Math 140

Lecture 21

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with modifications by T. Milev

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Outline

(5.2) The Definite Integral

Definition (Definite Integral)

- Let f be a function defined for $a \leq x \leq b$.
- Divide the interval $[a, b]$ into n subintervals of equal width $\Delta x = (b - a)/n$.
- Let $x_0(= a), x_1, x_2, \ldots, x_n(= b)$ be the endpoints of these subintervals.
- Let $x_1^*, x_2^*, \ldots, x_n^*$ be any sample points in these subintervals; that is, x_i^* is in $[x_{i-1}, x_i]$.

Then the definite integral of f from a to b is

$$\int_a^b f(x)\mathrm{d}x = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x,$$

provided that the limit exists. If the limit exists, we say that f is integrable on $[a, b]$.

$$\int_a^b f(x)\mathrm{d}x = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\Delta x,$$

- $\int$ is called the integration sign.
- $f(x)$ is called the integrand.
- a and b are called the limits of integration.

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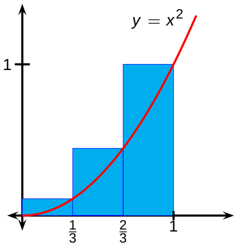
- $f(x)$ is called the integrand.
- a and b are called the limits of integration.
- The definite integral is a number. It does not depend on x. We could use any variable instead of x.

$$\int_a^b f(x)\mathrm{d}x = \int_a^b f(t)\mathrm{d}t = \int_a^b f(r)\mathrm{d}r = \int_a^b f(\theta)\mathrm{d}\theta$$

- We know already that if $f(x)$ is always positive, then $\int_a^b f(x)\mathrm{d}x$ is the area under the curve.



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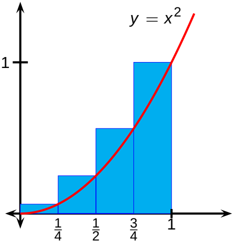

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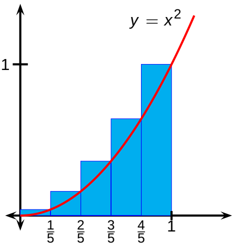

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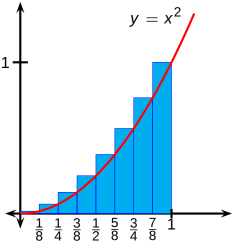

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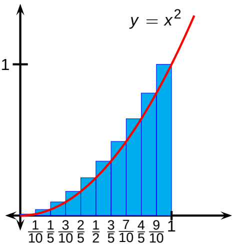

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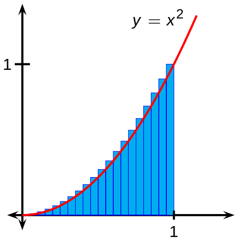

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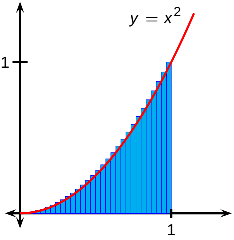

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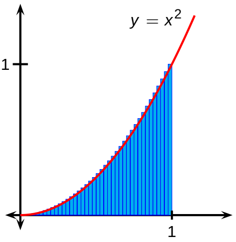

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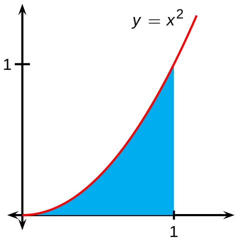

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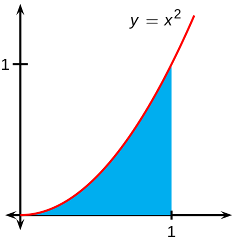

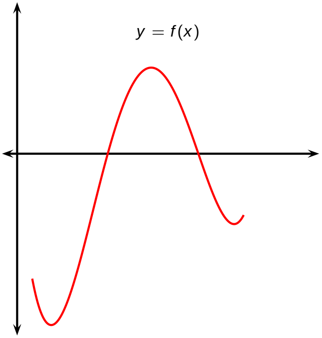

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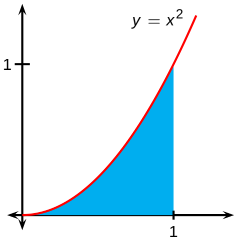

