Math 140

Lecture 19

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with modifications by T. Milev

University of Massachusetts Boston

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Outline

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Optimization Problems

The methods we have learned for finding extreme values of functions have applications to real life.

The basic steps are always the same:

1. Draw a picture of the problem.
2. Assign variable names to all of the quantities involved.
3. Find a formula that expresses the desired quantity in terms of the other quantities.
4. If the desired quantity has been expressed as a function of more than one variable, use formulas to eliminate all but one of these variables.
5. Now use calculus to find the maximum (or minimum) value of the desired quantity.

Example

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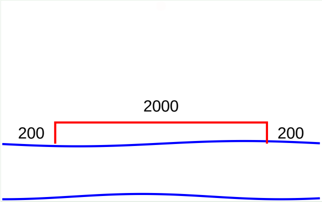

$$\text{Area} = 200 \cdot 2000 = 400,000 \text{ft}^2$$

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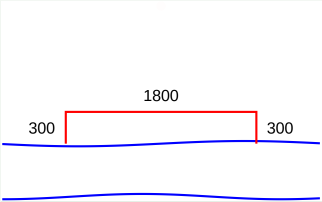

Area $= 300 \cdot 1800 = 540,000\text{ft}^2$

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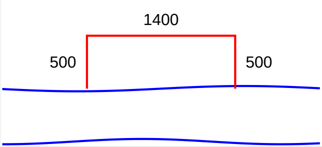

$\text{Area} = 500 \cdot 1400 = 700,000 \text{ft}^2$

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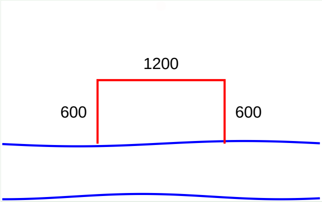

Area $= 600 \cdot 1200 = 720,000 \text{ft}^2$

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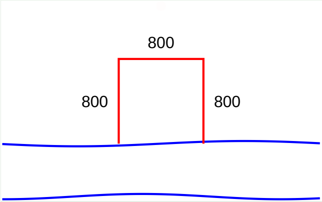

$\text{Area} = 800 \cdot 800 = 640,000\text{ft}^2$

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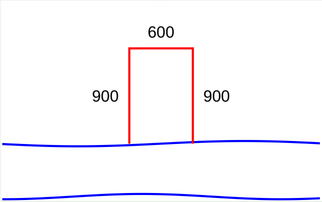

Area $= 900 \cdot 600 = 540,000\text{ft}^2$

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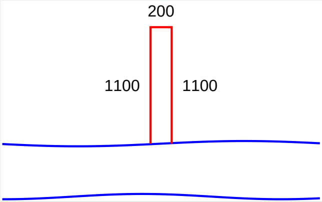

Area $= 1100 \cdot 200 = 220,000\text{ft}^2$

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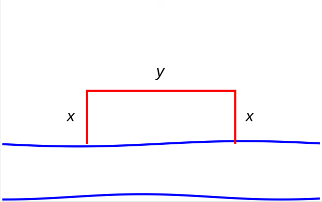

Let x and y denote the depth and width of the rectangle (in feet). Let A be its area.

Area $= A = xy$

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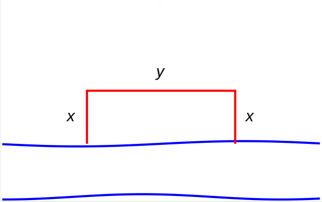

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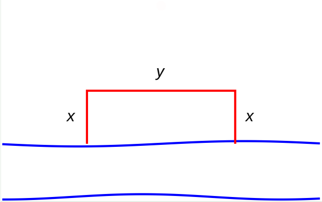

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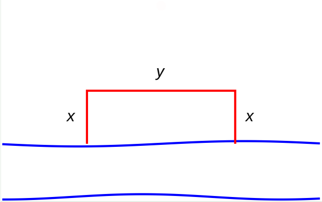

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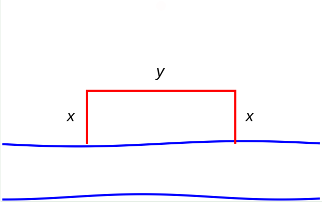

