Math 140

Lecture 18

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with modifications by T. Milev

University of Massachusetts Boston

April 16, 2013

Outline

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- $f'(x) = 3x^2 + 4$.
- Therefore $f'(x)$ is always positive.
- Contradiction.

Theorem (The Mean Value Theorem)

Let f be a function that is continuous on $[a, b]$ and differentiable on (a, b). Then there is a number c in (a, b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

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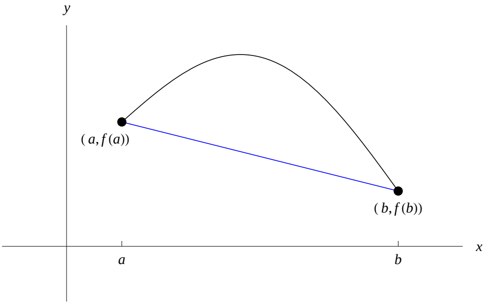

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- Slope: $m =$

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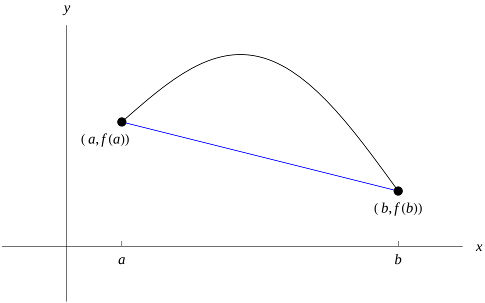

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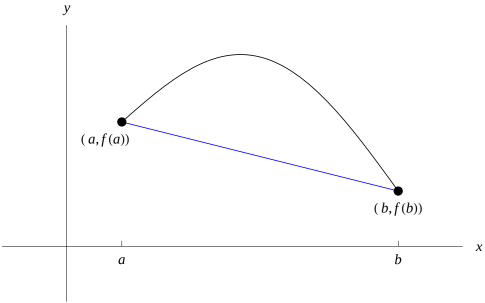

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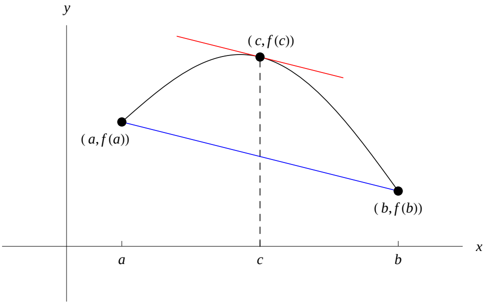

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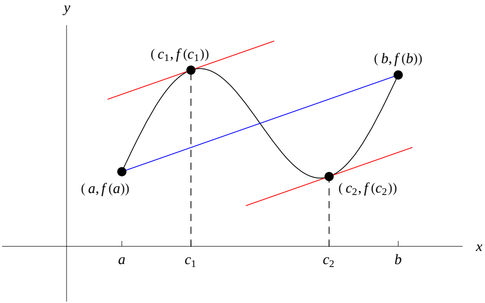

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- Slope: $m = \frac{f(b)-f(a)}{b-a}$.
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- Maybe more than one number.

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- Therefore f is differentiable on (x_1, x_2) and continuous on $[x_1, x_2]$.

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$$f'(c)(x_2 - x_1) = f(x_2) - f(x_1)$$

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If $f'(x) = 0$ for all x in an interval (a, b), then f is constant on (a, b).

Proof.

- Let x_1 and x_2 be any numbers in (a, b) with $x_1 < x_2$.
- f is differentiable on (a, b).
- Therefore f is differentiable on (x_1, x_2) and continuous on $[x_1, x_2]$.
- Mean Value Theorem: There exists c in (x_1, x_2) such that

$$\begin{aligned} f'(c) &= \frac{f(x_2) - f(x_1)}{x_2 - x_1} \\ f'(c)(x_2 - x_1) &= f(x_2) - f(x_1) \\ 0 &= f(x_2) - f(x_1) \end{aligned}$$

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Therefore f is constant on (a, b). □

Corollary

If $f'(x) = g'(x)$ *for all* x *in an interval* (a, b), *then* $f - g$ *is constant on* (a, b); *that is,* $f(x) = g(x) + c$ *where* c *is constant.*

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Corollary

If $f'(x) = g'(x)$ *for all* x *in an interval* (a, b)*, then* $f - g$ *is constant on* (a, b)*; that is,* $f(x) = g(x) + c$ *where* c *is constant.*

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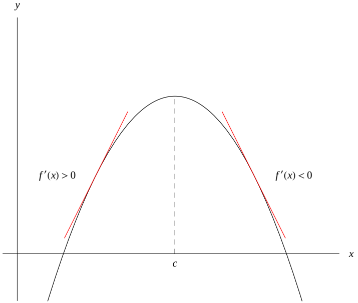
- By the previous theorem, F is constant, so $f - g$ is constant. □

What Does f' Say About f?



- Consider the graph on the left.
- $f'(x) > 0$ to the left of c and $f'(x) < 0$ to the right of c.
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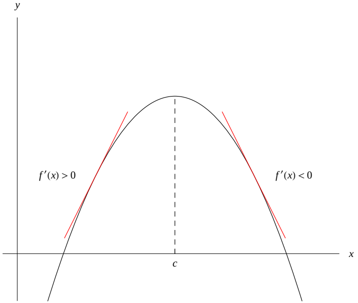

- Consider the graph on the left.
- $f'(x) > 0$ to the left of c and $f'(x) < 0$ to the right of c.
- f is increasing to the left of c and decreasing to the right of c.
- This property holds more generally:

Increasing/Decreasing Test

1. If $f'(x) > 0$ on an interval, then f is increasing on that interval.
2. If $f'(x) < 0$ on an interval, then f is decreasing on that interval.

Example

Find where the function $f(x) = 3x^4 + 8x^3 - 18x^2 + 6$ is increasing and where it is decreasing.

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Find where the function $f(x) = 3x^4 + 8x^3 - 18x^2 + 6$ is increasing and where it is decreasing.

$f'(x) = 12x^3 + 24x^2 - 36x = 12x(x^2 + 2x - 3) = 12x(x+3)(x-1)$

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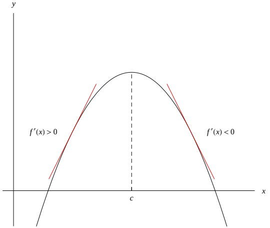
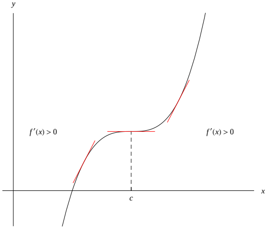
- Recall that if f has a local maximum at c, then c must be a critical number for f, but if c is a critical number for f, it is not necessarily a local maximum.





