Math 140

Lecture 20

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Outline

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1. Antiderivatives
   - Rectilinear Motion
2. Areas and Distances
   - The Area Problem

Antiderivatives

Definition (Antiderivative)

A function F is called an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I.

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- $\frac{1}{3}x^3 + 2$ will also work.
- Any function of the form $H(x) = \frac{1}{3}x^3 + C$, where C is a constant, is an antiderivative of f.

Theorem

If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is

$$F(x) + C$$

where C is an arbitrary constant.

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$$G(x) = \begin{cases} \ln|x| + C_1 & \text{if} \quad x > 0 \\ \ln|x| + C_2 & \text{if} \quad x < 0 \end{cases}$$

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Function	Particular Antiderivative
$cf(x)$	
$f(x) + g(x)$	
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Find the antiderivative:

$$\begin{aligned} g'(x) &= 4\sin x + 2x^4 - \frac{1}{\sqrt{x}} \\ g(x) &= 4(-\cos x) + 2\frac{x^5}{5} - \frac{x^{1/2}}{\frac{1}{2}} + C \\ &= -4\cos x + \frac{2}{5}x^5 - 2\sqrt{x} + C \end{aligned}$$

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$$\begin{aligned} f(x) &= \frac{x^{-1/2}}{-\frac{1}{2}} + C \\ &= -\frac{2}{\sqrt{x}} + C \end{aligned}$$

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Therefore

$$f(x) = -\frac{2}{\sqrt{x}} + 3.$$

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- Velocity is the antiderivative of acceleration.
- If we know the acceleration and the initial values $s(0)$ and $v(0)$ for position and velocity, then we can find $s(t)$ by antidifferentiating twice.

An object near the Earth is subject to a gravitational force that produces a downward acceleration of 32 ft/s^2 (or 9.8 m/s^2).

Example

A ball is thrown upward with a speed of 48 ft/s from the edge of a cliff 432 ft above the ground. Find its height above the ground t seconds later.

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The Area Problem

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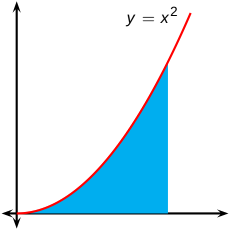

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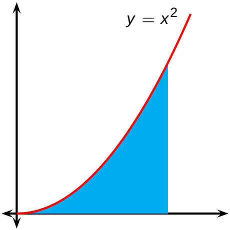

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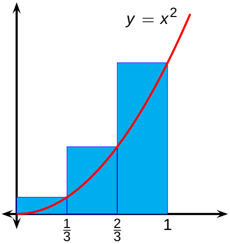

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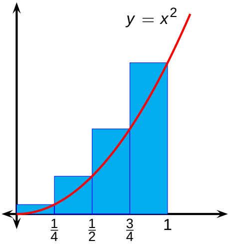

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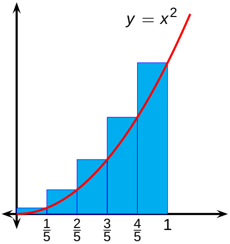

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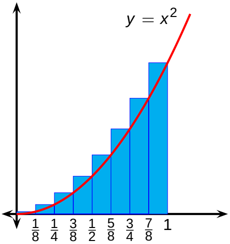

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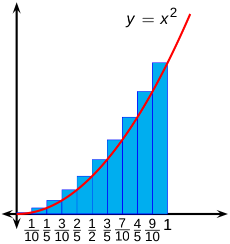

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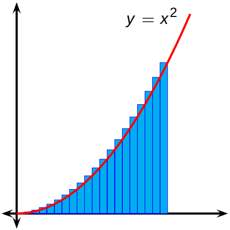

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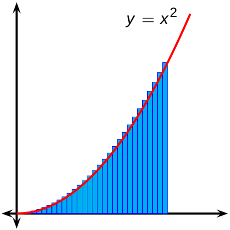

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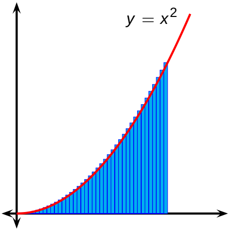

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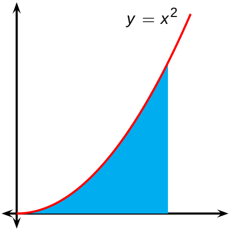