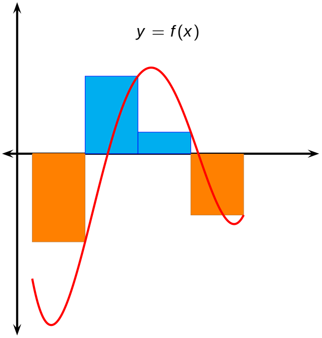

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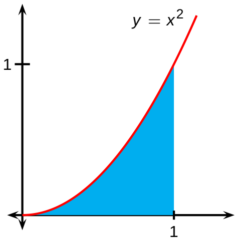

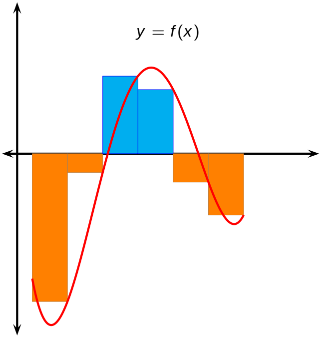

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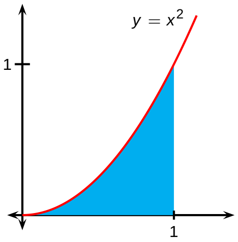

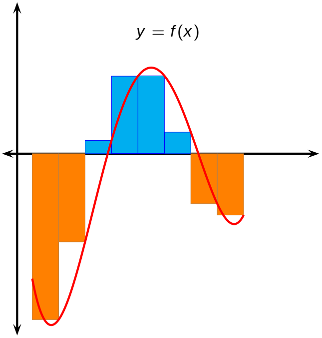

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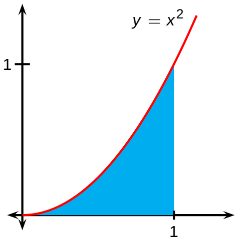

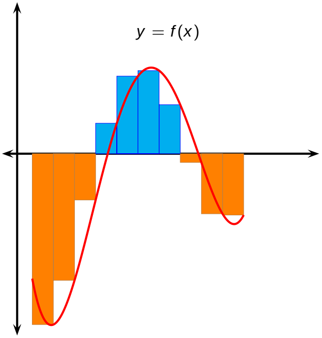

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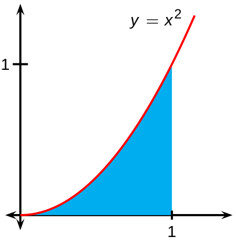

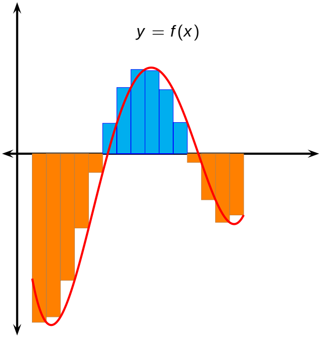

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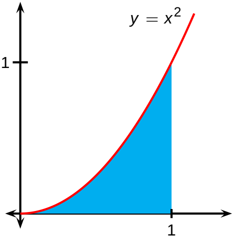

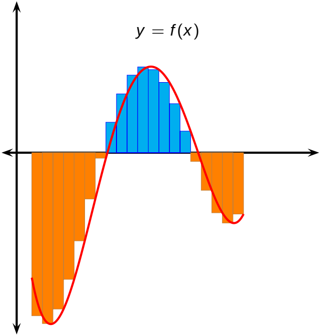

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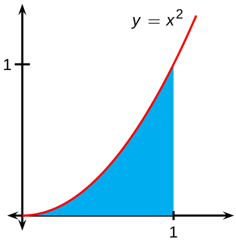