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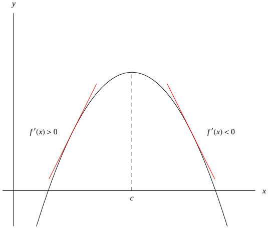

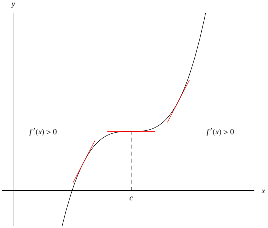

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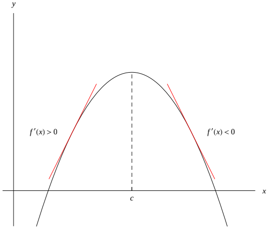

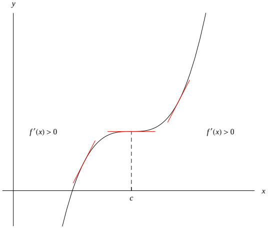

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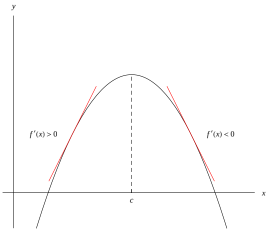

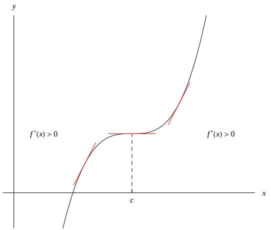

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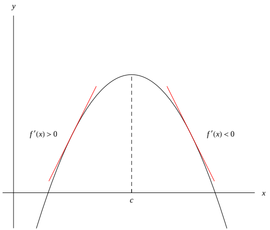

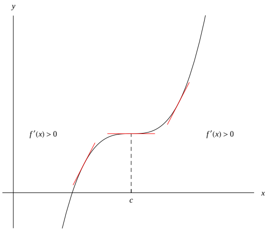

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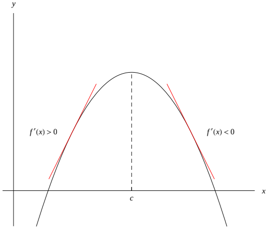
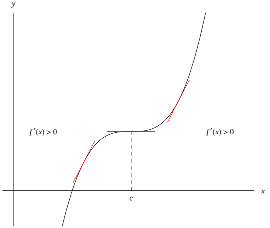
- This suggests a way of testing for local maxima/minima.





The First Derivative Test

Suppose that c is a critical number of a continuous function f.

1. If f' changes from positive to negative at c, then f has a local maximum at c.
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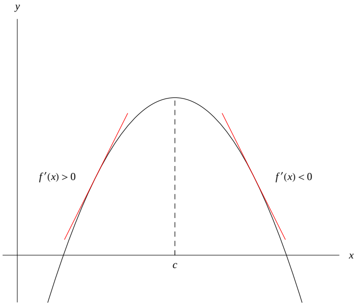

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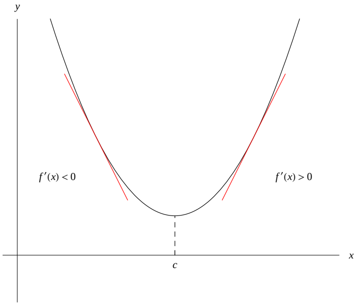

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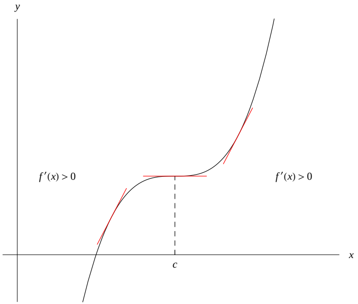
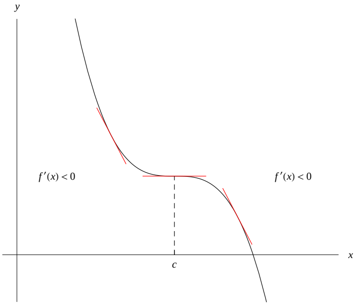
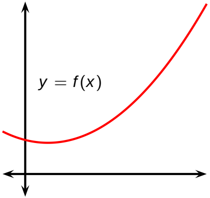
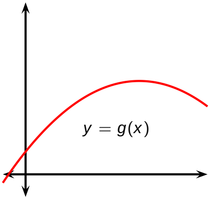




What Does f'' Say About f?

f and g are both increasing functions on (a, b), but they look different because they bend in different directions.





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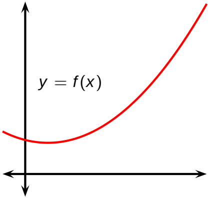

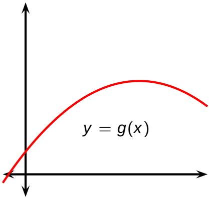

Definition (Concave Up/Concave Down)

A function is called concave up/down if the line segment between any two points lies above/below its graph. Suppose f is a differentiable function. If f lies above all of its tangents on an interval I, then we call it concave up on I. If f lies below all of its tangents on I, it we call it concave down on I.

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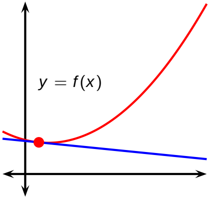

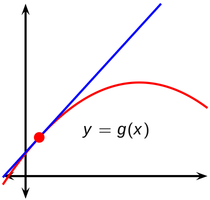

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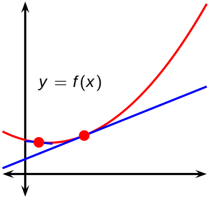

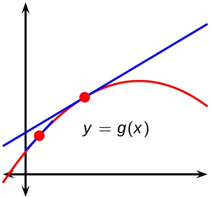

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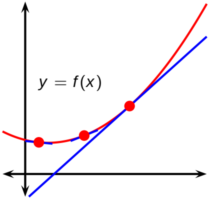

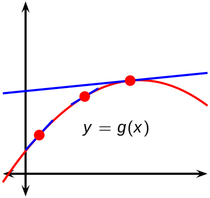

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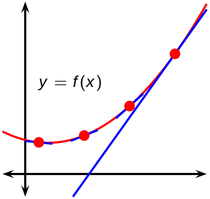