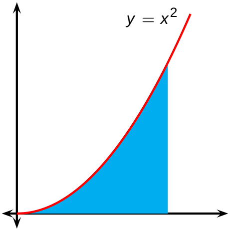

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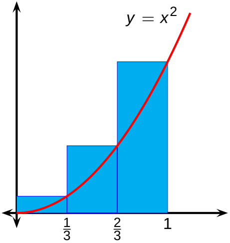

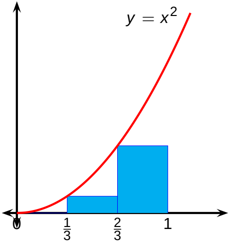

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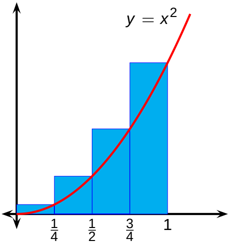

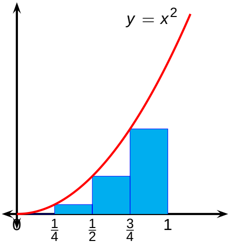

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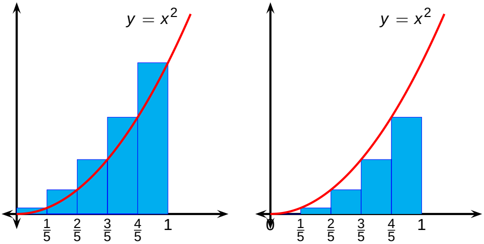

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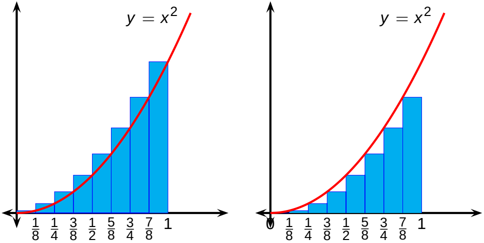

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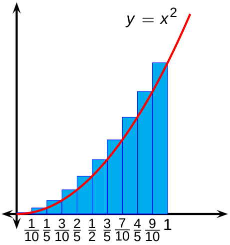

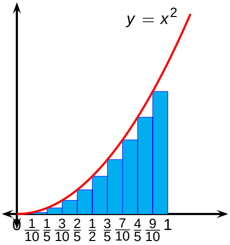

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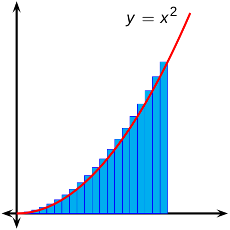

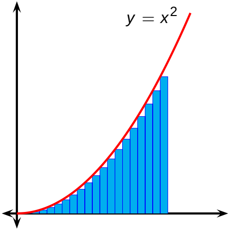

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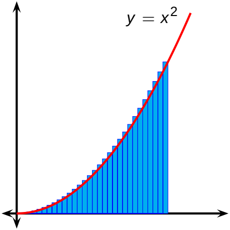

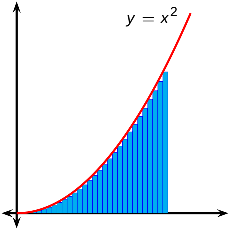

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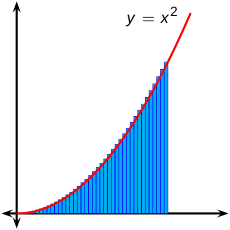
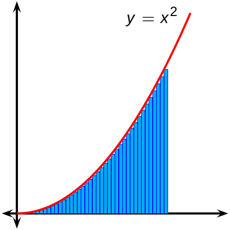
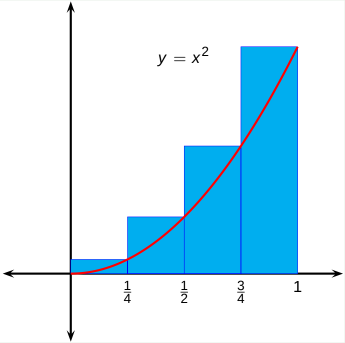




Example

Find the sum of the areas of the four approximating rectangles obtained using right endpoints.



Example

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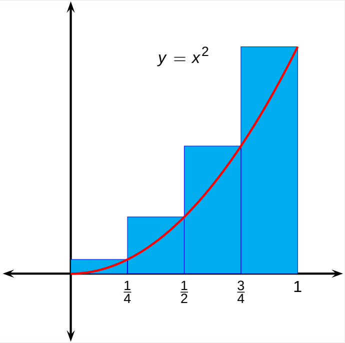
- Let R_4 denote the sum of the areas of the rectangles.



