Math 140
Lecture 16

Greg Maloney

with modifications by T. Milev

University of Massachusetts Boston

April 11, 2013

Outline

1 (2.8) Related Rates

Outline

1. (2.8) Related Rates
2. (3.1) Maximum and Minimum Values
   - The Extreme Value Theorem

Related Rates

- Suppose we are pumping a balloon with air.
- The balloon's volume is increasing.
- The balloon's radius is increasing.
- The rates of increase of these quantities are related to one another.

Related Rates

- Suppose we are pumping a balloon with air.
- The balloon's volume is increasing.
- The balloon's radius is increasing.
- The rates of increase of these quantities are related to one another.
- It is easier to measure the rate of increase of volume.

Related Rates

- Suppose we are pumping a balloon with air.
- The balloon's volume is increasing.
- The balloon's radius is increasing.
- The rates of increase of these quantities are related to one another.
- It is easier to measure the rate of increase of volume.
- In a related rates problem, the idea is to compute the rate of change of one quantity in terms of the rate of change of another (which may be more easily measured).

Related Rates

- Suppose we are pumping a balloon with air.
- The balloon's volume is increasing.
- The balloon's radius is increasing.
- The rates of increase of these quantities are related to one another.
- It is easier to measure the rate of increase of volume.
- In a related rates problem, the idea is to compute the rate of change of one quantity in terms of the rate of change of another (which may be more easily measured).
- Procedure:
  1. Find an equation relating the two quantities.
  2. Use the Chain Rule to differentiate both sides with respect to time.

Example

Air is being pumped into a balloon such that its volume changes at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the diameter is 50 cm?

Example

Air is being pumped into a balloon such that its volume changes at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the diameter is 50 cm?

- Let V denote the balloon's volume.
- Let r denote its radius.

Example

Air is being pumped into a balloon such that its volume changes at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the diameter is 50 cm?

- Let V denote the balloon's volume.
- Let r denote its radius.
- Given:
- Unknown:

Example

Air is being pumped into a balloon such that its volume changes at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the diameter is 50 cm?

- Let V denote the balloon's volume.
- Let r denote its radius.
- Given:
- Unknown:

Example

Air is being pumped into a balloon such that its volume changes at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the diameter is 50 cm?

- Let V denote the balloon's volume.
- Let r denote its radius.
- Given: $\frac{dV}{dt} = 100$ cm^3/s.
- Unknown:

Example

Air is being pumped into a balloon such that its volume changes at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the diameter is 50 cm?

- Let V denote the balloon's volume.
- Let r denote its radius.
- Given: $\frac{dV}{dt} = 100$ cm^3/s.
- Unknown:

Example

Air is being pumped into a balloon such that its volume changes at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the diameter is 50 cm?

- Let V denote the balloon's volume.
- Let r denote its radius.
- Given: $\frac{dV}{dt} = 100$ cm^3/s.
- Unknown: $\frac{dr}{dt}$ when $r = 25$ cm.

Example

Air is being pumped into a balloon such that its volume changes at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the diameter is 50 cm?

- Let V denote the balloon's volume.
- Let r denote its radius.
- Given: $\frac{\mathrm{d}V}{\mathrm{d}t} = 100$ cm^3/s.
- Unknown: $\frac{\mathrm{d}r}{\mathrm{d}t}$ when $r = 25$ cm.

1. Find an equation relating the two quantities.
2. Use the Chain Rule to differentiate both sides.

Example

Air is being pumped into a balloon such that its volume changes at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the diameter is 50 cm?

- Let V denote the balloon's volume.
- Let r denote its radius.
- Given: $\frac{dV}{dt} = 100 \text{ cm}^3/\text{s}$.
- Unknown: $\frac{dr}{dt}$ when $r = 25$ cm.

1. Find an equation relating the two quantities.
2. Use the Chain Rule to differentiate both sides.

Example

Air is being pumped into a balloon such that its volume changes at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the diameter is 50 cm?

- Let V denote the balloon's volume.
- Let r denote its radius.
- Given: $\frac{dV}{dt} = 100$ cm^3/s.
- Unknown: $\frac{dr}{dt}$ when $r = 25$ cm.

$$V = \frac{4}{3}\pi r^3$$

1. Find an equation relating the two quantities.
2. Use the Chain Rule to differentiate both sides.

Example

Air is being pumped into a balloon such that its volume changes at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the diameter is 50 cm?

- Let V denote the balloon's volume.
- Let r denote its radius.
- Given: $\frac{dV}{dt} = 100$ cm^3/s.
- Unknown: $\frac{dr}{dt}$ when $r = 25$ cm.

1. Find an equation relating the two quantities.
2. Use the Chain Rule to differentiate both sides.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$$

Example

Air is being pumped into a balloon such that its volume changes at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the diameter is 50 cm?

- Let V denote the balloon's volume.
- Let r denote its radius.
- Given: $\frac{dV}{dt} = 100$ cm^3/s.
- Unknown: $\frac{dr}{dt}$ when $r = 25$ cm.

1. Find an equation relating the two quantities.
2. Use the Chain Rule to differentiate both sides.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$$

$$\frac{dV}{dt} = \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right)\frac{dr}{dt}$$

Example

Air is being pumped into a balloon such that its volume changes at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the diameter is 50 cm?

- Let V denote the balloon's volume.
- Let r denote its radius.
- Given: $\frac{dV}{dt} = 100$ cm^3/s.
- Unknown: $\frac{dr}{dt}$ when $r = 25$ cm.

1. Find an equation relating the two quantities.
2. Use the Chain Rule to differentiate both sides.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$$

$$\frac{dV}{dt} = \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right)\frac{dr}{dt}$$

$$\frac{dV}{dt} = \qquad \frac{dr}{dt}$$

Example

Air is being pumped into a balloon such that its volume changes at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the diameter is 50 cm?

- Let V denote the balloon's volume.
- Let r denote its radius.
- Given: $\frac{dV}{dt} = 100$ cm^3/s.
- Unknown: $\frac{dr}{dt}$ when $r = 25$ cm.