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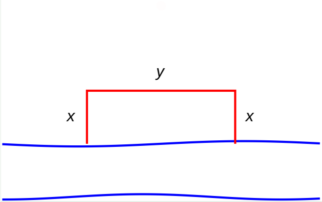

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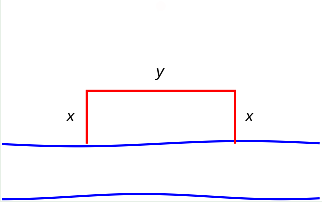

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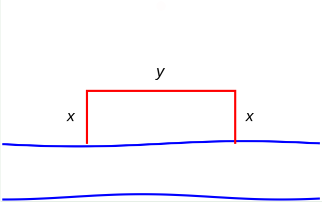

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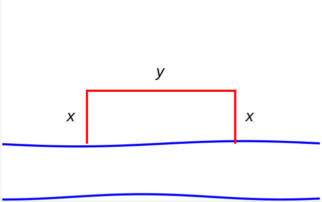

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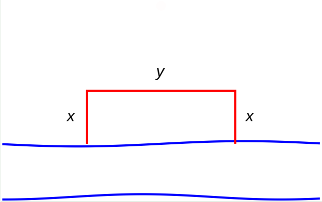

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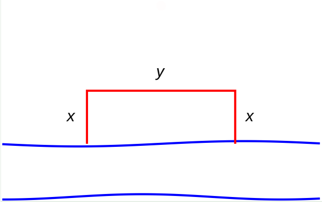

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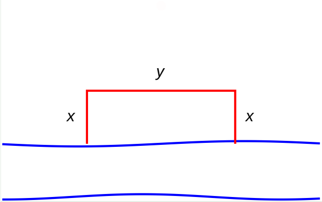

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x	$A(x)$
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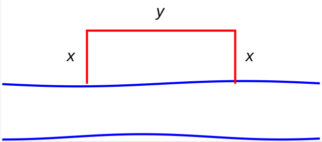

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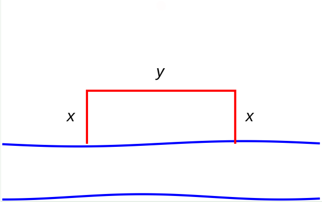

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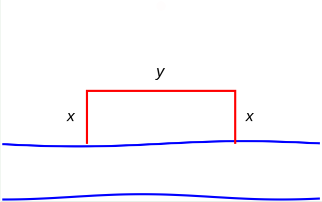

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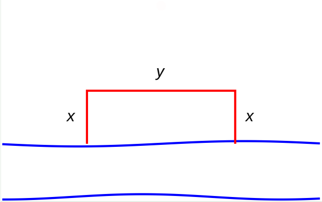

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x	$A(x)$
0	0
600	720,000
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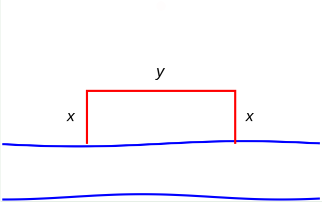

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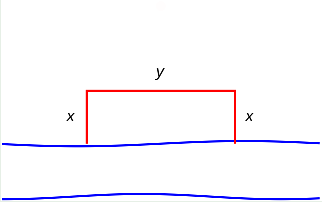

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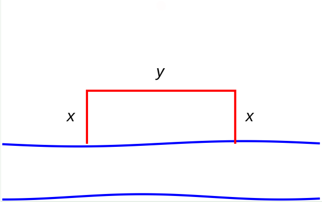

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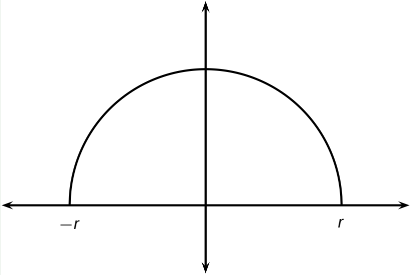
Therefore the maximum area occurs when $x = 600$ft and $y = 1200$ft.

Example

Find the largest possible area of a rectangle inscribed in a semicircle of radius r.



