Math 140

Lecture 22

Greg Maloney

with modifications by T. Milev

University of Massachusetts Boston

April 30, 2013

Outline

Theorem (The Evaluation Theorem)

If f is continuous on $[a, b]$, then

$$\int_a^b f(x)dx = F(b) - F(a),$$

where F is any antiderivative of f.

Example

Evaluate the integral $\int_{-2}^{1} x^3 \, \mathrm{d}x$.

Example

Evaluate the integral $\int_{-2}^{1} x^3 \, \mathrm{d}x$.

- x^3 is continuous on $[-2, 1]$ (in fact, it's continuous everywhere).

Example

Evaluate the integral $\int_{-2}^{1} x^3 \, \mathrm{d}x$.

- x^3 is continuous on $[-2, 1]$ (in fact, it's continuous everywhere).
- An antiderivative is $F(x) =$

Example

Evaluate the integral $\int_{-2}^{1} x^3 \, \mathrm{d}x$.

- x^3 is continuous on $[-2, 1]$ (in fact, it's continuous everywhere).
- An antiderivative is $F(x) = \frac{1}{4}x^4$.

Example

Evaluate the integral $\int_{-2}^{1} x^3 \, dx$.

- x^3 is continuous on $[-2, 1]$ (in fact, it's continuous everywhere).
- An antiderivative is $F(x) = \frac{1}{4}x^4$.

$$\int_{-2}^{1} x^3 \, dx = F(1) - F(-2)$$

Example

Evaluate the integral $\int_{-2}^{1} x^3 \, dx$.

- x^3 is continuous on $[-2, 1]$ (in fact, it's continuous everywhere).
- An antiderivative is $F(x) = \frac{1}{4}x^4$.

$$\int_{-2}^{1} x^3 \, dx = F(1) - F(-2) = \frac{1}{4}(1)^4 - \frac{1}{4}(-2)^4$$

Example

Evaluate the integral $\int_{-2}^{1} x^3 \, \mathrm{d}x$.

- x^3 is continuous on $[-2, 1]$ (in fact, it's continuous everywhere).
- An antiderivative is $F(x) = \frac{1}{4}x^4$.

$$\int_{-2}^{1} x^3 \, \mathrm{d}x = F(1) - F(-2) = \frac{1}{4}(1)^4 - \frac{1}{4}(-2)^4 = \frac{1}{4} - \frac{16}{4} = -\frac{15}{4}$$

We often use the notation

$$F(x)]_a^b = F(b) - F(a)$$

or

$$[F(x)]_a^b = F(b) - F(a)$$

Therefore we can write

$$\int_a^b f(x)\mathrm{d}x = F(x)]_a^b$$

or

$$\int_a^b f(x)\mathrm{d}x = [F(x)]_a^b$$

Example

Find the area under the parabola $y = x^2$ from 0 to 1.

Example

Find the area under the parabola $y = x^2$ from 0 to 1.

- x^2 is continuous on $[0, 1]$ (in fact, it's continuous everywhere).

Example

Find the area under the parabola $y = x^2$ from 0 to 1.

- x^2 is continuous on $[0, 1]$ (in fact, it's continuous everywhere).
- An antiderivative is

Example

Find the area under the parabola $y = x^2$ from 0 to 1.

- x^2 is continuous on $[0, 1]$ (in fact, it's continuous everywhere).
- An antiderivative is $\frac{1}{3}x^3$.

Example

Find the area under the parabola $y = x^2$ from 0 to 1.

- x^2 is continuous on $[0, 1]$ (in fact, it's continuous everywhere).
- An antiderivative is $\frac{1}{3}x^3$.

$$\int_0^1 x^2 \, dx = \left[\frac{1}{3}x^3\right]_0^1$$

Example

Find the area under the parabola $y = x^2$ from 0 to 1.