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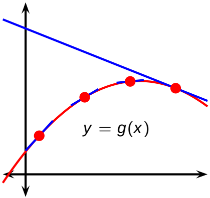

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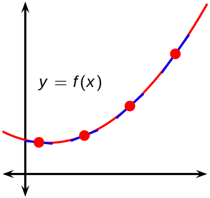

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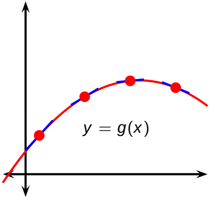

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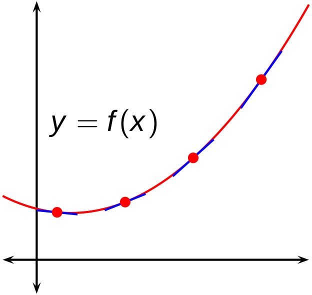
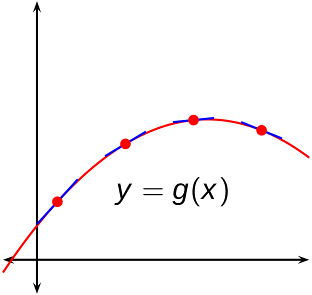
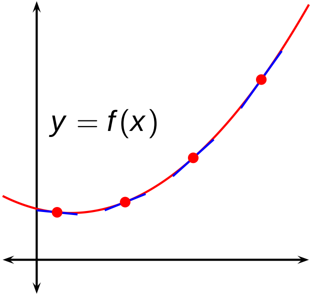
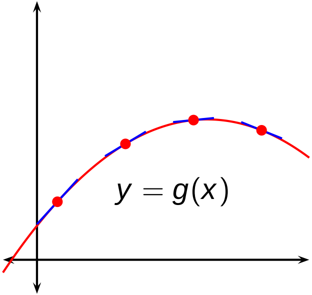




- In the graph of f the slopes of the tangent lines increase as we move from left to right.





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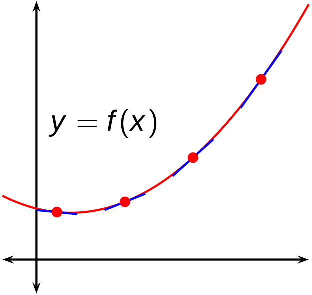
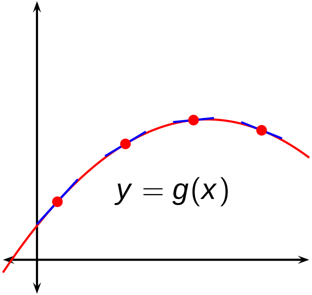
- This means f' is an increasing function.





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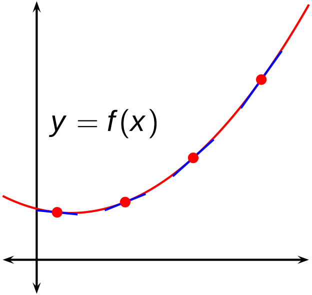
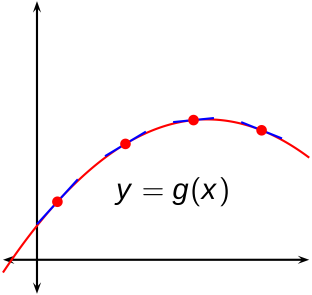
- This means f'' is positive on (a, b).





- In the graph of f the slopes of the tangent lines increase as we move from left to right.
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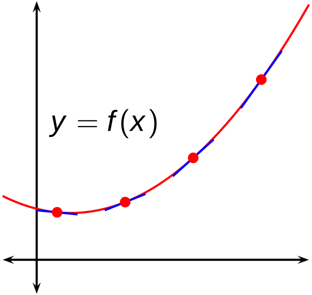
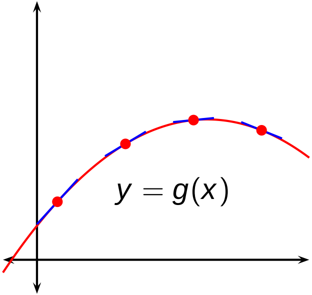
- Similarly g'' is negative on (a, b).





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- Similarly g'' is negative on (a, b).

Concavity Test

1. If $f''(x) > 0$ for all x in I, then the graph of f is concave up on I.
2. If $f''(x) < 0$ for all x in I, then the graph of f is concave down on I.

Definition (Inflection Point)

Let f be a twice differentiable function. A point P on a curve $y = f(x)$ is called an inflection point if f changes from concave up to concave down or from concave down to concave up at P.

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Another way of saying this is that P is an inflection point if f'' changes signs at P.

This gives us a new way of checking if critical points are local maxima or local minima:

The Second Derivative Test
Suppose f'' is continuous near c.

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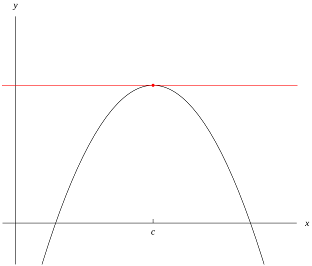

- $f'(c) = 0$, so f has a horizontal tangent at c.
- $f''(c) < 0$, so f is concave down near c.
- This means f lies below its horizontal tangent.
- This means $f(c)$ is a local maximum.

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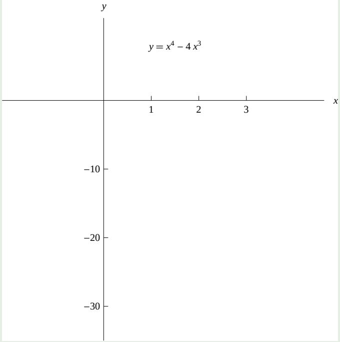
Discuss the curve $y = f(x) = x^4 - 4x^3$ with respect to concavity, points of inflection, and local maxima and minima. Sketch the curve.