1. Find an equation relating the two quantities.
2. Use the Chain Rule to differentiate both sides.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$$

$$\frac{dV}{dt} = \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right)\frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

Example

Air is being pumped into a balloon such that its volume changes at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the diameter is 50 cm?

- Let V denote the balloon's volume.
- Let r denote its radius.
- Given: $\frac{dV}{dt} = 100$ cm^3/s.
- Unknown: $\frac{dr}{dt}$ when $r = 25$ cm.

1. Find an equation relating the two quantities.
2. Use the Chain Rule to differentiate both sides.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$$

$$\frac{dV}{dt} = \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right)\frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{4\pi r^2}\frac{dV}{dt}$$

Example

Air is being pumped into a balloon such that its volume changes at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the diameter is 50 cm?

- Let V denote the balloon's volume.
- Let r denote its radius.
- Given: $\frac{dV}{dt} = 100$ cm^3/s.
- Unknown: $\frac{dr}{dt}$ when $r = 25$ cm.

1. Find an equation relating the two quantities.
2. Use the Chain Rule to differentiate both sides.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$$

$$\frac{dV}{dt} = \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right)\frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{4\pi r^2}\frac{dV}{dt}$$

$$\frac{dr}{dt} = \frac{1}{4\pi(25\mathrm{cm})^2} 100\frac{\mathrm{cm}^3}{\mathrm{s}}$$

Example

Air is being pumped into a balloon such that its volume changes at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the diameter is 50 cm?

- Let V denote the balloon's volume.
- Let r denote its radius.
- Given: $\frac{dV}{dt} = 100$ cm^3/s.
- Unknown: $\frac{dr}{dt}$ when $r = 25$ cm.

1. Find an equation relating the two quantities.
2. Use the Chain Rule to differentiate both sides.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$$

$$\frac{dV}{dt} = \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right)\frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{4\pi r^2}\frac{dV}{dt}$$

$$\frac{dr}{dt} = \frac{1}{4\pi(25\mathrm{cm})^2}100\frac{\mathrm{cm}^3}{\mathrm{s}} = \frac{1}{25\pi}\mathrm{cm/s}$$

Example

A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom is 6 ft from the wall?

Example

A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom is 6 ft from the wall?

- Let y be the distance from the top to the ground.
- Let x be the distance from the bottom to the wall.

Example

A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom is 6 ft from the wall?

- Let y be the distance from the top to the ground.
- Let x be the distance from the bottom to the wall.
- Given:
- Unknown:

Example

A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom is 6 ft from the wall?

- Let y be the distance from the top to the ground.
- Let x be the distance from the bottom to the wall.
- Given:
- Unknown:

Example

A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom is 6 ft from the wall?

- Let y be the distance from the top to the ground.
- Let x be the distance from the bottom to the wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown:

Example

A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom is 6 ft from the wall?

- Let y be the distance from the top to the ground.
- Let x be the distance from the bottom to the wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown:

Example

A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom is 6 ft from the wall?

- Let y be the distance from the top to the ground.
- Let x be the distance from the bottom to the wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown: $\frac{dy}{dt}$ when $x = 6$ ft.

Example

A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom is 6 ft from the wall?

- Let y be the distance from the top to the ground.
- Let x be the distance from the bottom to the wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown: $\frac{dy}{dt}$ when $x = 6$ ft.

1. Find an equation relating the two quantities.
2. Use the Chain Rule to differentiate both sides.

Example

A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom is 6 ft from the wall?

- Let y be the distance from the top to the ground.
- Let x be the distance from the bottom to the wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown: $\frac{dy}{dt}$ when $x = 6$ ft.

1. Find an equation relating the two quantities.
2. Use the Chain Rule to differentiate both sides.

Example

A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom is 6 ft from the wall?

- Let y be the distance from the top to the ground.
- Let x be the distance from the bottom to the wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown: $\frac{dy}{dt}$ when $x = 6$ ft.

$$x^2 + y^2 = 100$$

1. Find an equation relating the two quantities.
2. Use the Chain Rule to differentiate both sides.

Example

A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom is 6 ft from the wall?

- Let y be the distance from the top to the ground.
- Let x be the distance from the bottom to the wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown: $\frac{dy}{dt}$ when $x = 6$ ft.

$$x^2 + y^2 = 100$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

1. Find an equation relating the two quantities.
2. Use the Chain Rule to differentiate both sides.

Example

A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom is 6 ft from the wall?

- Let y be the distance from the top to the ground.
- Let x be the distance from the bottom to the wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown: $\frac{dy}{dt}$ when $x = 6$ ft.

$$x^2 + y^2 = 100$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

1. Find an equation relating the two quantities.
2. Use the Chain Rule to differentiate both sides.

Example

A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom is 6 ft from the wall?

- Let y be the distance from the top to the ground.
- Let x be the distance from the bottom to the wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown: $\frac{dy}{dt}$ when $x = 6$ ft.

1. Find an equation relating the two quantities.
2. Use the Chain Rule to differentiate both sides.

$$x^2 + y^2 = 100$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

$$\frac{dy}{dt} = -\frac{\quad}{\quad}.$$

Example

A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom is 6 ft from the wall?

- Let y be the distance from the top to the ground.
- Let x be the distance from the bottom to the wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown: $\frac{dy}{dt}$ when $x = 6$ ft.

1. Find an equation relating the two quantities.
2. Use the Chain Rule to differentiate both sides.

$$x^2 + y^2 = 100$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

$$\frac{dy}{dt} = -\underline{\quad\quad}.$$

Example

A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom is 6 ft from the wall?

- Let y be the distance from the top to the ground.
- Let x be the distance from the bottom to the wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown: $\frac{dy}{dt}$ when $x = 6$ ft.

1. Find an equation relating the two quantities.
2. Use the Chain Rule to differentiate both sides.

$$x^2 + y^2 = 100$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

$$\frac{dy}{dt} = -\frac{}{} \cdot 1 \text{ ft/s}$$

Example

A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom is 6 ft from the wall?