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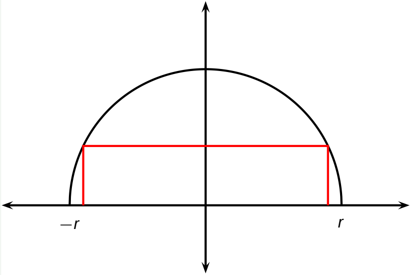

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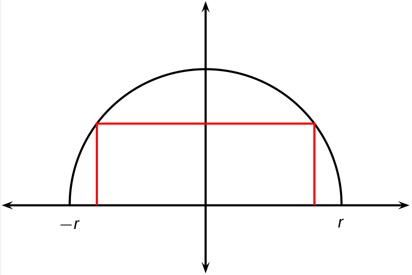

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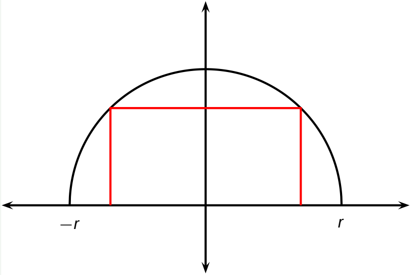

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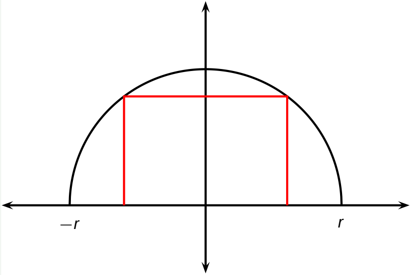

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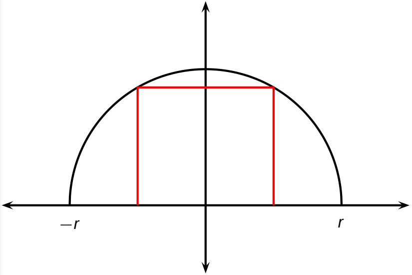

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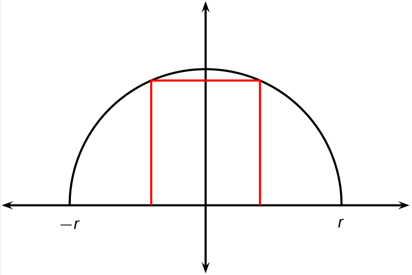

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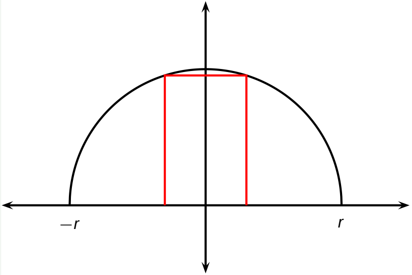

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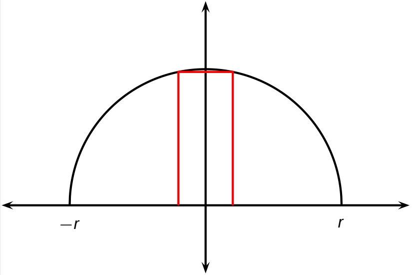

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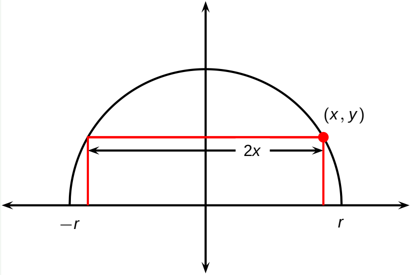

Let the semicircle have center at the origin. Let (x, y) be the coordinates of the top right corner of the rectangle. Let A be its area.

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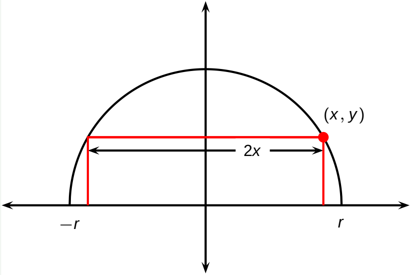

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$$A = 2xy$$

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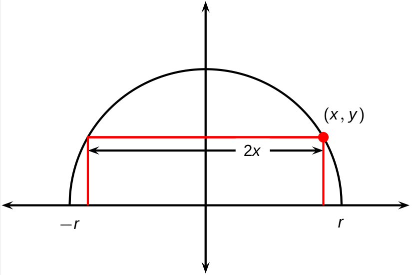

Let the semicircle have center at the origin. Let (x, y) be the coordinates of the top right corner of the rectangle. Let A be its area.