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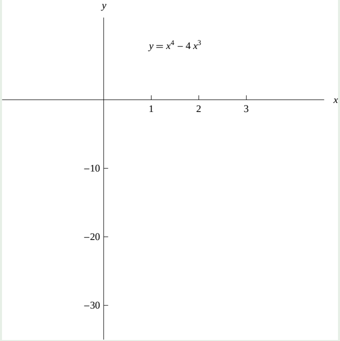

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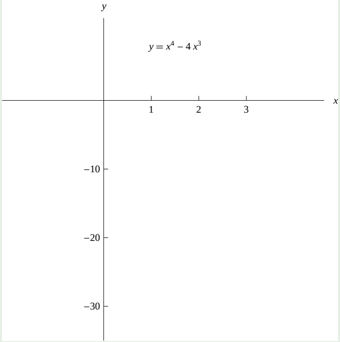

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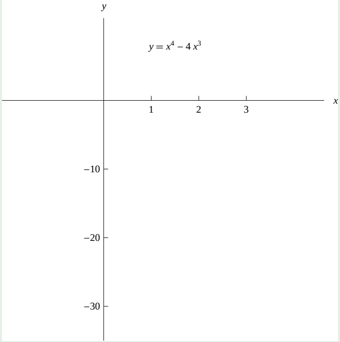

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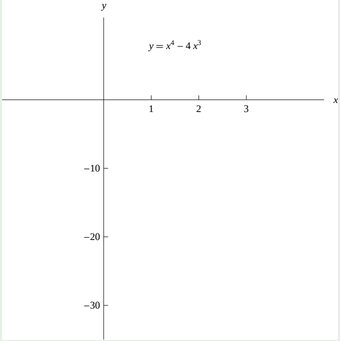

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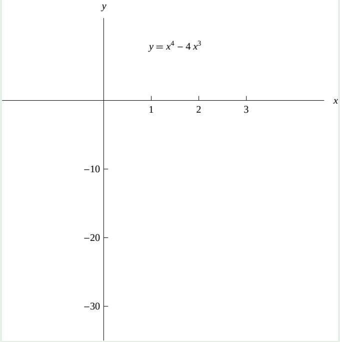

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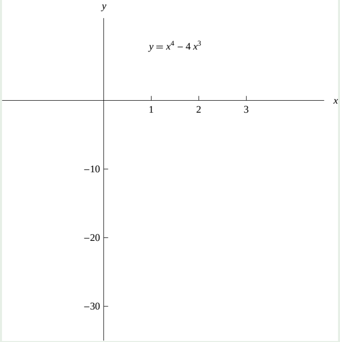

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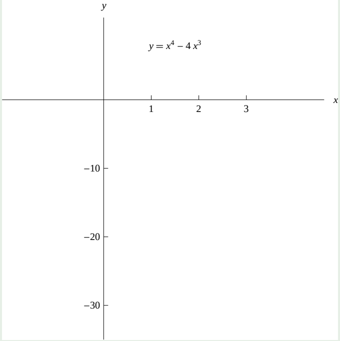

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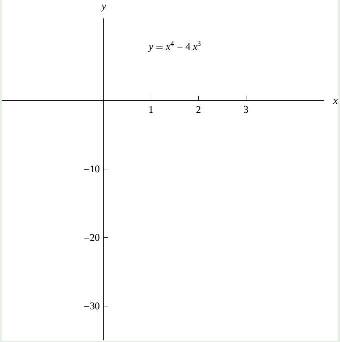

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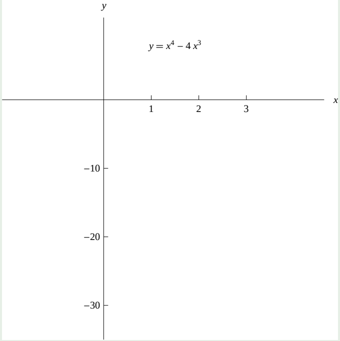

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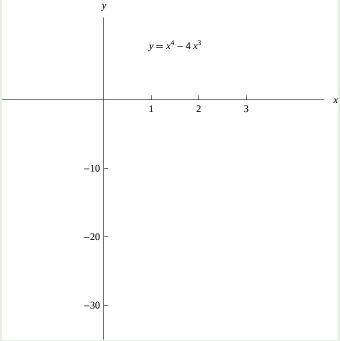

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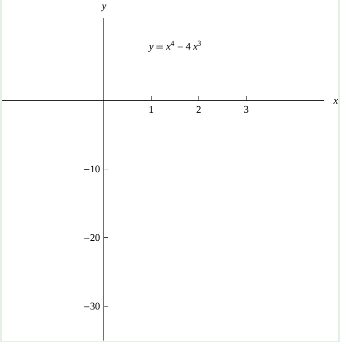

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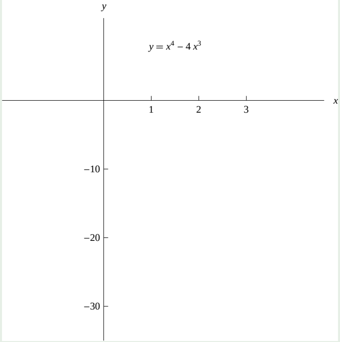

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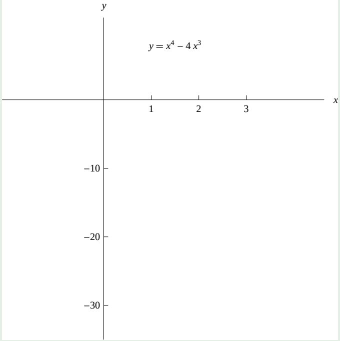

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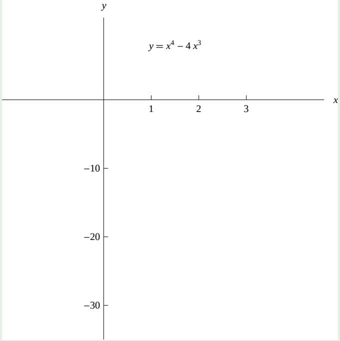

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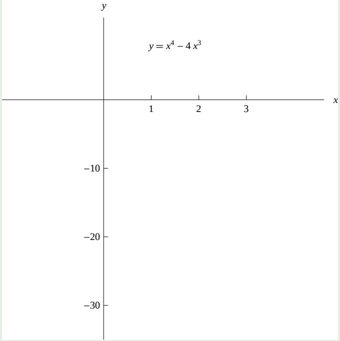

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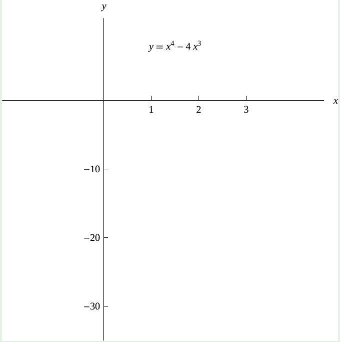