- Let y be the distance from the top to the ground.
- Let x be the distance from the bottom to the wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown: $\frac{dy}{dt}$ when $x = 6$ ft.

$$x^2 + y^2 = 100$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

$$\frac{dy}{dt} = -\frac{}{} \cdot 1 \text{ ft/s}$$

1. Find an equation relating the two quantities.
2. Use the Chain Rule to differentiate both sides.

Example

A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom is 6 ft from the wall?

- Let y be the distance from the top to the ground.
- Let x be the distance from the bottom to the wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown: $\frac{dy}{dt}$ when $x = 6$ ft.

1. Find an equation relating the two quantities.
2. Use the Chain Rule to differentiate both sides.

$$x^2 + y^2 = 100$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

$$\frac{dy}{dt} = -\frac{6\text{ ft}}{} \cdot 1\text{ ft/s}$$

Example

A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom is 6 ft from the wall?

- Let y be the distance from the top to the ground.
- Let x be the distance from the bottom to the wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown: $\frac{dy}{dt}$ when $x = 6$ ft.

1. Find an equation relating the two quantities.
2. Use the Chain Rule to differentiate both sides.

$$x^2 + y^2 = 100$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

$$\frac{dy}{dt} = -\frac{6\text{ ft}}{} \cdot 1\text{ ft/s}$$

Example

A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom is 6 ft from the wall?

- Let y be the distance from the top to the ground.
- Let x be the distance from the bottom to the wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown: $\frac{dy}{dt}$ when $x = 6$ ft.
- Pythagorean Theorem: $y = 8$.

1. Find an equation relating the two quantities.
2. Use the Chain Rule to differentiate both sides.

$$x^2 + y^2 = 100$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

$$\frac{dy}{dt} = -\frac{6 \text{ ft}}{8 \text{ ft}} \cdot 1 \text{ ft/s}$$

Example

A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom is 6 ft from the wall?

- Let y be the distance from the top to the ground.
- Let x be the distance from the bottom to the wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown: $\frac{dy}{dt}$ when $x = 6$ ft.
- Pythagorean Theorem: $y = 8$.

1. Find an equation relating the two quantities.
2. Use the Chain Rule to differentiate both sides.

$$x^2 + y^2 = 100$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

$$\frac{dy}{dt} = -\frac{6\text{ ft}}{8\text{ ft}} \cdot 1\text{ ft/s}$$

$$= -3/4\text{ ft/s}.$$

Example

A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom is 6 ft from the wall?

- Let y be the distance from the top to the ground.
- Let x be the distance from the bottom to the wall.
- Given: $\frac{dx}{dt} = 1$ ft/s.
- Unknown: $\frac{dy}{dt}$ when $x = 6$ ft.
- Pythagorean Theorem: $y = 8$.

1. Find an equation relating the two quantities.
2. Use the Chain Rule to differentiate both sides.

$$x^2 + y^2 = 100$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

$$\frac{dy}{dt} = -\frac{6\text{ ft}}{8\text{ ft}} \cdot 1\text{ ft/s}$$

$$= -3/4\text{ ft/s}.$$

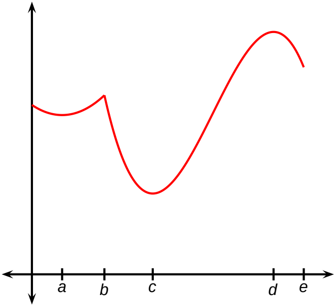
Therefore the top of the ladder is falling at a rate of $3/4$ ft/s.

Maximum and Minimum Values

Many real-world problems involve optimization (finding the best possible way of doing something). Examples include

- What shape of can minimizes manufacturing costs?
- What is the maximum acceleration of a space shuttle?
- What is the maximum load an elevator can carry?

Questions like these can be reduced to finding maximum or minimum values of a function.

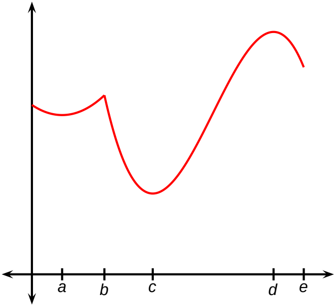


Definition (Absolute Maximum or Minimum)

A function f has an absolute maximum (or global maximum) at c if $f(c) \geq f(x)$ for all x in the domain of f. The number $f(c)$ is called the maximum value of f.

Likewise, f has an absolute minimum at c if $f(c) \leq f(x)$ for all x in the domain of f. $f(c)$ is called the minimum value of f.

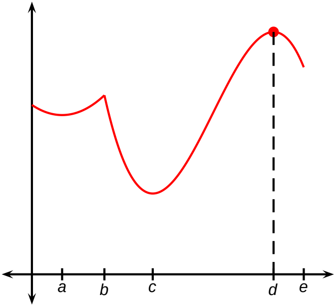
Maximum and minimum values of f are called extreme values.



- Absolute maximum at .
- Absolute minimum at .

Definition (Absolute Maximum or Minimum)

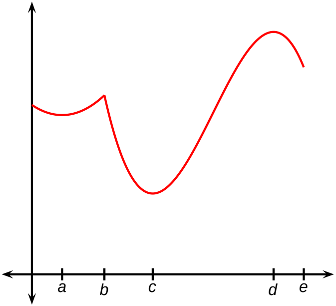
A function f has an absolute maximum (or global maximum) at c if $f(c) \geq f(x)$ for all x in the domain of f. The number $f(c)$ is called the maximum value of f.
Likewise, f has an absolute minimum at c if $f(c) \leq f(x)$ for all x in the domain of f. $f(c)$ is called the minimum value of f.
Maximum and minimum values of f are called extreme values.



- Absolute maximum at d.
- Absolute minimum at .