Example

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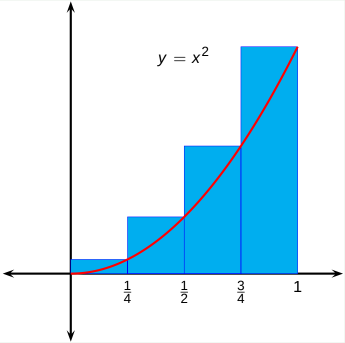
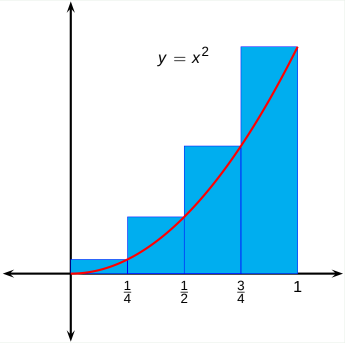
- Let R_4 denote the sum of the areas of the rectangles.
- Each rectangle has width



Example

Find the sum of the areas of the four approximating rectangles obtained using right endpoints.

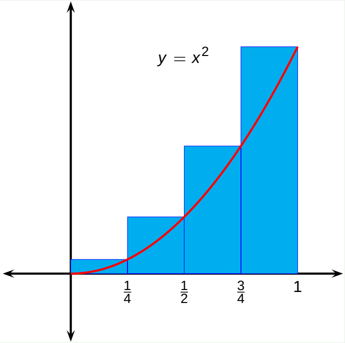
- Let R_4 denote the sum of the areas of the rectangles.
- Each rectangle has width $\frac{1}{4}$.



Example

Find the sum of the areas of the four approximating rectangles obtained using right endpoints.

- Let R_4 denote the sum of the areas of the rectangles.
- Each rectangle has width $\frac{1}{4}$.
- The heights are



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Find the sum of the areas of the four approximating rectangles obtained using right endpoints.

- Let R_4 denote the sum of the areas of the rectangles.
- Each rectangle has width $\frac{1}{4}$.
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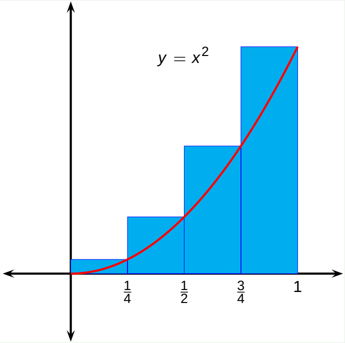

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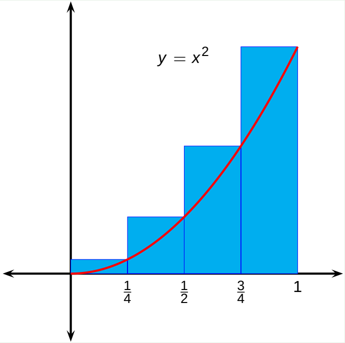

$$R_4 = \frac{1}{4} \cdot \left(\frac{1}{4}\right)^2 + \frac{1}{4} \cdot \left(\frac{1}{2}\right)^2 + \frac{1}{4} \cdot \left(\frac{3}{4}\right)^2 + \frac{1}{4} \cdot (1)^2$$

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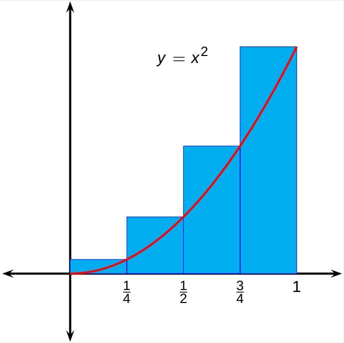

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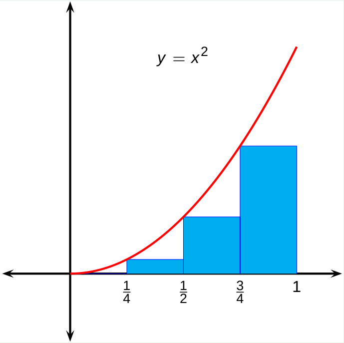
- A similar calculation works for L_4, the sum of the areas of the left endpoint rectangles.



