Math 140
Lecture 11

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Outline

Theorem (The Power Rule (General Version))

If n is any real number, then

$$\frac{d}{dx}(x^n) = nx^{n-1}.$$

Example (Power Rule, negative exponent)

Differentiate $y = \frac{1}{x}$.

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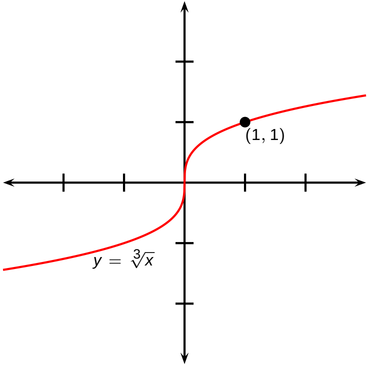
$$y = x^{-1}.$$

$$\text{Power Rule:} \quad \frac{\mathrm{d}y}{\mathrm{d}x} = (-1)x^{-2}$$

$$= -\frac{1}{x^2}.$$

Example (Calculating the tangent line using the Power Rule)

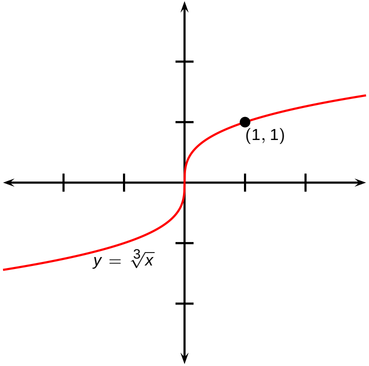
Find an equation for the tangent line to the parabola $y = \sqrt[3]{x}$ at the point $P = (1, 1)$.



Example (Calculating the tangent line using the Power Rule)

Find an equation for the tangent line to the parabola $y = \sqrt[3]{x}$ at the point $P = (1, 1)$.

Here $a = 1$ and $f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$.



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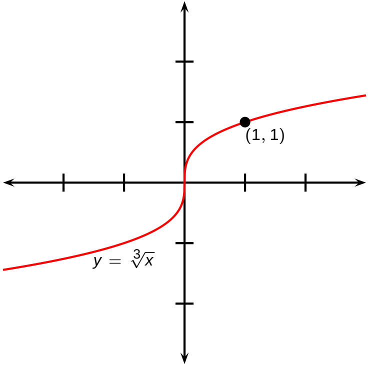

Here $a = 1$ and $f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$.

$$f'(x) = \frac{1}{3}x^{\frac{1}{3}-1}$$

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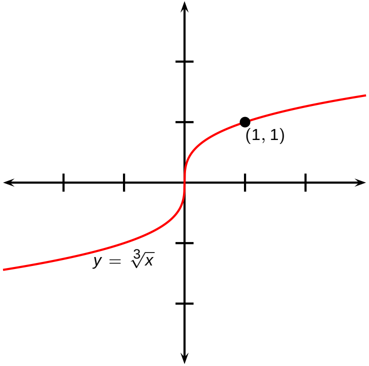

$$f'(x) = \frac{1}{3}x^{\frac{1}{3}-1}$$
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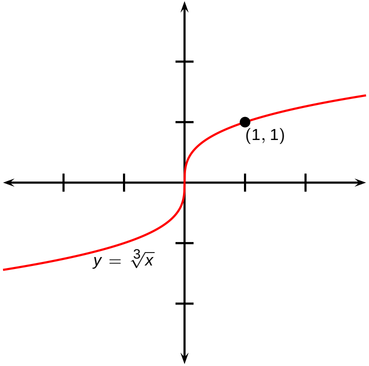

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$$\begin{aligned} f'(x) &= \frac{1}{3}x^{\frac{1}{3}-1} \\ &= \frac{1}{3}x^{\frac{-2}{3}} \\ &= \frac{1}{3\sqrt[3]{x^2}}. \end{aligned}$$

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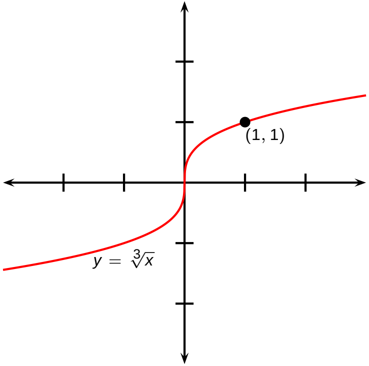