- x^2 is continuous on $[0, 1]$ (in fact, it's continuous everywhere).
- An antiderivative is $\frac{1}{3}x^3$.

$$\int_0^1 x^2 \, dx = \left[\frac{1}{3}x^3\right]_0^1 = \frac{1}{3}(1)^3 - \frac{1}{3}(0)^3$$

Example

Find the area under the parabola $y = x^2$ from 0 to 1.

- x^2 is continuous on $[0, 1]$ (in fact, it's continuous everywhere).
- An antiderivative is $\frac{1}{3}x^3$.

$$\int_0^1 x^2 \, dx = \left[\frac{1}{3}x^3\right]_0^1 = \frac{1}{3}(1)^3 - \frac{1}{3}(0)^3 = \frac{1}{3}$$

Example

Find the area under the cosine curve from 0 to b, where $0 \leq b \leq \pi/2$.

Example

Find the area under the cosine curve from 0 to b, where $0 \leq b \leq \pi/2$.

- $\cos x$ is continuous on $[0, \pi/2]$ (in fact, it's continuous everywhere).

Example

Find the area under the cosine curve from 0 to b, where $0 \leq b \leq \pi/2$.

- $\cos x$ is continuous on $[0, \pi/2]$ (in fact, it's continuous everywhere).
- An antiderivative is

Example

Find the area under the cosine curve from 0 to b, where $0 \leq b \leq \pi/2$.

- $\cos x$ is continuous on $[0, \pi/2]$ (in fact, it's continuous everywhere).
- An antiderivative is $\sin x$.

Example

Find the area under the cosine curve from 0 to b, where $0 \leq b \leq \pi/2$.

- $\cos x$ is continuous on $[0, \pi/2]$ (in fact, it's continuous everywhere).
- An antiderivative is $\sin x$.

$$\int_0^b \cos x \, dx = [\sin x]_0^b$$

Example

Find the area under the cosine curve from 0 to b, where $0 \leq b \leq \pi/2$.

- $\cos x$ is continuous on $[0, \pi/2]$ (in fact, it's continuous everywhere).
- An antiderivative is $\sin x$.

$$\int_0^b \cos x \, dx = [\sin x]_0^b = \sin(b) - \sin(0)$$

Example

Find the area under the cosine curve from 0 to b, where $0 \leq b \leq \pi/2$.

- $\cos x$ is continuous on $[0, \pi/2]$ (in fact, it's continuous everywhere).
- An antiderivative is $\sin x$.

$$\int_0^b \cos x \, dx = [\sin x]_0^b = \sin(b) - \sin(0) = \sin b$$

Indefinite Integrals

- The Evaluation Theorem establishes a connection between antiderivatives and definite integrals.
- It says that $\int_a^b f(x)\mathrm{d}x$ equals $F(b) - F(a)$, where F is an antiderivative of f.
- We need convenient notation for writing antiderivatives.
- This is what the indefinite integral is.

Definition (Indefinite Integral)

The indefinite integral of f is another way of saying the antiderivative of f, and is written $\int f(x)\mathrm{d}x$. In other words,

$$\int f(x)\mathrm{d}x = F(x) \qquad \text{means} \qquad F'(x) = f(x).$$

Example

$$\int x^4 \mathrm{d}x =$$

Example

$$\int x^4 \mathrm{d}x = \frac{x^5}{5}$$

Example

$$\int x^4 dx = \frac{x^5}{5} + C$$

Example

$$\int x^4 \mathrm{d}x = \frac{x^5}{5} + C$$

because

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{x^5}{5} + C\right) = x^4.$$

Example

$$\int x^4 dx = \frac{x^5}{5} + C$$

because

$$\frac{d}{dx}\left(\frac{x^5}{5} + C\right) = x^4.$$

- This shows that the indefinite integral is a whole family of functions.