Definition (Absolute Maximum or Minimum)

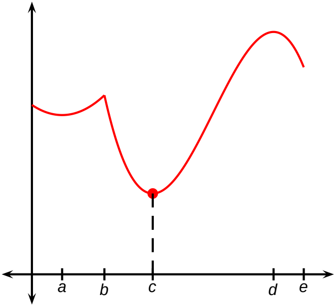
A function f has an absolute maximum (or global maximum) at c if $f(c) \geq f(x)$ for all x in the domain of f. The number $f(c)$ is called the maximum value of f.
Likewise, f has an absolute minimum at c if $f(c) \leq f(x)$ for all x in the domain of f. $f(c)$ is called the minimum value of f.
Maximum and minimum values of f are called extreme values.



- Absolute maximum at d.
- Absolute minimum at .

Definition (Absolute Maximum or Minimum)

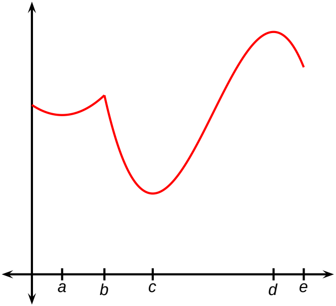
A function f has an absolute maximum (or global maximum) at c if $f(c) \geq f(x)$ for all x in the domain of f. The number $f(c)$ is called the maximum value of f.
Likewise, f has an absolute minimum at c if $f(c) \leq f(x)$ for all x in the domain of f. $f(c)$ is called the minimum value of f.
Maximum and minimum values of f are called extreme values.



- Absolute maximum at d.
- Absolute minimum at c.

Definition (Absolute Maximum or Minimum)

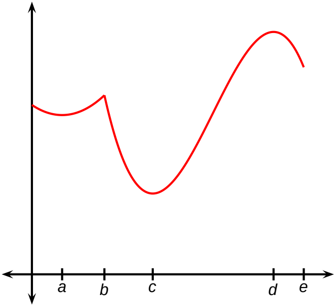
A function f has an absolute maximum (or global maximum) at c if $f(c) \geq f(x)$ for all x in the domain of f. The number $f(c)$ is called the maximum value of f.
Likewise, f has an absolute minimum at c if $f(c) \leq f(x)$ for all x in the domain of f. $f(c)$ is called the minimum value of f.
Maximum and minimum values of f are called extreme values.



- Absolute maximum at d.
- Absolute minimum at c.

Definition (Local Maximum or Minimum)

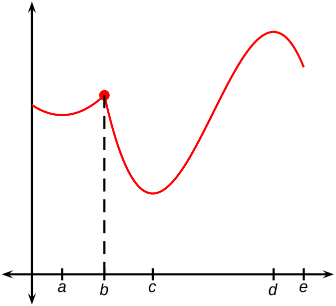
A function f has a local maximum at c if $f(c) \geq f(x)$ for all x in some open interval containing c. Similarly, f has a local minimum at c if $f(c) \leq f(x)$ for all x in some open interval containing c.



- Absolute maximum at d.
- Absolute minimum at c.
- Local maximum at
- Local minimum at

Definition (Local Maximum or Minimum)

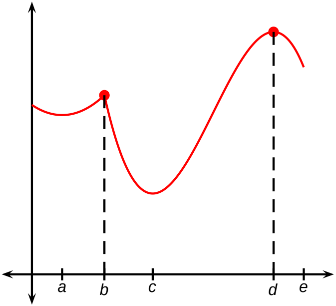
A function f has a local maximum at c if $f(c) \geq f(x)$ for all x in some open interval containing c. Similarly, f has a local minimum at c if $f(c) \leq f(x)$ for all x in some open interval containing c.



- Absolute maximum at d.
- Absolute minimum at c.
- Local maximum at b,
- Local minimum at

Definition (Local Maximum or Minimum)

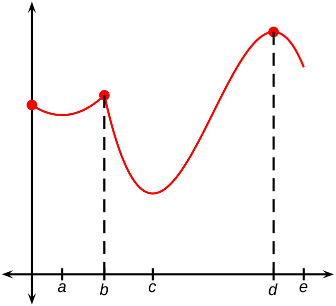
A function f has a local maximum at c if $f(c) \geq f(x)$ for all x in some open interval containing c. Similarly, f has a local minimum at c if $f(c) \leq f(x)$ for all x in some open interval containing c.



- Absolute maximum at d.
- Absolute minimum at c.
- Local maximum at b, d
- Local minimum at

Definition (Local Maximum or Minimum)

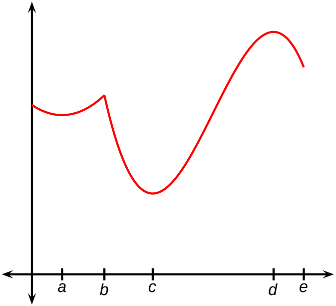
A function f has a local maximum at c if $f(c) \geq f(x)$ for all x in some open interval containing c. Similarly, f has a local minimum at c if $f(c) \leq f(x)$ for all x in some open interval containing c.



- Absolute maximum at d.
- Absolute minimum at c.
- Local maximum at b, d and 0.
- Local minimum at

Definition (Local Maximum or Minimum)

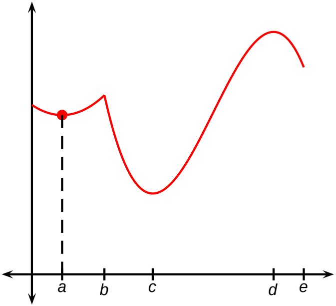
A function f has a local maximum at c if $f(c) \geq f(x)$ for all x in some open interval containing c. Similarly, f has a local minimum at c if $f(c) \leq f(x)$ for all x in some open interval containing c.