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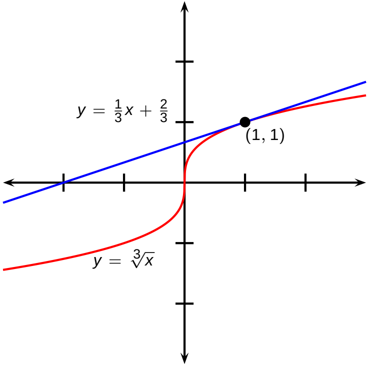

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Point-slope form: $y - 1 = \frac{1}{3}(x - 1)$, or $y = \frac{1}{3}x + \frac{2}{3}$.

Theorem (The Constant Multiple Rule)

If c is a constant and f is a differentiable function, then

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x).$$

Proof.

□

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$$= \lim_{h\to 0}\frac{c(f(x+h)-f(x))}{h}$$

$$\text{Limit Law 3:} \quad = c\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

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&= c
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Example (Constant Multiple Rule, Power Rule)

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$$= \frac{10x^4}{7}.$$

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Constant Multiple Rule: $= (2\pi)\frac{d}{dx}\left(x^{-4}\right)$

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$$= -\frac{8\pi}{x^5}.$$

Theorem (The Sum Rule)

If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x).$$

Proof.

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Then $$F'(x) = \lim_{h\to 0}\frac{F(x+h)-F(x)}{h}$$
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\text{Let} \quad F(x) &= f(x)+g(x). \\
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&= \lim_{h\to 0}\frac{[f(x+h)+g(x+h)]-[f(x)+g(x)]}{h} \\
&= \lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h}+\frac{g(x+h)-g(x)}{h}\right] \\
\text{Limit Law 1:} \quad &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}+\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}
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&= \lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h}+\frac{g(x+h)-g(x)}{h}\right] \\
\text{Limit Law 1:} \quad &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} + \lim_{h\to 0}\frac{g(x+h)-g(x)}{h} \\
&= f'(x)+g'(x).
\end{aligned}$$

□

The Sum Rule can be extended to any number of summands. For instance, using the theorem twice, we get

$$(f+g+h)' = [(f+g)+h]' = (f+g)'+h' = f'+g'+h'.$$

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$$(f + g + h)' = [(f + g) + h]' = (f + g)' + h' = f' + g' + h'.$$

By writing $f - g$ as $f + (-1)g$ and applying the Sum Rule and the Constant Multiple Rule, we get

Theorem (The Difference Rule)

If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x).$$

The Constant Multiple Rule, the Sum Rule, the Difference Rule, and the Power Rule can be combined to differentiate any polynomial.

Example (Derivative of a Polynomial)

If $y = x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5,$

Then $\frac{dy}{dx} =$

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Then
$$\frac{dy}{dx} = \frac{d}{dx}\left(x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5\right)$$
$$= \frac{d}{dx}\left(x^{16}\right) + \frac{d}{dx}\left(2\sqrt{3}x^7\right) - \frac{d}{dx}\left(4x^3\right) + \frac{d}{dx}\left(\frac{x}{8}\right) - \frac{d}{dx}(5)$$

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\text{Then} \quad \frac{dy}{dx} &= \frac{d}{dx}\left(x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5\right) \\
&= \frac{d}{dx}\left(x^{16}\right) + \frac{d}{dx}\left(2\sqrt{3}x^7\right) - \frac{d}{dx}\left(4x^3\right) + \frac{d}{dx}\left(\frac{x}{8}\right) - \frac{d}{dx}(5) \\
&= \frac{d}{dx}\left(x^{16}\right) + 2\sqrt{3}\frac{d}{dx}\left(x^7\right) - 4\frac{d}{dx}\left(x^3\right) + \frac{1}{8}\frac{d}{dx}(x) - \frac{d}{dx}(5)
\end{aligned}$$

