Review problems for April 9 Exam
Math 140

Instructor. Todor Milev

The exam will be closed book, no calculators allowed. Try to solve all theoretical problems without using the lectures/textbook. If you get stuck, read the lectures/textbook, but close the textbook/lectures when going back to the problem. Finally, compare what you wrote with the lectures/textbook.

Problem 1 *Find the equation of the tangent line to the function*

1. $y = x^3 + 2x^2 + 3x + 1$ *at the point* $(-1, -1)$. — *answer:* $y = 2x + 1$.
2. $y = x^3 + 2x^2 - 3x + 1$ *at the point* $(1, 1)$. — *answer:* $y = 4x - 3$.

Problem 2 *Compute the limit.*

1. $\lim\limits_{x\to 0} \frac{x^2}{\sin^2(2x)}$. — *answer:* $\frac{1}{4}$.
2. $\lim\limits_{x\to 0} \frac{x^2}{\cos 4x - 1}$. — *answer:* $-\frac{1}{8}$.
3. $\lim\limits_{x\to 0} \frac{x^2}{\cos 6x - 1}$. — *answer:* $-\frac{1}{18}$.

Problem 3 *Compute the derivative of the function.*

- $f(x) = \frac{1+x}{1+\frac{2}{x}}$. — *answer:* $\frac{4x+x^2+2}{(2+x)^2}$.
- $f(x) = \frac{1+x}{1+\frac{3}{x}}$. — *answer:* $\frac{6x+x^2+3}{(3+x)^2}$.

Problem 4 *Compute the derivative of the function.*

1. 2^{3^x}. — *answer:* $2^{3^x} 3^x (\ln 2)(\ln 3)$.
2. 3^{2^x}. — *answer:* $3^{2^x} 2^x (\ln 2)(\ln 3)$.

Problem 5 *Compute the derivative of the function.*

1. $\sec^2(3x^2)$. — *answer:* $12 \frac{x \sin(3x^2)}{(\cos(3x^2))^3}$.
2. $\csc^2(3x^2)$. — *answer:* $-12 \frac{x \cos(3x^2)}{(\sin(3x^2))^3}$.

Problem 6 *Use implicit differentiation to express* $\frac{dy}{dx}$ *via* y *and* x*, where* x *and* y *satisfy the following relation.*

1. $x^4(x + y) = y^2(3x - y)$.
2. $2x^3 + x^2 y - xy^3 = 2$.

Problem 7 *Use implicit differentiation to find an equation of the tangent line to the curve at the given point.*

- $x^{2/3} + y^{2/3} = 4$ *at* $(-3\sqrt{3}, 1)$.
- $y^2(y^2 - 4) = x^2(x^2 - 5)$ *at* $(0, -2)$.

Problem 8 *Prove that* $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$. *Compute* $\lim\limits_{x\to\infty}\left(1+\frac{2}{x}\right)^x$.

answer: e^2.

Problem 9

1. *Define concave up and concave down function. Define what is the connection between concave up/down function and the notion of derivative.*
2. *Define differentiable function at a point. Define derivative at a point.*
3. *Give example of non-differentiable function. Motivate your answer.*

Problem 10

1. *For integer* n*, prove the power rule* $\frac{d}{dx}(x^n) = nx^{n-1}$ *using the definition of derivative.*
2. *Let* $f(x) = a^x$. *Prove that* $\frac{d}{dx}(a^x) = a^x f'(0)$ *using the definition of derivative* ($f'(x) = \lim\limits_{h\to 0}\frac{f(x+h)-f(x)}{h}$).
3. *Prove the product rule* $\frac{d}{dx}(fg) = \frac{df}{dx}g + f\frac{dg}{dx}$ *using the definition of derivative.*

Problem 11 *Prove that* $(\sin x)' = \cos x$.