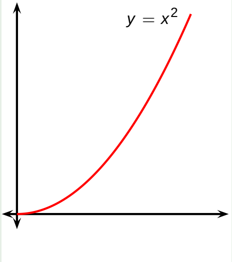
$$R_4 = \frac{1}{4} \cdot \left(\frac{1}{4}\right)^2 + \frac{1}{4} \cdot \left(\frac{1}{2}\right)^2 + \frac{1}{4} \cdot \left(\frac{3}{4}\right)^2 + \frac{1}{4} \cdot (1)^2 = \frac{15}{32} = 0.46875$$

$$L_4 = \frac{1}{4} \cdot (0)^2 + \frac{1}{4} \cdot \left(\frac{1}{4}\right)^2 + \frac{1}{4} \cdot \left(\frac{1}{2}\right)^2 + \frac{1}{4} \cdot \left(\frac{3}{4}\right)^2 = \frac{7}{32} = 0.21875$$

Example

For the region S underneath the parabola $y = x^2$ from 0 to 1, show that the area under the approximating rectangles approaches $\frac{1}{3}$, that is,

$$\lim_{n\to\infty} R_n = \frac{1}{3}.$$



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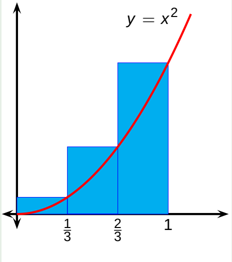

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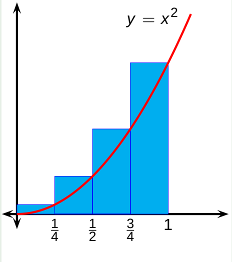

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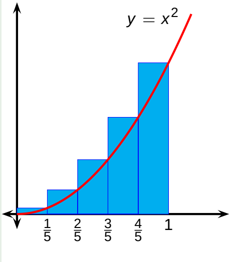

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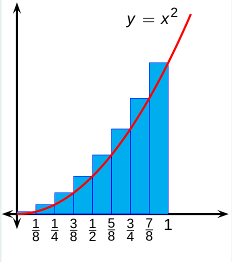

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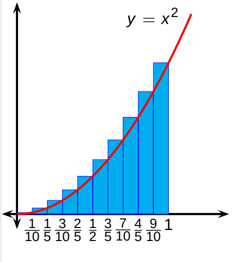

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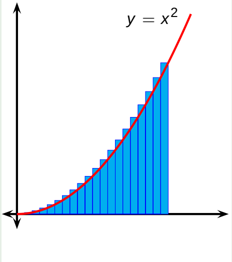

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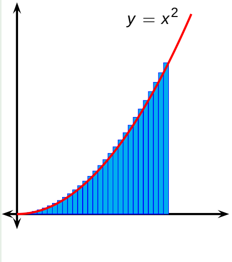

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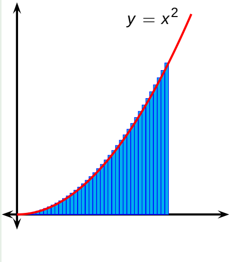

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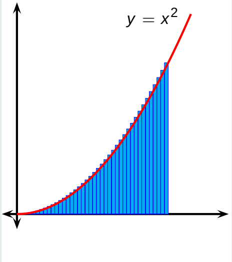

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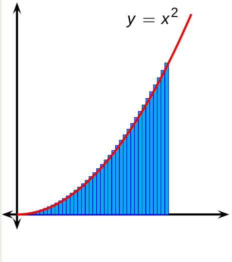

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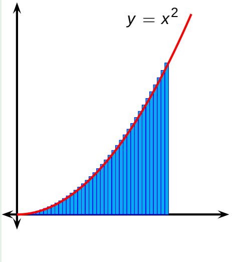

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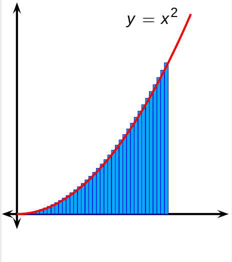

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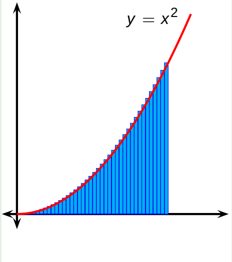

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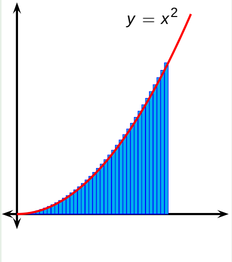