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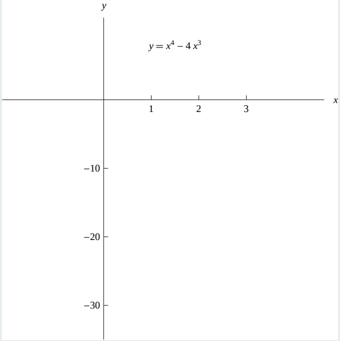

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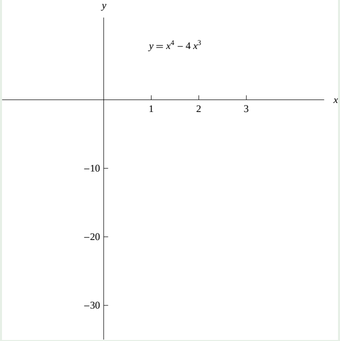

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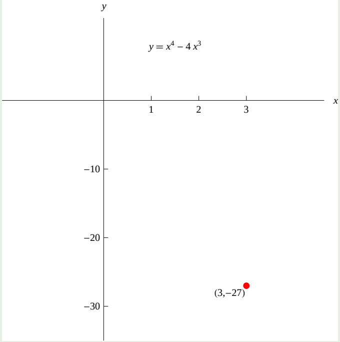

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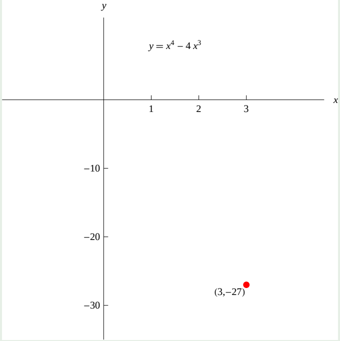

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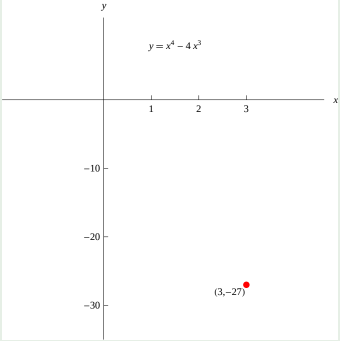

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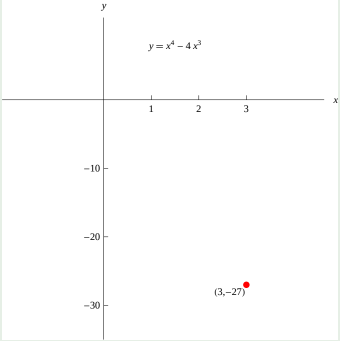

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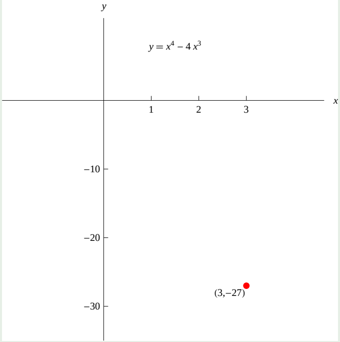

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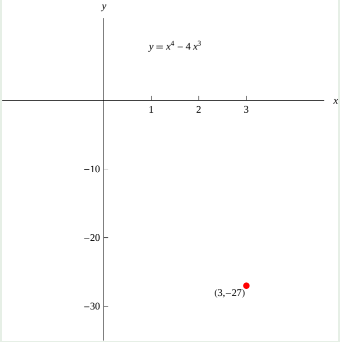

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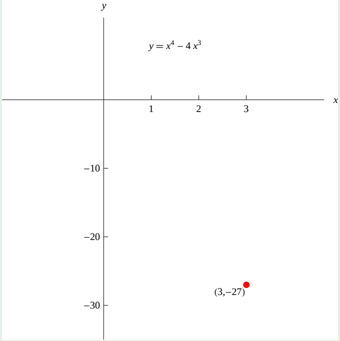

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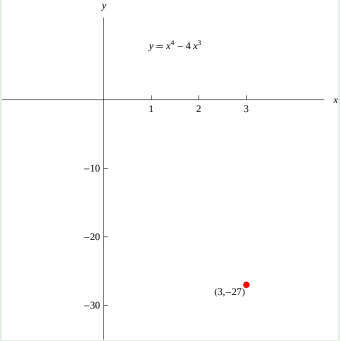

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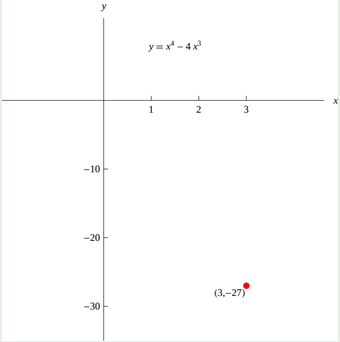

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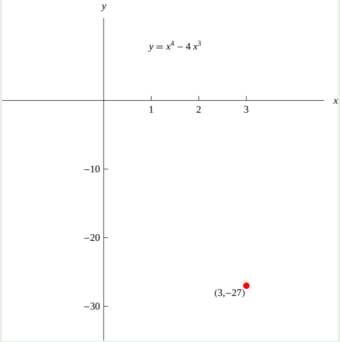

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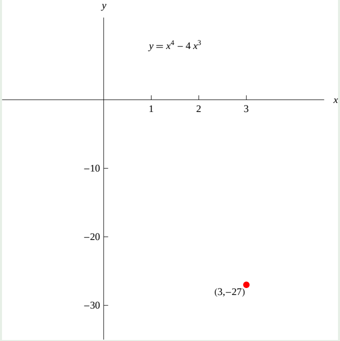

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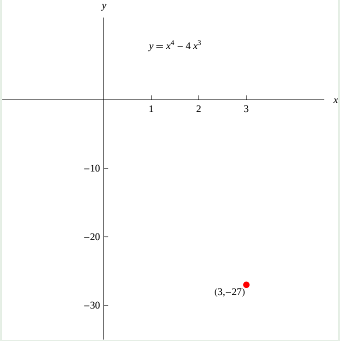

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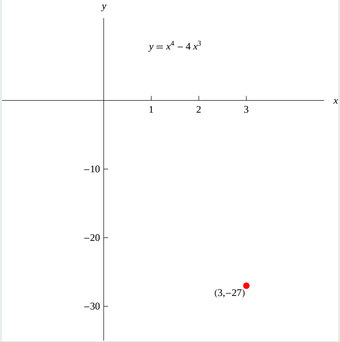