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Then
$$\begin{aligned}
\frac{dy}{dx} &= \frac{d}{dx}\left(x^{16} + 2\sqrt{3}x^7 - 4x^3 + \frac{x}{8} - 5\right) \\
&= \frac{d}{dx}\left(x^{16}\right) + \frac{d}{dx}\left(2\sqrt{3}x^7\right) - \frac{d}{dx}\left(4x^3\right) + \frac{d}{dx}\left(\frac{x}{8}\right) - \frac{d}{dx}(5) \\
&= \frac{d}{dx}\left(x^{16}\right) + 2\sqrt{3}\frac{d}{dx}\left(x^7\right) - 4\frac{d}{dx}\left(x^3\right) + \frac{1}{8}\frac{d}{dx}(x) - \frac{d}{dx}(5) \\
&= (\qquad) + 2\sqrt{3}\left(\qquad\right) - 4\left(\qquad\right) + \frac{1}{8}(\ ) - (\ )
\end{aligned}$$

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Differentiate $$v = \frac{3\sqrt{x} - \sqrt[3]{x}}{x}.$$

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We have shown that, if $f(x) = a^x$ is differentiable at 0, then it is differentiable everywhere, and

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$$\lim_{h\to 0} \frac{2^h - 1}{h} \approx 0.693147, \qquad \lim_{h\to 0} \frac{3^h - 1}{h} \approx 1.098612.$$

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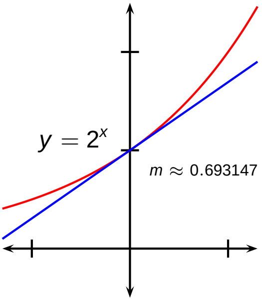
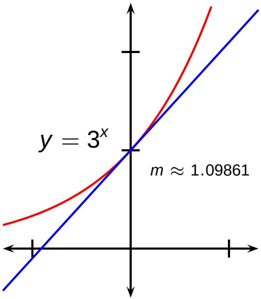
$$\frac{\mathrm{d}}{\mathrm{d}x}(2^x) \approx (0.69)2^x, \qquad \frac{\mathrm{d}}{\mathrm{d}x}(3^x) \approx (1.10)3^x.$$

$$\text{If} \quad f(x) = a^x, \quad \text{then} \quad f'(x) = f'(0)a^x.$$

The simplest differential formula occurs when $f'(0) = 1$. Since $\lim_{h\to 0} \frac{2^h-1}{h} \approx 0.69$ and $\lim_{h\to 0} \frac{3^h-1}{h} \approx 1.10$, we expect there is a number a between 2 and 3 such that $\lim_{h\to 0} \frac{a^h-1}{h} = 1$.





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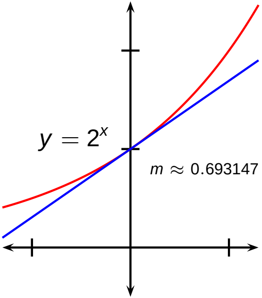
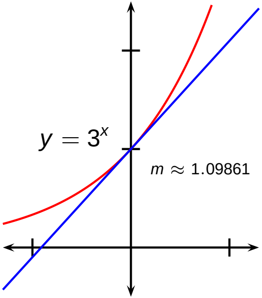
Definition (e)

e is the number such that $\lim_{h\to 0} \frac{e^h-1}{h} = 1$.





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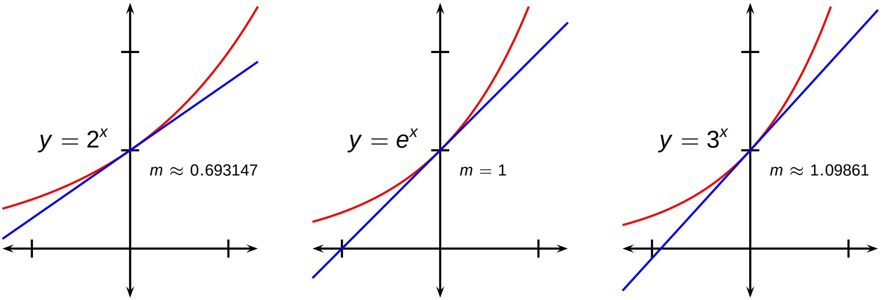

Definition (Natural Exponential Function)

e^x is called the natural exponential function. Its derivative is

$$\frac{\mathrm{d}}{\mathrm{d}x}e^x = e^x.$$

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Differentiate $y = e^x + x^7.$

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