- Absolute maximum at d.
- Absolute minimum at c.
- Local maximum at b, d and 0.
- Local minimum at

Definition (Local Maximum or Minimum)

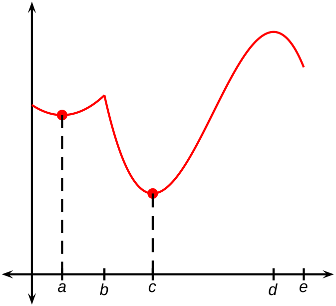
A function f has a local maximum at c if $f(c) \geq f(x)$ for all x in some open interval containing c. Similarly, f has a local minimum at c if $f(c) \leq f(x)$ for all x in some open interval containing c.



- Absolute maximum at d.
- Absolute minimum at c.
- Local maximum at b, d and 0.
- Local minimum at a,

Definition (Local Maximum or Minimum)

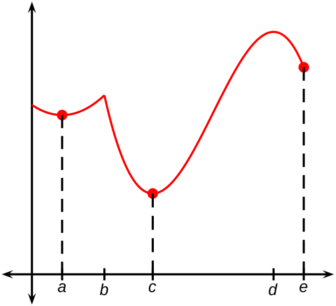
A function f has a local maximum at c if $f(c) \geq f(x)$ for all x in some open interval containing c. Similarly, f has a local minimum at c if $f(c) \leq f(x)$ for all x in some open interval containing c.



- Absolute maximum at d.
- Absolute minimum at c.
- Local maximum at b, d and 0.
- Local minimum at a, c and

Definition (Local Maximum or Minimum)

A function f has a local maximum at c if $f(c) \geq f(x)$ for all x in some open interval containing c. Similarly, f has a local minimum at c if $f(c) \leq f(x)$ for all x in some open interval containing c.



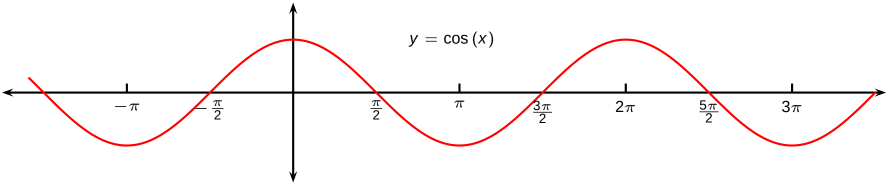
- Absolute maximum at d.
- Absolute minimum at c.
- Local maximum at b, d and 0.
- Local minimum at a, c and e.

Definition (Local Maximum or Minimum)

A function f has a local maximum at c if $f(c) \geq f(x)$ for all x in some open interval containing c. Similarly, f has a local minimum at c if $f(c) \leq f(x)$ for all x in some open interval containing c.

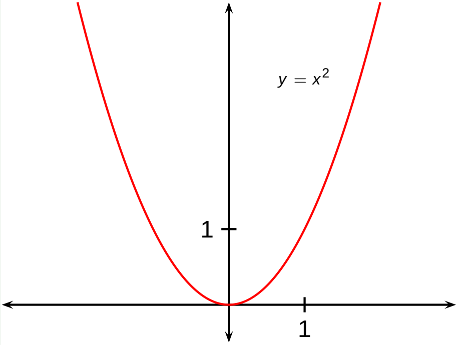
Example

The function f takes on its maximum value (local maximum and absolute maximum) of 1 infinitely many times, since $\cos 2n\pi = 1$ for any integer n.
Likewise, it takes on its minimum value of -1 infinitely many times, because $\cos(2n+1)\pi = -1$ for all integers n.



Example

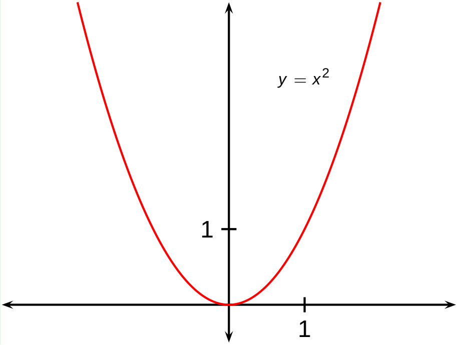
Consider the function $y = x^2$.



- Absolute maximum:
- Absolute minimum:
- Local maximum:
- Local minimum:

Example

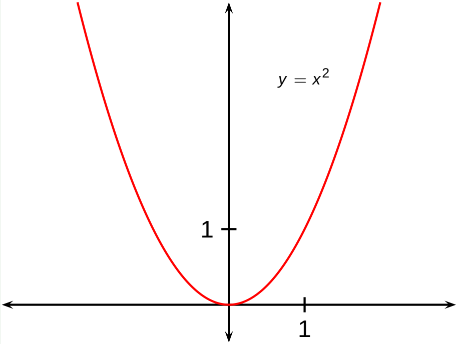
Consider the function $y = x^2$.



- Absolute maximum:
- Absolute minimum:
- Local maximum:
- Local minimum:

Example

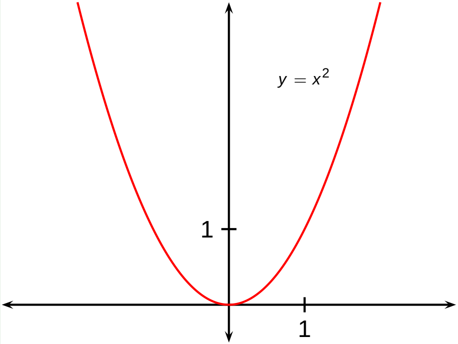
Consider the function $y = x^2$.



- Absolute maximum: None
- Absolute minimum:
- Local maximum:
- Local minimum:

Example

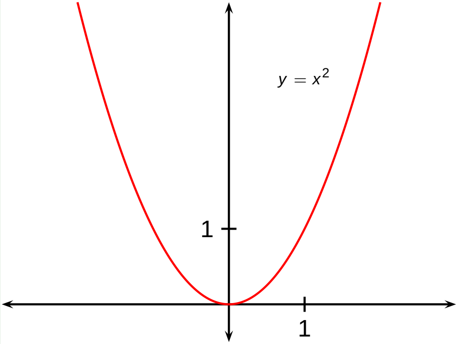
Consider the function $y = x^2$.