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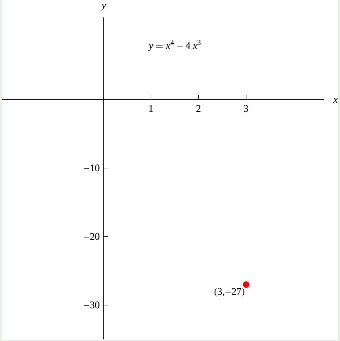

Interval	$f''(x)$	Concave
$(-\infty, 0)$	$+$	
$(0, 2)$	$-$	
$(2, \infty)$		

- $f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$.
- $f''(x) = 12x^2 - 24x = 12x(x-2)$.
- Critical numbers: 0 and 3.
- $f''(0) = 0$ and $f''(3) = 36 > 0$.
- Second Derivative Test:
- Local minimum at 3. $f(3) = -27$.
- No information about 0.
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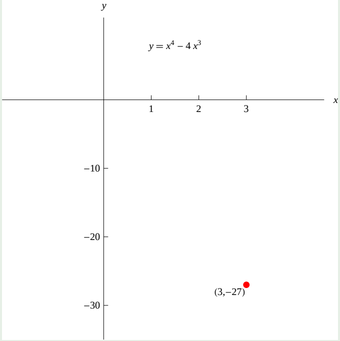

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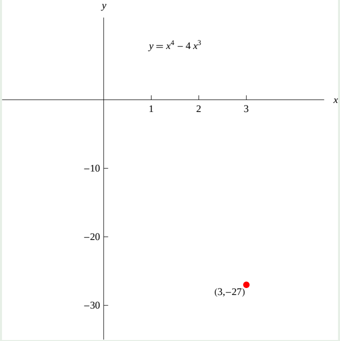

Interval	$f''(x)$	Concave
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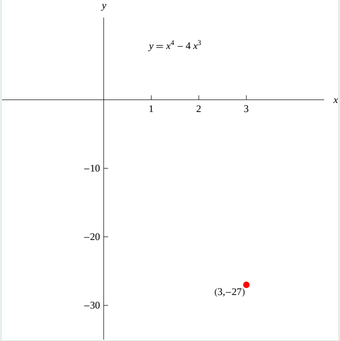

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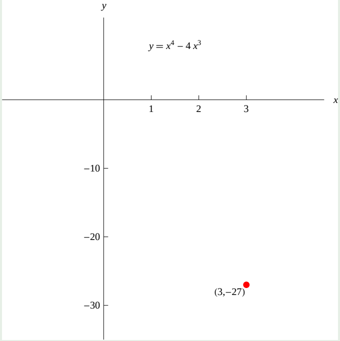

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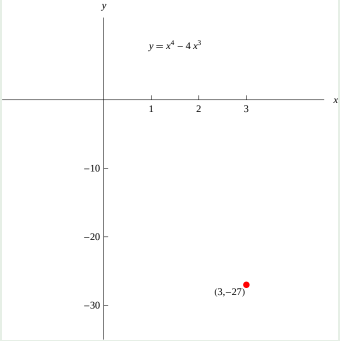

Interval	$f''(x)$	Concave
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- No information about 0.
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- No local max or min at 0.
- Inflection points: 0 and 2

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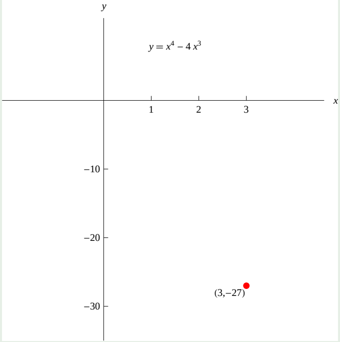

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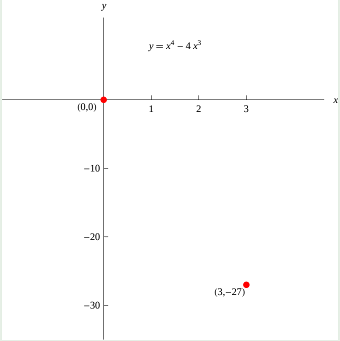

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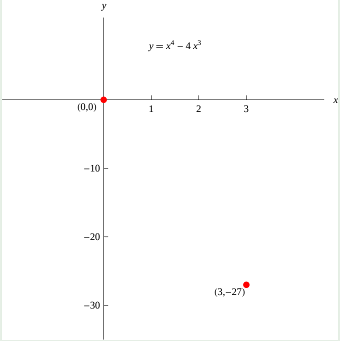

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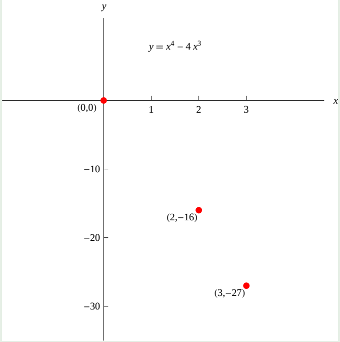

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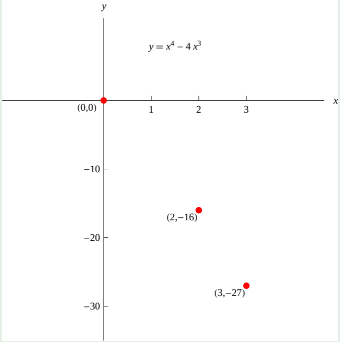

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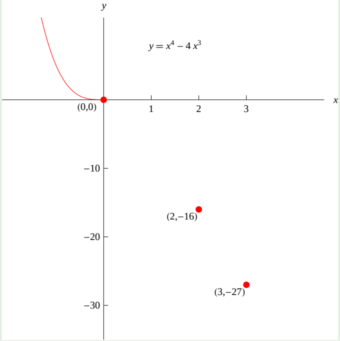

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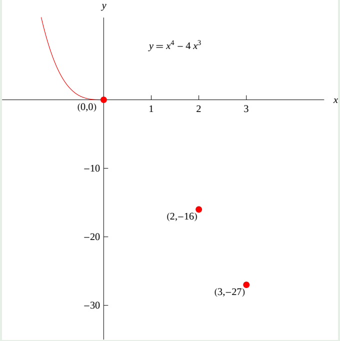

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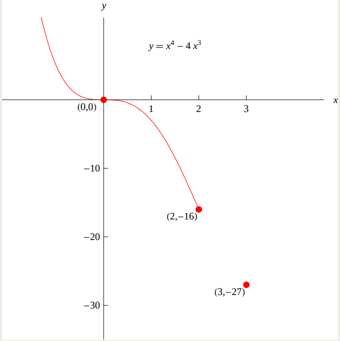

