

Problem 3 Select one problem from the following. Evaluate the limit if it exists.

1. $\lim_{x \rightarrow 1} \frac{3x^2+4x-7}{x^3-x}$ Answer. 5.
2. $\lim_{x \rightarrow -1} \frac{2x^2-3x-5}{x^3-x}$ Answer. $-\frac{7}{2}$.

Problem 4 Evaluate the limit if it exists.

1. $\lim_{x \rightarrow 3^+} \frac{\sqrt{\frac{x^2}{9}-1}}{2x^2-3x-9}$. Answer. ∞ .
2. $\lim_{x \rightarrow -2^-} \frac{\sqrt{\frac{x^2}{4}-1}}{2x^2+3x-2}$ Answer. ∞ .

Problem 5 Use the intermediate value theorem to show that the equation has a solution in the interval $(-1, 1)$.

1. $e^{2x} - 2 = \sin x$.
2. $e^{2x} - 2 = \cos x$.

Problem 6 Solve the equation.

1. $e^{4x} + 3e^{2x} - 4 = 0$. Answer: $x = 0$.
2. $e^{6x} + 4e^{3x} - 5 = 0$. Answer: $x = 0$.

Problem 7 Find the inverse function f^{-1} . Plot the function $f(x)$. Plot the function $f^{-1}(x)$.

1. $f(x) = x^2 + 2x - 2$, $x \geq -1$.
2. $f(x) = x^2 + x - 2$, $x \geq -\frac{1}{2}$.

Problem 8 Compute the limit.

1. $\lim_{x \rightarrow -\infty} \sqrt{x^2 + x} - \sqrt{x^2 - x}$. Answer -1.
2. $\lim_{x \rightarrow \infty} \sqrt{x^2 + 2x} - \sqrt{x^2 - 2x}$. Answer 2.

Problem 9 Compute the limit.

1. $\lim_{x \rightarrow 1} \frac{\sin(\frac{\pi x}{2})}{(2x^2-x-1)(x-2)(x+2)}$. Answer DNE.
2. $\lim_{x \rightarrow 1} \frac{\cos(\pi x)}{(3x^2-2x-1)(x-2)(x+2)}$. Answer DNE.