- Absolute maximum: None
- Absolute minimum:
- Local maximum:
- Local minimum:

Example

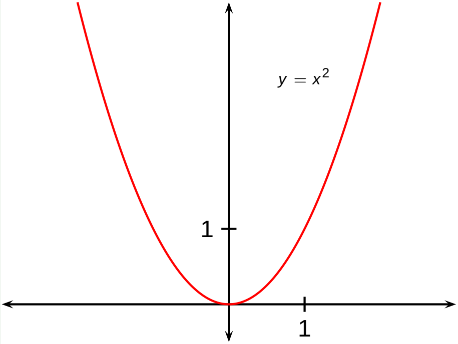
Consider the function $y = x^2$.



- Absolute maximum: None
- Absolute minimum: at 0
- Local maximum:
- Local minimum:

Example

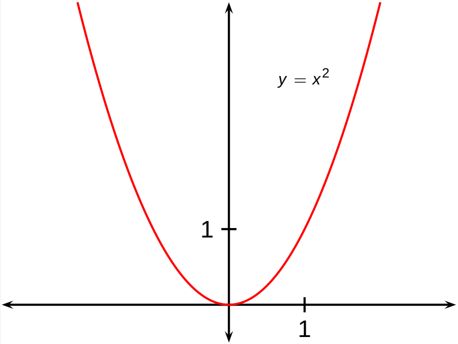
Consider the function $y = x^2$.



- Absolute maximum: None
- Absolute minimum: at 0
- Local maximum:
- Local minimum:

Example

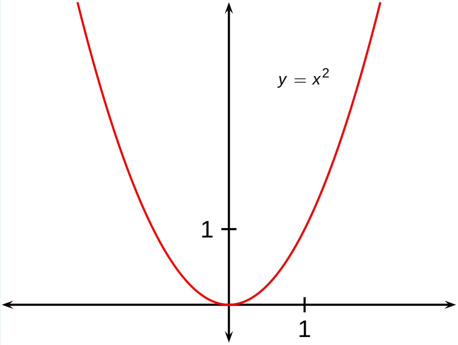
Consider the function $y = x^2$.



- Absolute maximum: None
- Absolute minimum: at 0
- Local maximum: None
- Local minimum:

Example

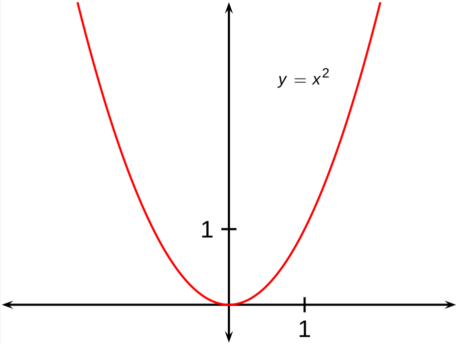
Consider the function $y = x^2$.



- Absolute maximum: None
- Absolute minimum: at 0
- Local maximum: None
- Local minimum:

Example

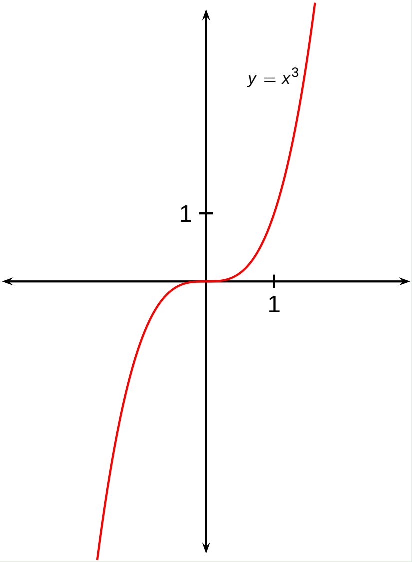
Consider the function $y = x^2$.



- Absolute maximum: None
- Absolute minimum: at 0
- Local maximum: None
- Local minimum: at 0

Example

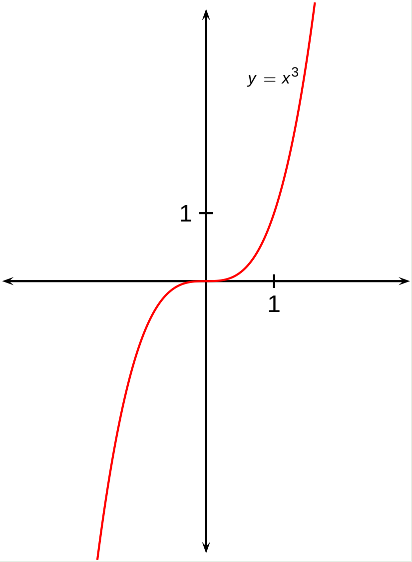
Consider the function $y = x^3$.



- Absolute maximum:
- Absolute minimum:
- Local maximum:
- Local minimum:

Example

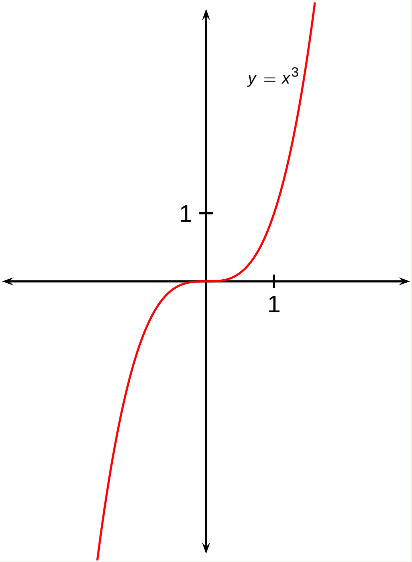
Consider the function $y = x^3$.