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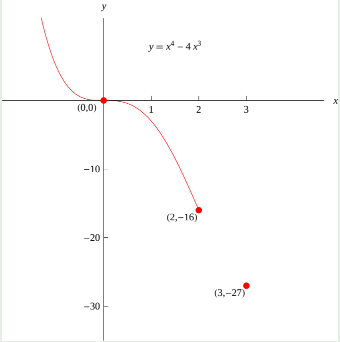

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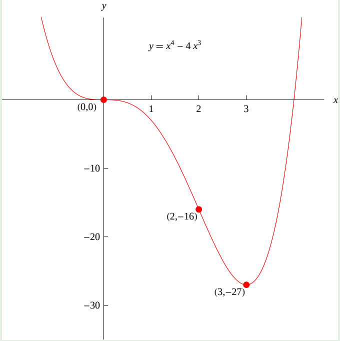

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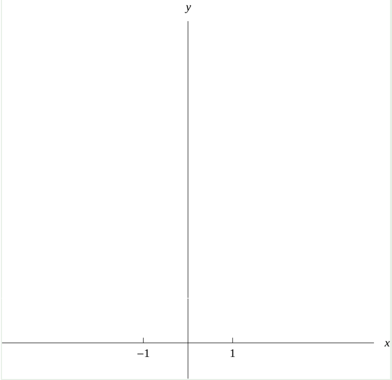
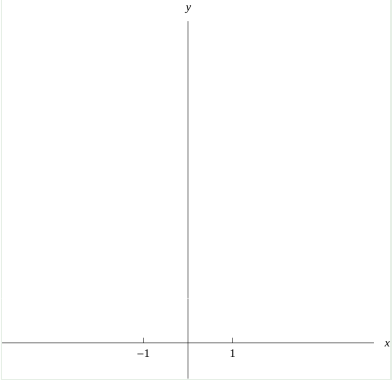
Example (Example 6, p. 277)



Draw the graph of $f(x) = e^{1/x}$.

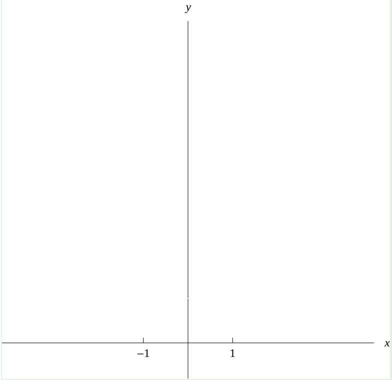
Example (Example 6, p. 277)



Draw the graph of $f(x) = e^{1/x}$.

- $f(x)$ is always positive.

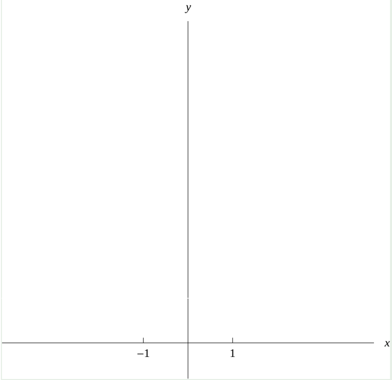
Example (Example 6, p. 277)



Draw the graph of $f(x) = e^{1/x}$.

- $f(x)$ is always positive.
- Domain: everything but 0.

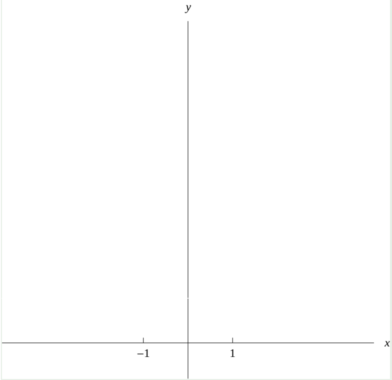
Example (Example 6, p. 277)



Draw the graph of $f(x) = e^{1/x}$.

- $f(x)$ is always positive.
- Domain: everything but 0.
- Check for vertical asymptote at 0.
- $t = 1/x$: $\lim_{x \to 0^+} e^{1/x}$
- $t = 1/x$: $\lim_{x \to 0^-} e^{1/x}$

Example (Example 6, p. 277)



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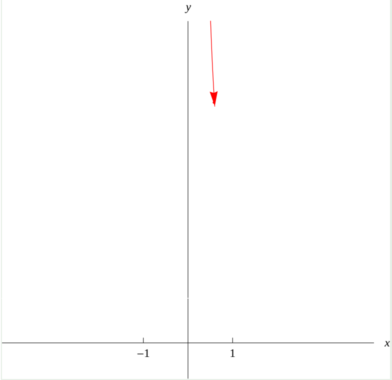
Example (Example 6, p. 277)



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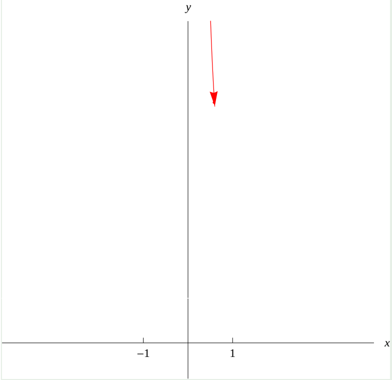
Example (Example 6, p. 277)



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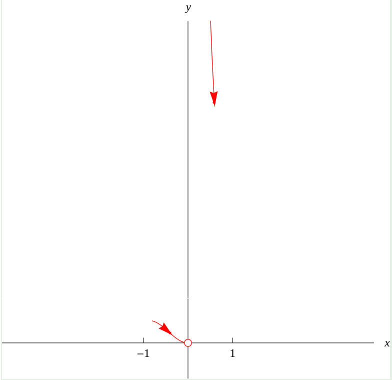
Example (Example 6, p. 277)



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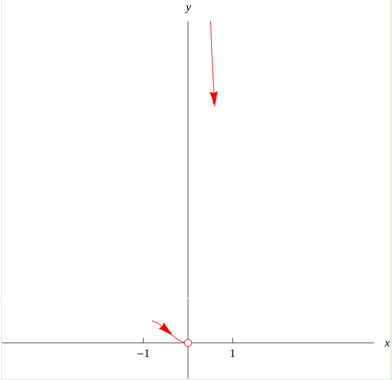

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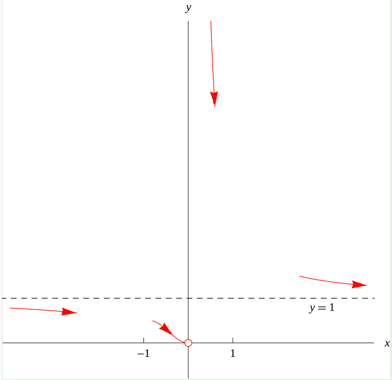
- $t = 1/x$: $\displaystyle\lim_{x\to 0^-} e^{1/x} = \lim_{t\to -\infty} e^t = 0$.
- As $x \to \pm\infty$, $1/x \to 0$.