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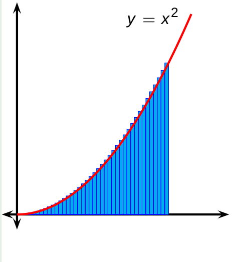

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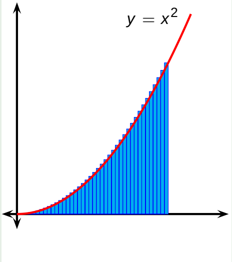

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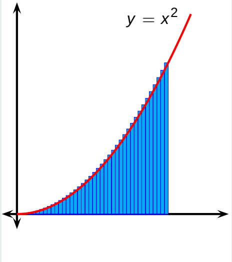

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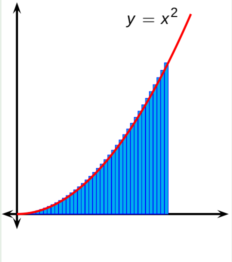

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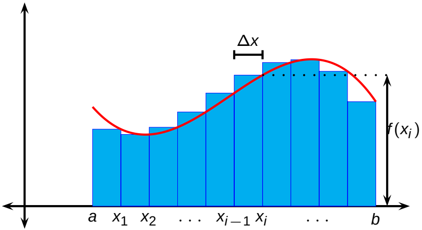
Estimate the area under the curve $y = f(x)$ between a and b.



- The right endpoints of the subintervals are

$$\begin{aligned} x_1 &= a + \Delta x \\ x_2 &= a + 2\Delta x \\ x_3 &= a + 3\Delta x \\ &\vdots \end{aligned}$$

- The width of the interval is $b - a$.
- The width of each of the n strips is $\Delta x = \frac{b-a}{n}$.
- The strips divide $[a, b]$ into n subintervals: $[x_0, x_1], [x_1, x_2], \dots, [x_{n-1}, x_n]$, where $x_0 = a$ and $x_n = b$.
- The height of the ith rectangle is $f(x_i)$.
- The area of the ith rectangle is $f(x_i)\Delta x$.

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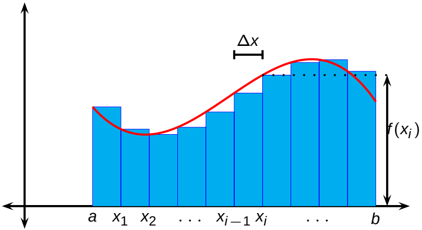

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- The left endpoints of the subintervals are

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Definition (Area Under a Curve)

The area of the region S that lies under the curve $y = f(x)$ is the limit of the sum of the areas of the approximating rectangles:

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty} [f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x]$$

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$$A = \lim_{n\to\infty} L_n = \lim_{n\to\infty} [f(x_0)\Delta x + f(x_1)\Delta x + \cdots + f(x_{n-1})\Delta x]$$
- We get the same answer if we use any number x_i^* in the interval $[x_{i-1}, x_i]$. x_i^* is called a sample point.
$$A = \lim_{n\to\infty} [f(x_1^*)\Delta x + f(x_2^*)\Delta x + \cdots + f(x_n^*)\Delta x]$$

Definition (Area Under a Curve)

The area of the region S that lies under the curve $y = f(x)$ is the limit of the sum of the areas of the approximating rectangles:

$$A = \lim_{n\to\infty} R_n = \lim_{n\to\infty}[f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x]$$

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Definition (Riemann Sum)

A Riemann sum is any sum of the form

$$f(x_1^*)\Delta x + f(x_2^*)\Delta x + \cdots + f(x_n^*)\Delta x.$$

$$\sum_{i=1}^{n} f(x_i)\Delta x = f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x$$

- We use sigma notation to write sums more compactly.

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Example

$$\sum_{i=1}^{4} 2i\Delta x = 2\Delta x + 4\Delta x + 6\Delta x + 8\Delta x$$

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$$\sum_{i=3}^{7} i^2\Delta x =$$

$$\sum_{i=1}^{n} f(x_i)\Delta x = f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x$$

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Example

$$\sum_{i=1}^{4} 2i\Delta x = 2\Delta x + 4\Delta x + 6\Delta x + 8\Delta x$$

$$\sum_{i=3}^{7} i^2\Delta x = 9\Delta x + 16\Delta x + 25\Delta x + 36\Delta x + 49\Delta x$$