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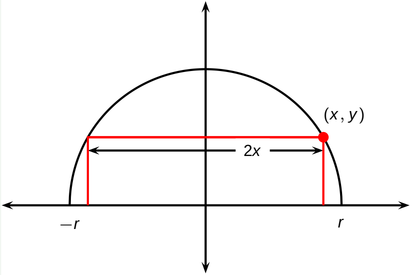

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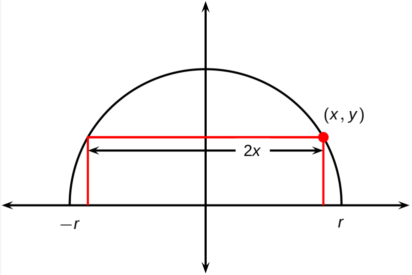

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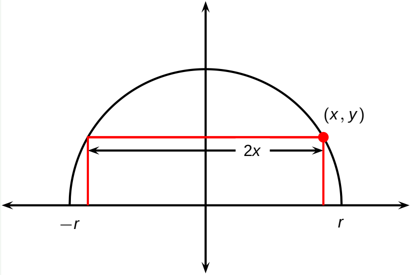

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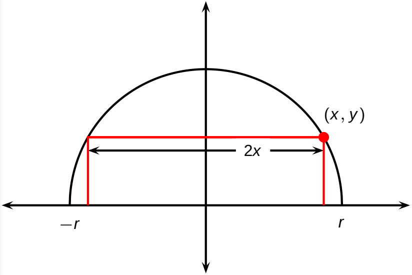

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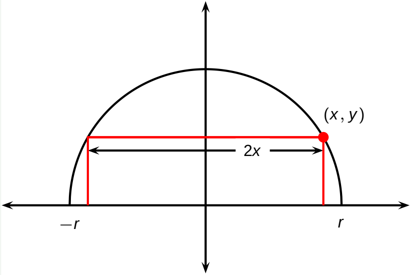

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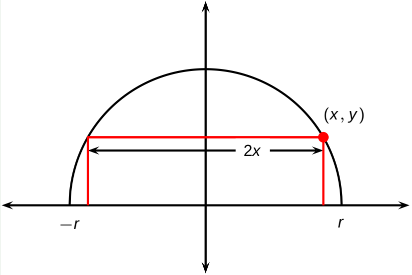

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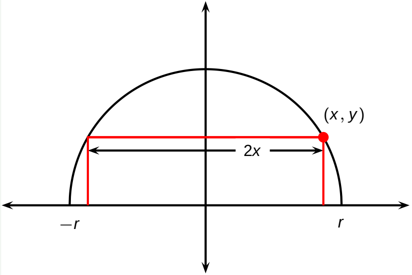

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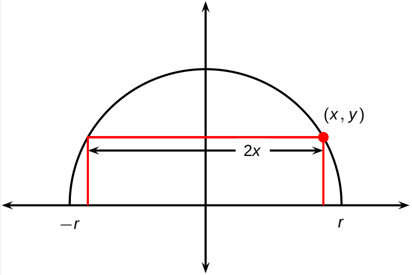

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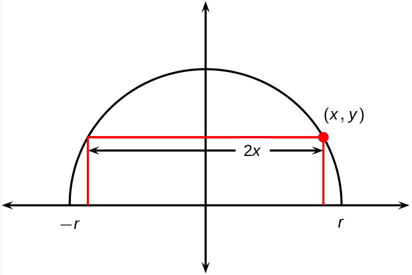

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There is a local max. here because $A(0) = 0 = A(r)$. Therefore the maximum area is $A(\frac{r}{\sqrt{2}}) = 2\frac{r}{\sqrt{2}}\sqrt{r^2 - \frac{r^2}{2}} = r^2$, achieved for $x = y = \frac{r}{\sqrt{2}}$

