Math 140

Lecture 14

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with modifications by T. Milev

University of Massachusetts Boston

March 28, 2013

Outline

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Implicit Differentiation

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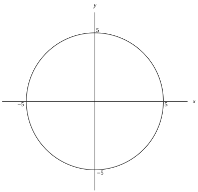

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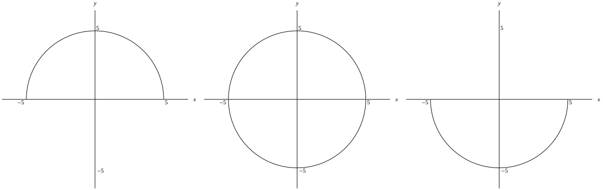

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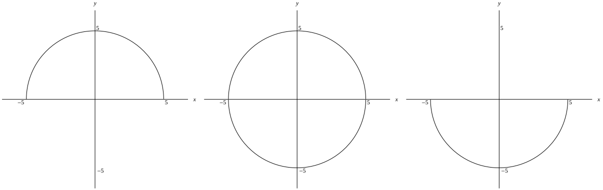

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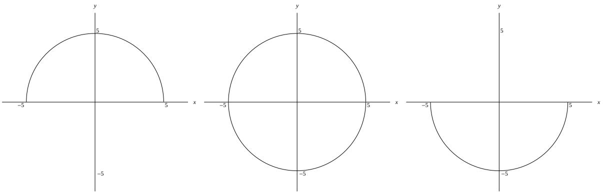
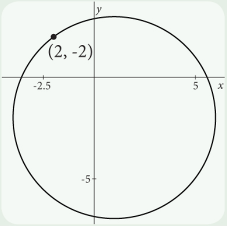
- Differentiate both sides with respect to x, and then solve for y'.



Example

Find an equation of the tangent line to $(x-1)^2 + (y+2)^2 = 25$ at $(-2, 2)$.



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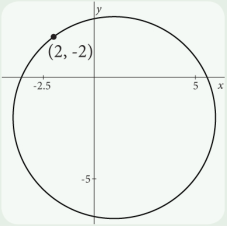

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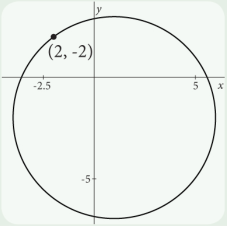

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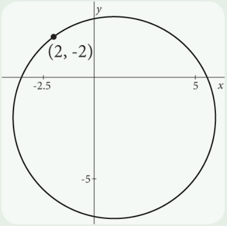

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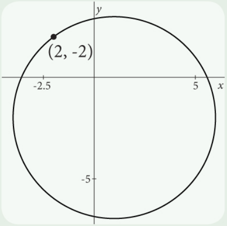

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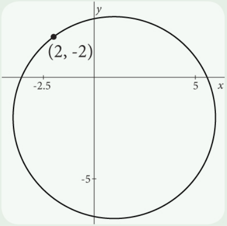

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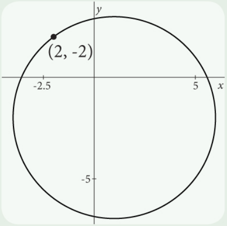

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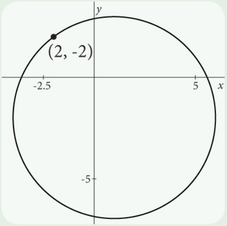

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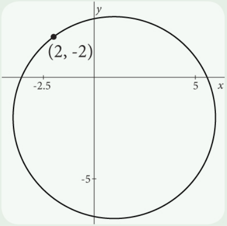

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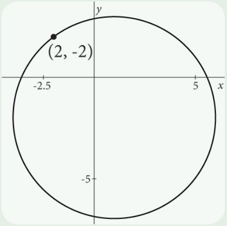

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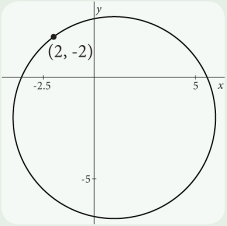

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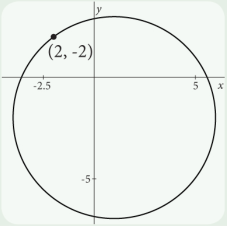

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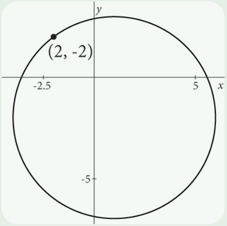

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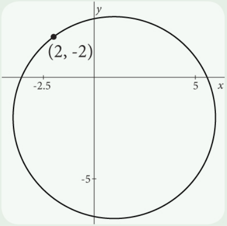

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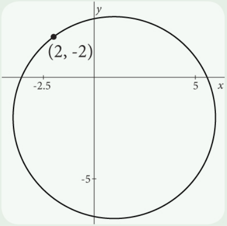

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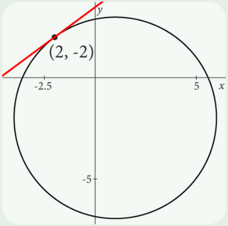

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Point-slope form:

$$y-2 = \frac{3}{4}(x+2)$$

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Factor: $y'(\cos(x+y) - 2y\cos x) = -y^2\sin x - \cos(x+y)$

Example

$$
\begin{aligned}
\text{Find } y' \text{ if } \sin(x+y) &= y^2 \cos x. \\
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y'\cos(x+y) - 2yy'\cos x &= -y^2 \sin x - \cos(x+y) \\
\text{Factor:} \quad y'(\cos(x+y) - 2y\cos x) &= -y^2 \sin x - \cos(x+y) \\
y' &= \frac{-y^2 \sin x - \cos(x+y)}{\cos(x+y) - 2y\cos x}.
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$$y'' = \frac{\mathrm{d}}{\mathrm{d}x}\left(-\frac{x^3}{y^3}\right)$$

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Find y'' if $x^4 + y^4 = 16$.

$$4x^3 + 4y^3y' = 0$$

$$y' = -\frac{x^3}{y^3}.$$

$$y'' = \frac{\mathrm{d}}{\mathrm{d}x}\left(-\frac{x^3}{y^3}\right) = -\frac{y^3\frac{\mathrm{d}}{\mathrm{d}x}(x^3) - x^3\frac{\mathrm{d}}{\mathrm{d}x}(y^3)}{(y^3)^2}$$

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Find y'' if $x^4 + y^4 = 16$.

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Derivatives of Logarithmic Functions

Theorem (The Derivative of $\log_a x$)

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}.$$

Proof.

$\square$

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Example (Chain Rule)

Differentiate $f(x) = \log_3(5^x + 1)$.

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$$\begin{aligned} \text{Differentiate} \quad & f(x) = \log_3(5^x + 1). \\ \text{Let} \quad & h(x) = \\ \text{Let} \quad & g(x) = \\ \text{Then} \quad & f(x) = g(h(x)). \end{aligned}$$

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&= \frac{5^x \ln 5}{(5^x + 1)\ln 3}.
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$\ln x = \log_e x$, so plug in $a = e$ to find the derivative of $\ln x$.

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$$= \frac{1}{x} \text{ if } x \neq 0.$$

Example (Logarithmic Differentiation)

Differentiate $y = \dfrac{(x-1)^{5/3}\sin^3 x}{\sqrt{e^x+1}}$.

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