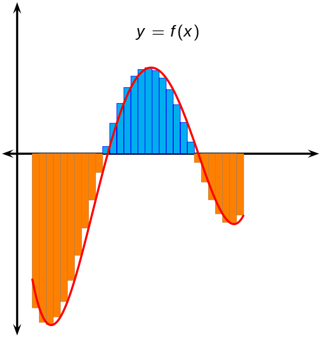

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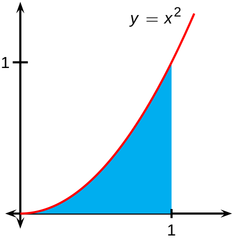

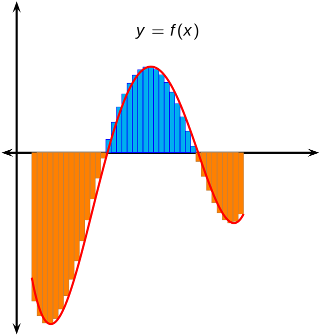

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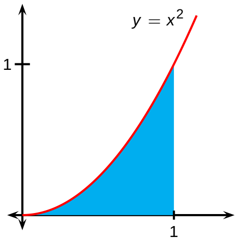

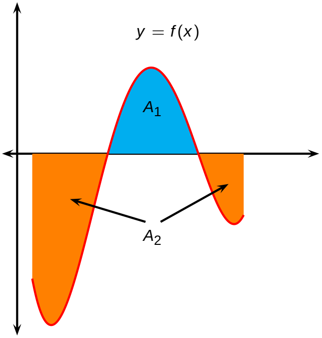

- What if $f(x)$ is sometimes negative?
- Then $\int_a^b f(x)\mathrm{d}x = A_1 - A_2$.
- A_1 is the area of the region above the x-axis and below the graph of f.
- A_2 is the area of the region below the x-axis and above the graph of f.

There are some formulas that are useful for evaluating integrals.

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$$\sum_{i=1}^{n} c = nc$$

$$\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$$

$$\sum_{i=1}^{n}(a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$$

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$$= \lim_{n\to\infty}\left(\frac{81}{n^4}\left[\frac{n(n+1)}{2}\right]^2 - \frac{54}{n^2}\frac{n(n+1)}{2}\right)$$

$$= \lim_{n\to\infty}\left[\frac{81}{4}\left(1+\frac{1}{n}\right)^2 - 27\left(1+\frac{1}{n}\right)\right]$$

Example

Evaluate $\int_0^3 (x^3 - 6x)\mathrm{d}x$. $\Delta x = \frac{b-a}{n} = \frac{3}{n}$.

$$\int_0^3 (x^3 - 6x)\mathrm{d}x = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{3i}{n}\right)\frac{3}{n}$$

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$$= \lim_{n\to\infty} \left[\frac{81}{4}\left(1 + \frac{1}{n}\right)^2 - 27\left(1 + \frac{1}{n}\right)\right] = \frac{81}{4} - 27 = -\frac{27}{4}$$

Properties of the Definite Integral

- So far when we have calculated $\int_a^b f(x)\mathrm{d}x$, we have assumed that $a < b$.
- The definition as a limit of Riemann sums will still work even if we don't assume this.

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Properties of the Integral

1. $\int_a^b c\mathrm{d}x = c(b-a)$, where c is any constant.
2. $\int_a^b [f(x)+g(x)]\mathrm{d}x = \int_a^b f(x)\mathrm{d}x + \int_a^b g(x)\mathrm{d}x$.
3. $\int_a^b cf(x)\mathrm{d}x = c\int_a^b f(x)\mathrm{d}x$, where c is any constant.
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Example

Use the properties of integrals to evaluate

$$\int_0^1 (4 + 3x^2)\mathrm{d}x$$

Example

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$$\int_0^1 (4 + 3x^2)\mathrm{d}x = \int_0^1 4\mathrm{d}x + \int_0^1 3x^2\mathrm{d}x \qquad \text{Property}$$