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Find the roots of these equations:

$$x^3 - 5x^2 - 6x = 0 \qquad\qquad 48x(1+x)^{60} - (1+x)^{60} + 1 = 0$$

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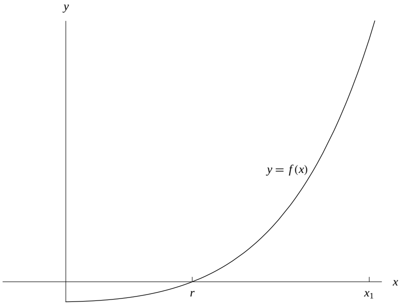

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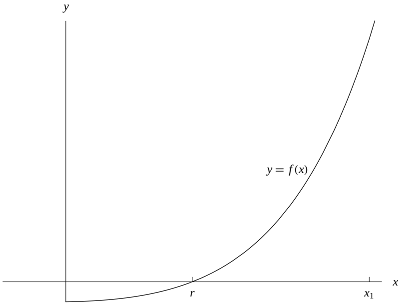

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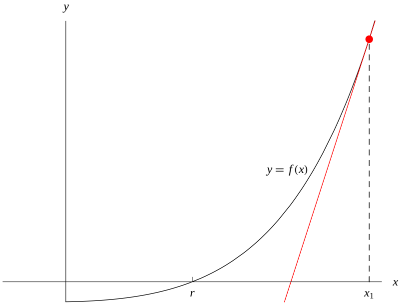

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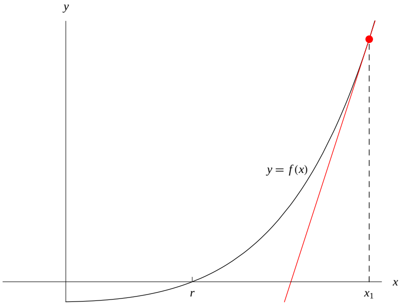

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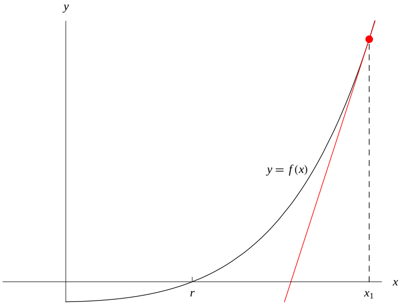

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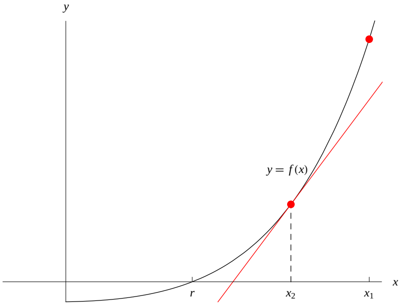

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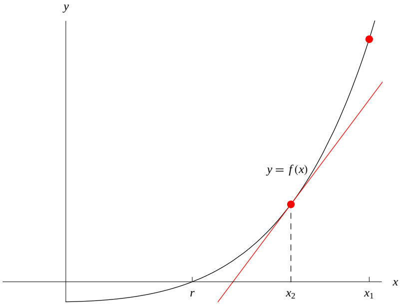

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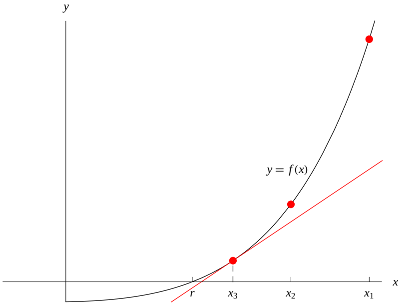

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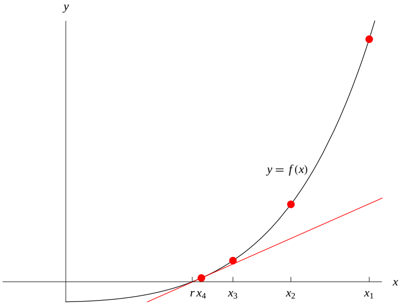

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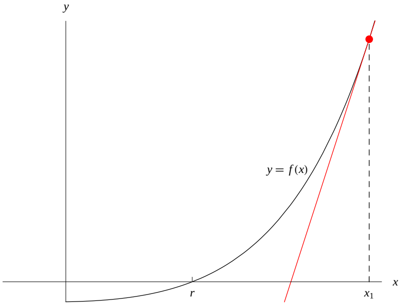