Example (Example 6, p. 277)



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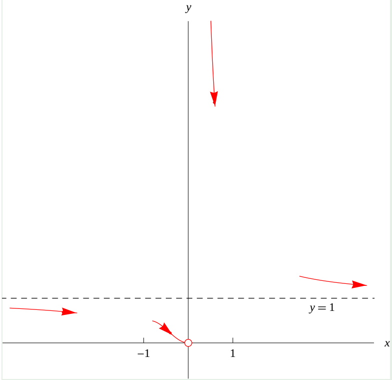
- $t = 1/x : \lim_{x\to 0^-} e^{1/x} = \lim_{t\to -\infty} e^t = 0$.
- As $x \to \pm\infty$, $1/x \to 0$.
- Therefore $\lim_{x\to\pm\infty} e^{1/x} = 1$
- $y = 1$ is a horizontal asymptote.

Example (Example 6, p. 277)



$$f'(x) = e^{1/x}(1/x)'$$

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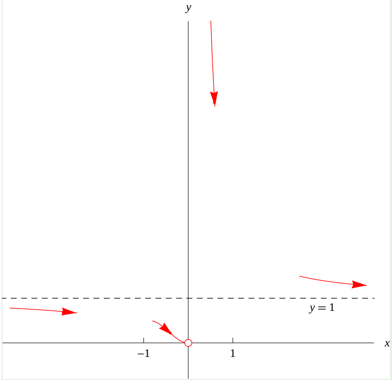

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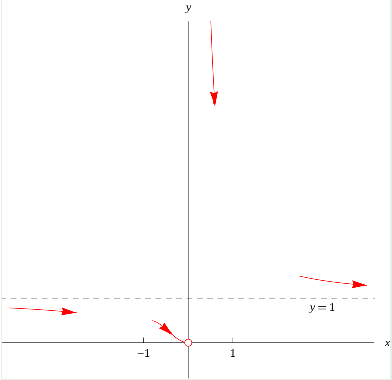

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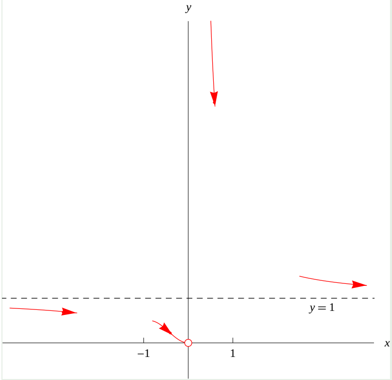

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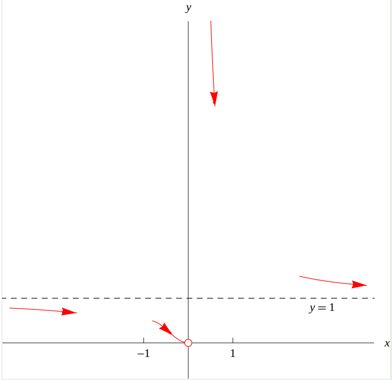

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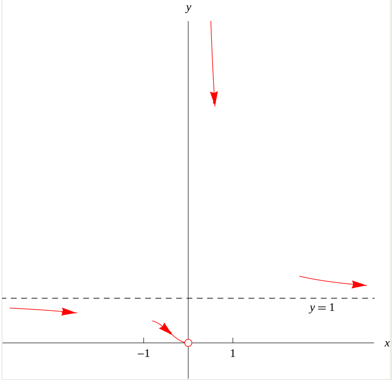

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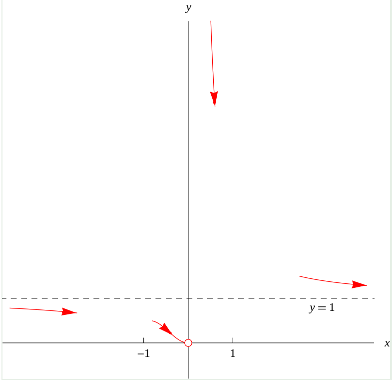

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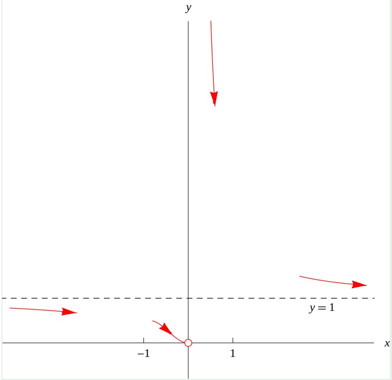

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Always decreasing.

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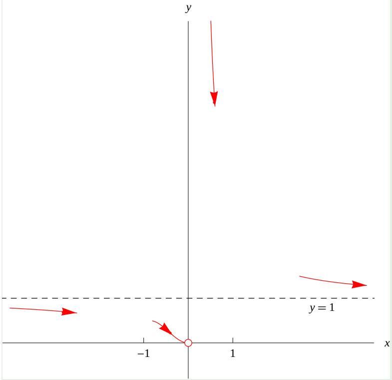

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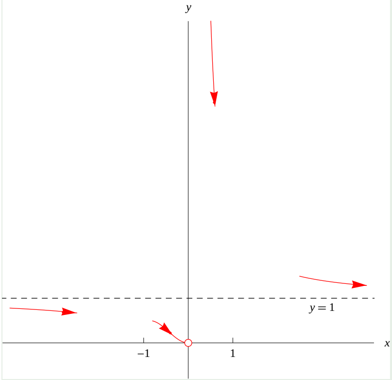

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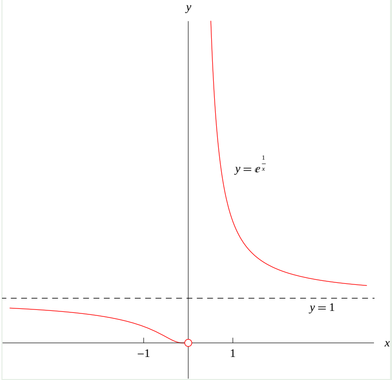

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