Example

$$\int x^4 dx = \frac{x^5}{5} + C$$

because

$$\frac{d}{dx}\left(\frac{x^5}{5} + C\right) = x^4.$$

- This shows that the indefinite integral is a whole family of functions.
- Example 1b, p. 318: the general antiderivative of $\frac{1}{x}$ is

$$F(x) = \begin{cases} \ln|x| + C_1 & \text{if} \quad x > 0 \\ \ln|x| + C_2 & \text{if} \quad x < 0 \end{cases}$$

Example

$$\int x^4 \mathrm{d}x = \frac{x^5}{5} + C$$

because

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{x^5}{5} + C\right) = x^4.$$

- This shows that the indefinite integral is a whole family of functions.
- Example 1b, p. 318: the general antiderivative of $\frac{1}{x}$ is

$$F(x) = \begin{cases} \ln|x| + C_1 & \text{if} \quad x > 0 \\ \ln|x| + C_2 & \text{if} \quad x < 0 \end{cases}$$

- We adopt the convention that the formula for an indefinite integral is only valid on one interval.

Example

$$\int x^4 dx = \frac{x^5}{5} + C$$

because

$$\frac{d}{dx}\left(\frac{x^5}{5} + C\right) = x^4.$$

- This shows that the indefinite integral is a whole family of functions.
- Example 1b, p. 318: the general antiderivative of $\frac{1}{x}$ is

$$F(x) = \begin{cases} \ln|x| + C_1 & \text{if} \quad x > 0 \\ \ln|x| + C_2 & \text{if} \quad x < 0 \end{cases}$$

- We adopt the convention that the formula for an indefinite integral is only valid on one interval.
- $\int \frac{1}{x} dx = \ln|x| + C$, and this is valid either on $(-\infty, 0)$ or $(0, \infty)$.

Example

Find the general indefinite integral.

$$\int (10x^4 - 2\sec^2 x)\mathrm{d}x$$

Example

Find the general indefinite integral.

$$\int (10x^4 - 2\sec^2 x) \mathrm{d}x = 10 \int x^4 \mathrm{d}x - 2 \int \sec^2 x \mathrm{d}x$$

Example

Find the general indefinite integral.

$$\int (10x^4 - 2\sec^2 x)\mathrm{d}x = 10\int x^4 \mathrm{d}x - 2\int \sec^2 x \mathrm{d}x$$

$$= 10 \qquad - 2$$

Example

Find the general indefinite integral.

$$\int (10x^4 - 2\sec^2 x) dx = 10 \int x^4 dx - 2 \int \sec^2 x dx$$

$$= 10 \frac{x^5}{5} - 2$$

Example

Find the general indefinite integral.

$$\int (10x^4 - 2\sec^2 x)\mathrm{d}x = 10\int x^4\mathrm{d}x - 2\int \sec^2 x\mathrm{d}x$$

$$= 10\frac{x^5}{5} - 2$$

Example

Find the general indefinite integral.

$$\int (10x^4 - 2\sec^2 x)\mathrm{d}x = 10\int x^4 \mathrm{d}x - 2\int \sec^2 x \mathrm{d}x$$

$$= 10\frac{x^5}{5} - 2\tan x$$

Example

Find the general indefinite integral.

$$\int (10x^4 - 2\sec^2 x)\mathrm{d}x = 10\int x^4 \mathrm{d}x - 2\int \sec^2 x \mathrm{d}x$$

$$= 10\frac{x^5}{5} - 2\tan x + C$$

Example

Find the general indefinite integral.

$$\begin{aligned}\int (10x^4 - 2\sec^2 x)\mathrm{d}x &= 10\int x^4 \mathrm{d}x - 2\int \sec^2 x \mathrm{d}x \\ &= 10\frac{x^5}{5} - 2\tan x + C \\ &= 2x^5 - 2\tan x + C\end{aligned}$$

Example

Find the general indefinite integral.

$$\int \frac{\cos\theta}{\sin^2\theta} \mathrm{d}\theta$$

Example

Find the general indefinite integral.

$$\int \frac{\cos\theta}{\sin^2\theta} d\theta = \int \left(\frac{1}{\sin\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right) d\theta$$

Example

Find the general indefinite integral.

$$\int \frac{\cos\theta}{\sin^2\theta} d\theta = \int \left(\frac{1}{\sin\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right) d\theta$$
$$= \int \qquad\qquad d\theta$$