- Absolute maximum:
- Absolute minimum:
- Local maximum:
- Local minimum:

Example

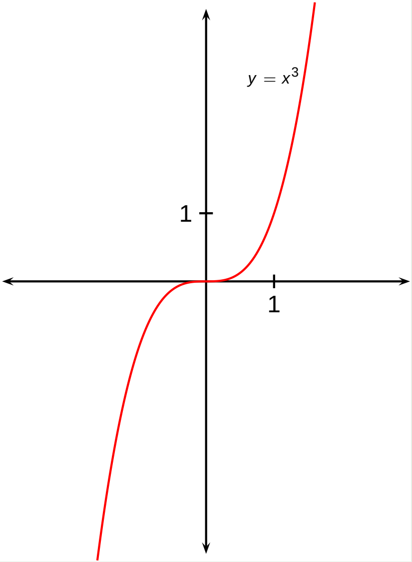
Consider the function $y = x^3$.



- Absolute maximum: None
- Absolute minimum:
- Local maximum:
- Local minimum:

Example

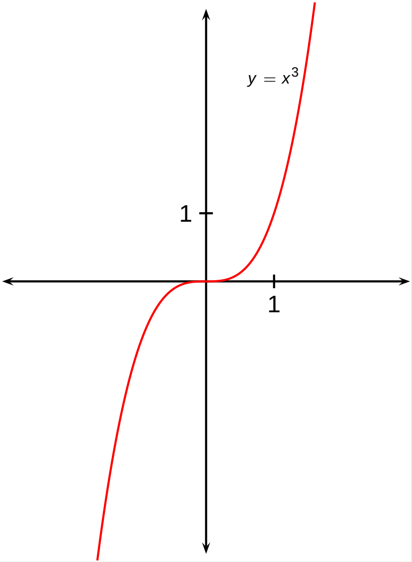
Consider the function $y = x^3$.



- Absolute maximum: None
- Absolute minimum:
- Local maximum:
- Local minimum:

Example

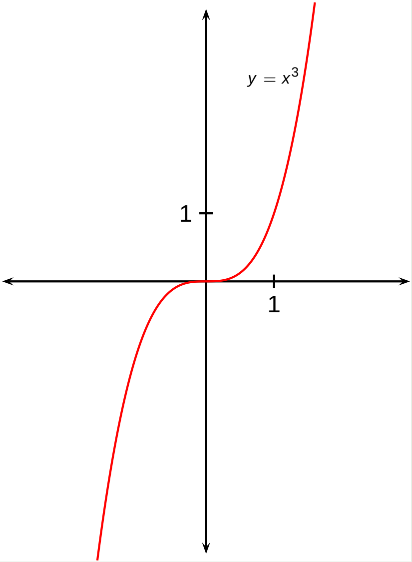
Consider the function $y = x^3$.



- Absolute maximum: None
- Absolute minimum: None
- Local maximum:
- Local minimum:

Example

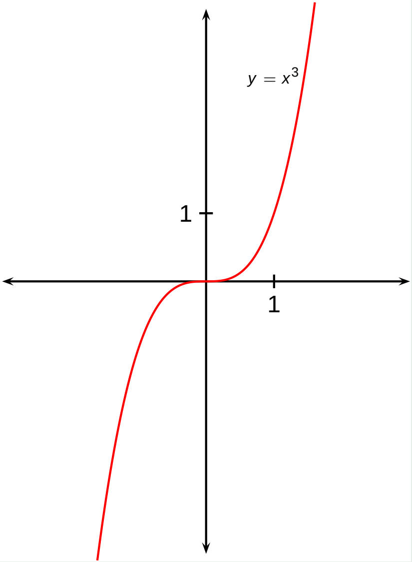
Consider the function $y = x^3$.



- Absolute maximum: None
- Absolute minimum: None
- Local maximum:
- Local minimum:

Example

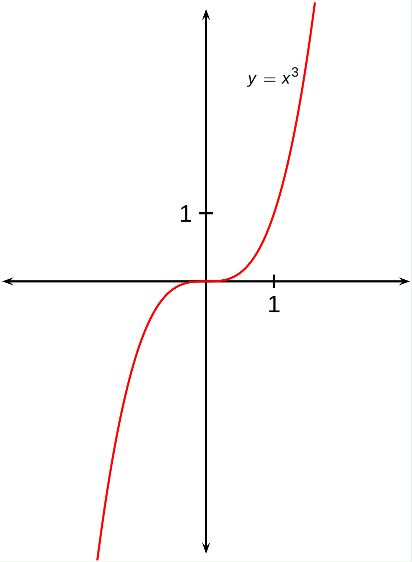
Consider the function $y = x^3$.



- Absolute maximum: None
- Absolute minimum: None
- Local maximum: None
- Local minimum:

Example

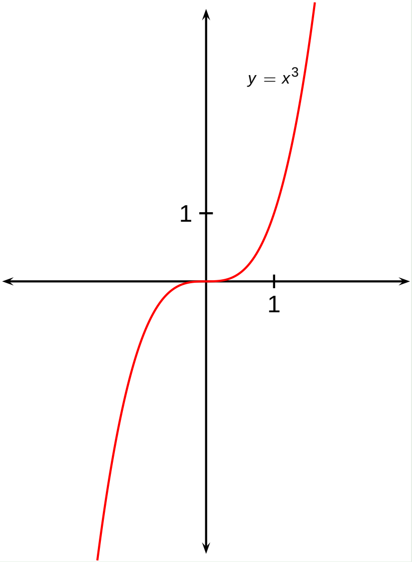
Consider the function $y = x^3$.



- Absolute maximum: None
- Absolute minimum: None
- Local maximum: None
- Local minimum:

Example

Consider the function $y = x^3$.



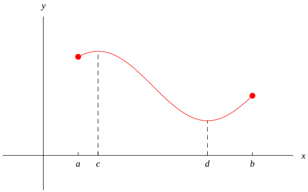
- Absolute maximum: None
- Absolute minimum: None
- Local maximum: None
- Local minimum: None

The Extreme Value Theorem

Recall that some functions (such as $y = \cos x$) have extreme values, while other functions (such as $y = x^3$) do not. The next theorem, which we will not prove, gives a condition under which f must have extreme values.