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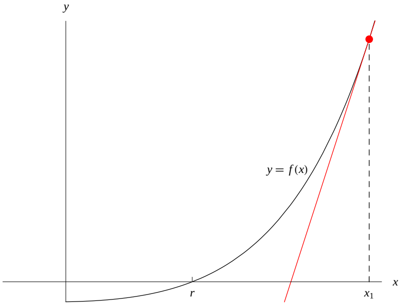

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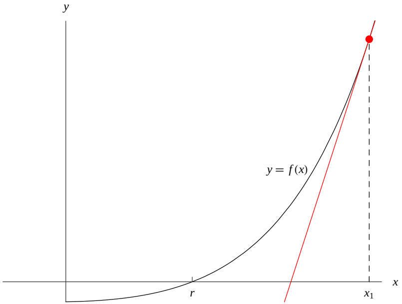

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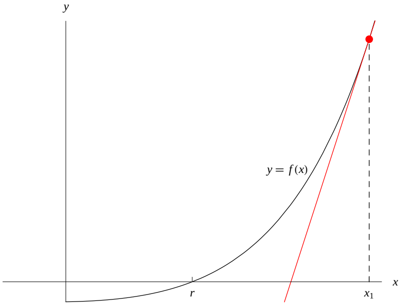

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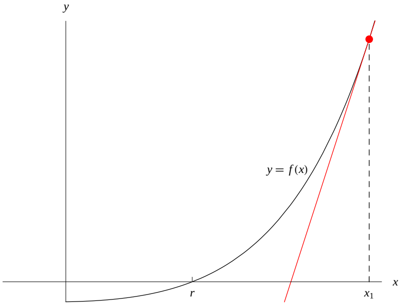

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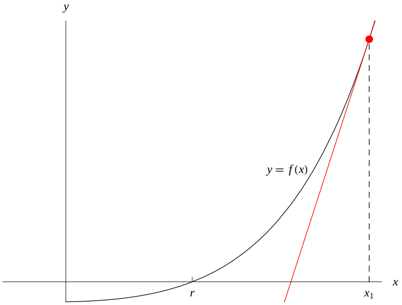

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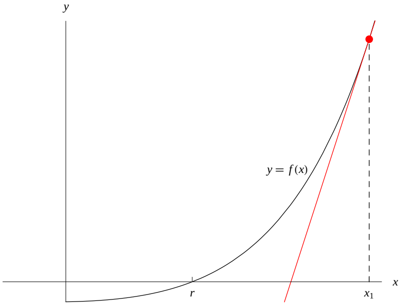

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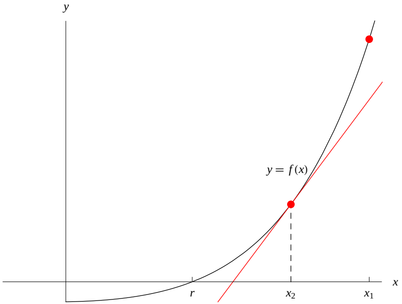

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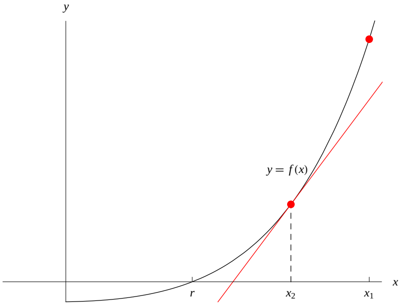

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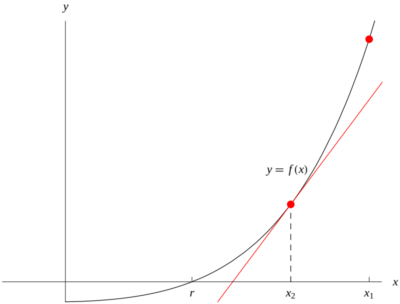