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Example

Use the properties of integrals to evaluate

$$\int_0^1 (4+3x^2)dx = \int_0^1 4dx + \int_0^1 3x^2 dx \qquad \text{Property 2}$$

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Use the properties of integrals to evaluate

$$\begin{aligned}\int_0^1 (4+3x^2)\mathrm{d}x &= \int_0^1 4\mathrm{d}x + \int_0^1 3x^2\mathrm{d}x && \text{Property 2}\\ &= \int_0^1 4\mathrm{d}x + 3\int_0^1 x^2\mathrm{d}x && \text{Property 3}\\ &= 4(1-0) + 3\int_0^1 x^2\mathrm{d}x && \text{Property 1}\end{aligned}$$

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$$= 4 + 3 \cdot$$

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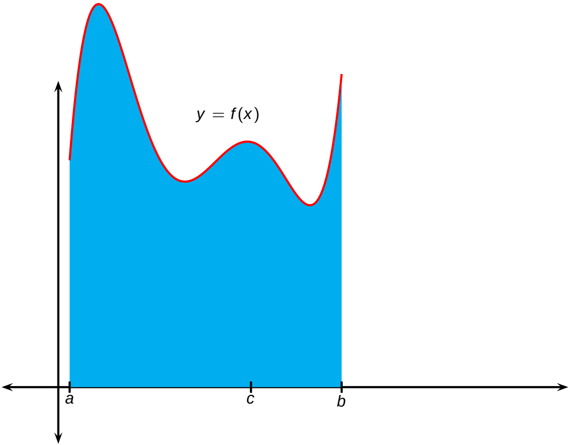
$$= 4 + 3\cdot\frac{1}{3} \qquad \text{From preceding lectures/slides}$$

$$= 5$$

Properties of the Integral

5. $\int_a^b f(x)\mathrm{d}x = \int_a^c f(x)\mathrm{d}x + \int_c^b f(x)\mathrm{d}x$



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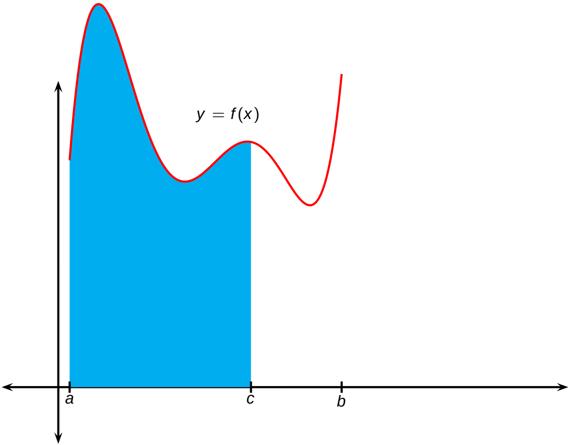

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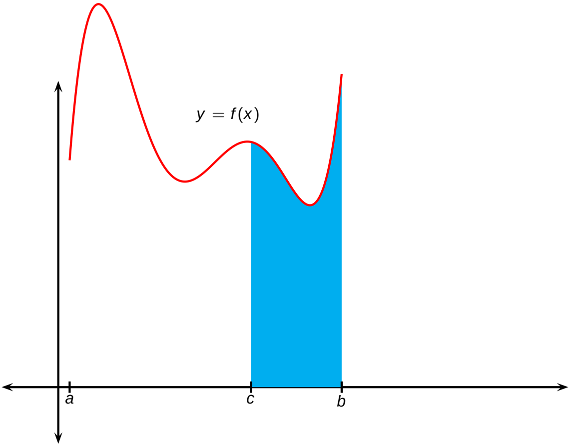

Example

If it is known that $\int_0^{10} f(x)\mathrm{d}x = 17$ and $\int_0^{8} f(x)\mathrm{d}x = 12$, then find $\int_8^{10} f(x)\mathrm{d}x$.