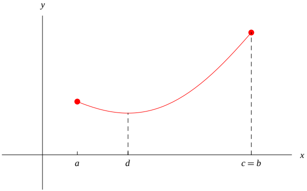
Theorem (The Extreme Value Theorem)

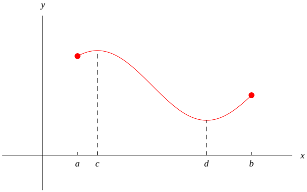
If f is continuous on a closed interval $[a, b]$, then f attains its absolute maximum value $f(c)$ and its absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.



Theorem (The Extreme Value Theorem)

If f is continuous on a closed interval $[a, b]$, then f attains its absolute maximum value $f(c)$ and its absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

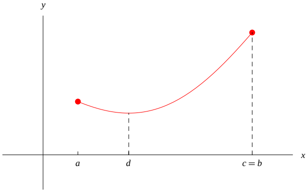


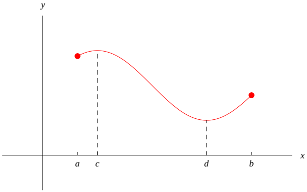


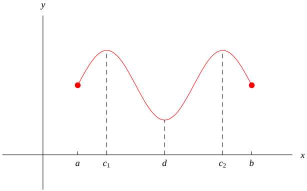
- Extreme values might happen at endpoints.

Theorem (The Extreme Value Theorem)

If f is continuous on a closed interval $[a, b]$, then f attains its absolute maximum value $f(c)$ and its absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.







- Extreme values might happen at endpoints.
- Extreme values might happen twice.

Theorem (The Extreme Value Theorem)

If f is continuous on a closed interval $[a, b]$, then f attains its absolute maximum value $f(c)$ and its absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

- Do we need all of the hypotheses of the theorem?

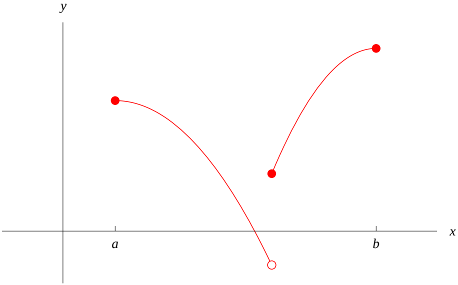
Theorem (The Extreme Value Theorem)

If f is continuous on a closed interval $[a, b]$, then f attains its absolute maximum value $f(c)$ and its absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

- Do we need all of the hypotheses of the theorem?
- Do we need f to be continuous?
- Do we need the interval to be closed?

Theorem (The Extreme Value Theorem)

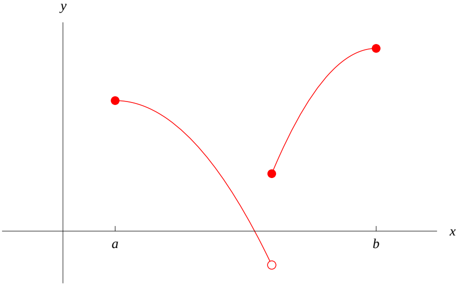
If f is continuous on a closed interval $[a, b]$, then f attains its absolute maximum value $f(c)$ and its absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.



- Do we need all of the hypotheses of the theorem?
- Do we need f to be continuous?
- Do we need the interval to be closed?

Theorem (The Extreme Value Theorem)

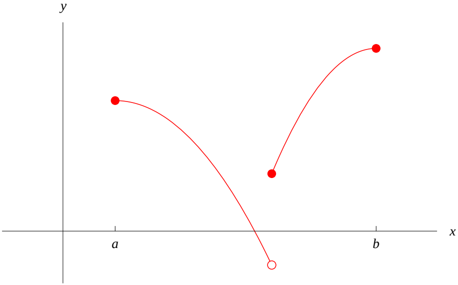
If f is continuous on a closed interval $[a, b]$, then f attains its absolute maximum value $f(c)$ and its absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

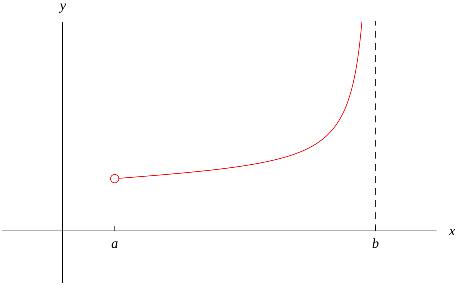


- Do we need all of the hypotheses of the theorem?
- Do we need f to be continuous? Yes.
- Do we need the interval to be closed?

Theorem (The Extreme Value Theorem)

If f is continuous on a closed interval $[a, b]$, then f attains its absolute maximum value $f(c)$ and its absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

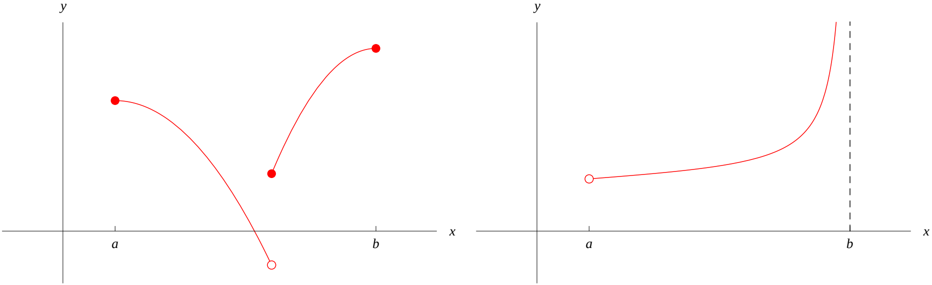




- Do we need all of the hypotheses of the theorem?
- Do we need f to be continuous? Yes.
- Do we need the interval to be closed?

Theorem (The Extreme Value Theorem)

If f is continuous on a closed interval $[a, b]$, then f attains its absolute maximum value $f(c)$ and its absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.



- Do we need all of the hypotheses of the theorem?
- Do we need f to be continuous? Yes.
- Do we need the interval to be closed? Yes.