Example

Find the general indefinite integral.

$$\int \frac{\cos\theta}{\sin^2\theta} d\theta = \int \left(\frac{1}{\sin\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right) d\theta$$
$$= \int \csc\theta \qquad d\theta$$

Example

Find the general indefinite integral.

$$\int \frac{\cos\theta}{\sin^2\theta} d\theta = \int \left(\frac{1}{\sin\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right) d\theta$$
$$= \int \csc\theta \qquad d\theta$$

Example

Find the general indefinite integral.

$$\int \frac{\cos\theta}{\sin^2\theta} d\theta = \int \left(\frac{1}{\sin\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right) d\theta$$
$$= \int \csc\theta \cot\theta d\theta$$

Example

Find the general indefinite integral.

$$\int \frac{\cos\theta}{\sin^2\theta} d\theta = \int \left(\frac{1}{\sin\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right) d\theta$$
$$= \int \csc\theta \cot\theta d\theta$$
$$=$$

Example

Find the general indefinite integral.

$$\int \frac{\cos\theta}{\sin^2\theta} d\theta = \int \left(\frac{1}{\sin\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right) d\theta$$
$$= \int \csc\theta \cot\theta d\theta$$
$$= -\csc\theta$$

Example

Find the general indefinite integral.

$$\int \frac{\cos\theta}{\sin^2\theta}d\theta = \int \left(\frac{1}{\sin\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right)d\theta$$
$$= \int \csc\theta\cot\theta d\theta$$
$$= -\csc\theta + C$$

Example

$$\int_0^3 (x^3 - 6x)\mathrm{d}x$$

Example

$$\int_0^3 (x^3 - 6x)\mathrm{d}x = \left[\int (x^3 - 6x)\mathrm{d}x\right]_0^3$$

Example

$$\int_0^3 (x^3 - 6x)\mathrm{d}x = \left[\int (x^3 - 6x)\mathrm{d}x\right]_0^3$$
$$= \left[\int x^3\mathrm{d}x - 6\int x\mathrm{d}x\right]_0^3$$

Example

$$\int_0^3 (x^3 - 6x)\mathrm{d}x = \left[\int (x^3 - 6x)\mathrm{d}x\right]_0^3$$
$$= \left[\int x^3\mathrm{d}x - 6\int x\mathrm{d}x\right]_0^3$$
$$= \left[\quad - 6 \quad\right]_0^3$$

Example

$$\int_0^3 (x^3 - 6x)\mathrm{d}x = \left[\int (x^3 - 6x)\mathrm{d}x\right]_0^3$$
$$= \left[\int x^3\mathrm{d}x - 6\int x\mathrm{d}x\right]_0^3$$
$$= \left[\frac{x^4}{4} - 6 \quad\right]_0^3$$

Example

$$\int_0^3 (x^3 - 6x)\mathrm{d}x = \left[\int (x^3 - 6x)\mathrm{d}x\right]_0^3$$
$$= \left[\int x^3\mathrm{d}x - 6\int x\mathrm{d}x\right]_0^3$$
$$= \left[\frac{x^4}{4} - 6 \qquad \right]_0^3$$

Example

$$\int_0^3 (x^3 - 6x)\mathrm{d}x = \left[\int (x^3 - 6x)\mathrm{d}x\right]_0^3$$
$$= \left[\int x^3\mathrm{d}x - 6\int x\mathrm{d}x\right]_0^3$$
$$= \left[\frac{x^4}{4} - 6\frac{x^2}{2}\right]_0^3$$

Example

$$\int_0^3 (x^3 - 6x)\mathrm{d}x = \left[\int (x^3 - 6x)\mathrm{d}x\right]_0^3$$
$$= \left[\int x^3\mathrm{d}x - 6\int x\mathrm{d}x\right]_0^3$$
$$= \left[\frac{x^4}{4} - 6\frac{x^2}{2}\right]_0^3$$
$$= \left(\frac{1}{4}\cdot 3^4 - 3\cdot 3^2\right) - \left(\frac{1}{4}\cdot 0^4 - 3\cdot 0^2\right)$$