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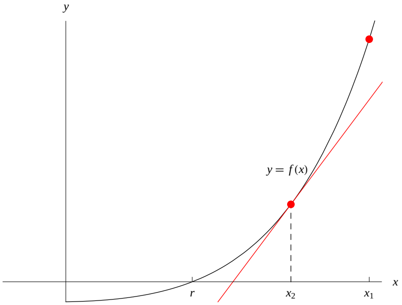

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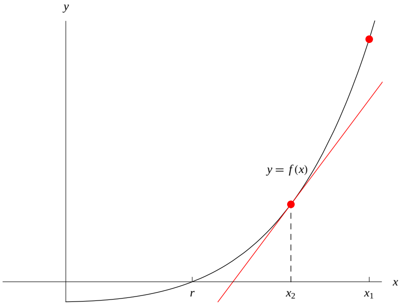
- Find the tangent to f at $(x_2, f(x_2))$.
- Call the x-intercept of this line x_3.

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

Equation: $y - f(x_2) = f'(x_2)(x - x_2)$

x-intercept: $0 - f(x_2) = f'(x_2)(x_3 - x_2)$

Goal: find a root r of $f(x)$.



- Pick a number x_1.
- Find the tangent to f at $(x_1, f(x_1))$.
- Call the x-intercept of this line x_2.
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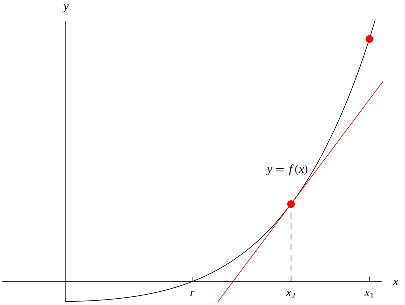

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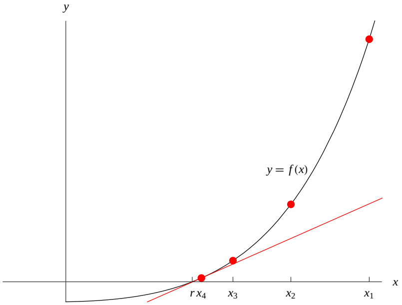

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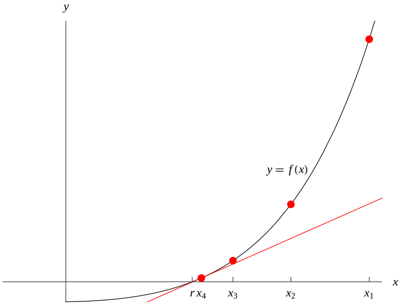

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$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

- Newton's Method gives us a sequence $x_1, x_2, x_3, \ldots$ of approximations to a root r of a function $f(x)$.
- If the nth approximation is x_n and $f'(x_n) \neq 0$, then the $(n+1)$st approximation is
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$
- If the numbers x_n become closer and closer to r as n becomes large, we say that the sequence converges to r.
- The sequence does not always converge.

Example (Newton's Method, Example 1, p. 313)

Starting with $x_1 = 2$, find the third approximation x_3 to the root of the equation $x^3 - 2x - 5 = 0$.

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Starting with $x_1 = 2$, find the third approximation x_3 to the root of the equation $x^3 - 2x - 5 = 0$.

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$$\begin{aligned} x_2 &= x_1 - \frac{{x_1}^3 - 2x_1 - 5}{3{x_1}^2 - 2} \\ &= (2) - \frac{(2)^3 - 2(2) - 5}{3(2)^2 - 2} \end{aligned}$$

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Example (Newton's Method)

Starting with $x_1 = 5$, use two steps of Newton's Method to approximate $\sqrt{28}$.

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Example (Newton's Method)

Starting with $x_1 = 5$, use two steps of Newton's Method to approximate $\sqrt{28}$.

$$f(x) = x^2 - 28.$$

$$f'(x) = 2x.$$

Newton's Method: $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^2 - 28}{}$$

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Starting with $x_1 = 5$, use two steps of Newton's Method to approximate $\sqrt{28}$.

$$f(x) = x^2 - 28.$$

$$f'(x) = 2x.$$

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$$\begin{aligned} x_2 &= x_1 - \frac{{x_1}^2 - 28}{2x_1} \\ &= (5) - \frac{(5)^2 - 28}{2(5)} \end{aligned}$$

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