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$$= \quad -$$

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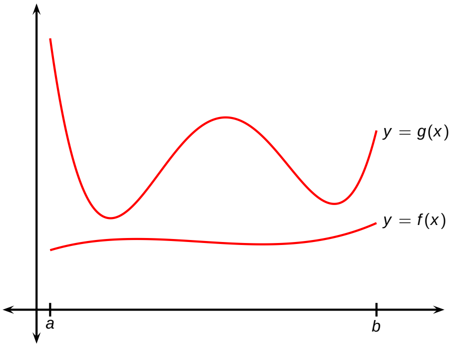
Comparison Properties of the Integral

6. If $f(x) \geq 0$ for all $a \leq x \leq b$, then $\displaystyle\int_a^b f(x)\mathrm{d}x \geq 0$.

Comparison Properties of the Integral

7. If $f(x) \leq g(x)$ for all $a \leq x \leq b$, then $\int_a^b f(x)dx \leq \int_a^b g(x)dx$.



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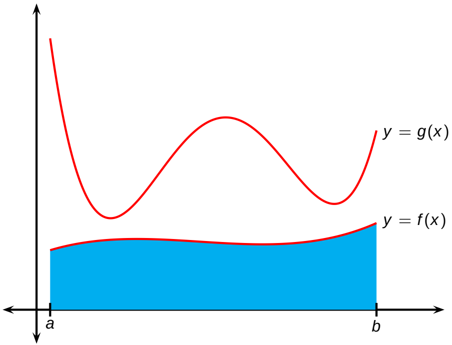

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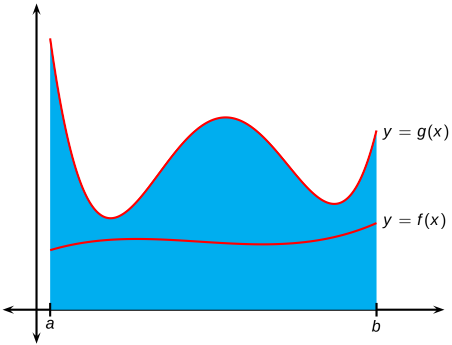
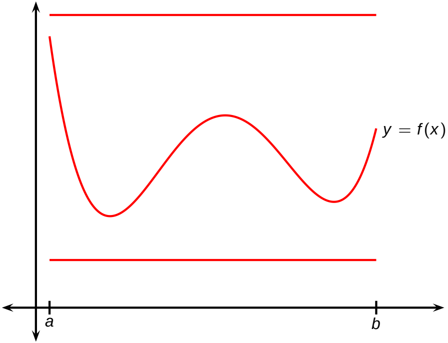


$$\int_a^b f(x)\mathrm{d}x \le \int_a^b g(x)\mathrm{d}x$$

Comparison Properties of the Integral

8. If $m \le f(x) \le M$ for all $a \le x \le b$, then

$$m(b-a) \le \int_a^b f(x)\mathrm{d}x \le M(b-a)$$



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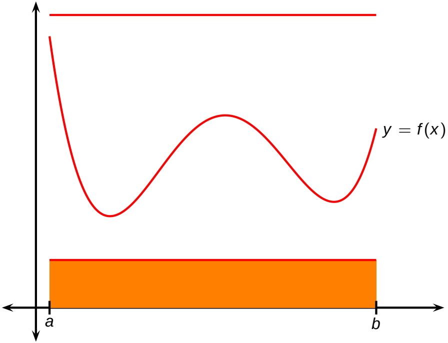

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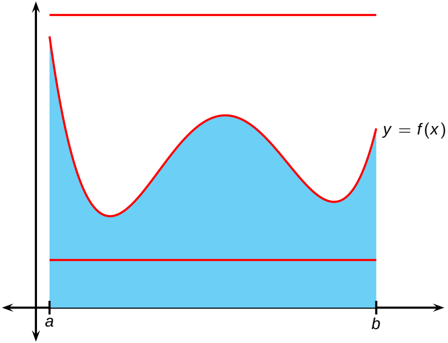

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