Example

$$\begin{aligned}
\int_0^3 (x^3 - 6x)\mathrm{d}x &= \left[\int (x^3 - 6x)\mathrm{d}x\right]_0^3 \\
&= \left[\int x^3\mathrm{d}x - 6\int x\mathrm{d}x\right]_0^3 \\
&= \left[\frac{x^4}{4} - 6\frac{x^2}{2}\right]_0^3 \\
&= \left(\frac{1}{4}\cdot 3^4 - 3\cdot 3^2\right) - \left(\frac{1}{4}\cdot 0^4 - 3\cdot 0^2\right) \\
&= \frac{81}{4} - 27 - 0 + 0 = -\frac{27}{4}.
\end{aligned}$$

Example

Evaluate: $$\int_1^9 \frac{2t^2 + t^2\sqrt{t} - 1}{t^2} \mathrm{d}t$$

Example

Evaluate: $$\int_1^9 \frac{2t^2 + t^2\sqrt{t} - 1}{t^2} \mathrm{d}t$$

$$= \int_1^9 (2 + t^{1/2} - t^{-2}) \mathrm{d}t$$

Example

Evaluate: $$\int_1^9 \frac{2t^2 + t^2\sqrt{t} - 1}{t^2} \mathrm{d}t$$

$$= \int_1^9 (2 + t^{1/2} - t^{-2}) \mathrm{d}t = \left[\int (2 + t^{1/2} - t^{-2}) \mathrm{d}t \right]_1^9$$

Example

Evaluate: $\displaystyle \int_1^9 \frac{2t^2 + t^2\sqrt{t} - 1}{t^2} \mathrm{d}t$

$$= \int_1^9 (2 + t^{1/2} - t^{-2}) \mathrm{d}t = \left[\int (2 + t^{1/2} - t^{-2}) \mathrm{d}t \right]_1^9$$

$$= \left[\int 2 \mathrm{d}t + \int t^{1/2} \mathrm{d}t - \int t^{-2} \mathrm{d}t \right]_1^9$$

Example

Evaluate: $\displaystyle\int_1^9 \frac{2t^2 + t^2\sqrt{t} - 1}{t^2} \mathrm{d}t$

$$= \int_1^9 (2 + t^{1/2} - t^{-2}) \mathrm{d}t = \left[\int (2 + t^{1/2} - t^{-2}) \mathrm{d}t\right]_1^9$$

$$= \left[\int 2\mathrm{d}t + \int t^{1/2}\mathrm{d}t - \int t^{-2}\mathrm{d}t\right]_1^9$$

$$= \left[\quad + \quad - \quad \right]_1^9$$

Example

Evaluate: $\displaystyle \int_1^9 \frac{2t^2 + t^2\sqrt{t} - 1}{t^2} dt$

$$= \int_1^9 (2 + t^{1/2} - t^{-2}) dt = \left[\int (2 + t^{1/2} - t^{-2}) dt \right]_1^9$$

$$= \left[\int 2 dt + \int t^{1/2} dt - \int t^{-2} dt \right]_1^9$$

$$= \left[2t + \quad - \quad \right]_1^9$$

Example

Evaluate: $\int_1^9 \frac{2t^2 + t^2\sqrt{t} - 1}{t^2} \mathrm{d}t$

$$= \int_1^9 (2 + t^{1/2} - t^{-2}) \mathrm{d}t = \left[\int (2 + t^{1/2} - t^{-2}) \mathrm{d}t \right]_1^9$$

$$= \left[\int 2 \mathrm{d}t + \int t^{1/2} \mathrm{d}t - \int t^{-2} \mathrm{d}t \right]_1^9$$

$$= \left[2t + \qquad - \qquad \right]_1^9$$

Example

Evaluate: $\displaystyle\int_1^9 \frac{2t^2 + t^2\sqrt{t} - 1}{t^2} \mathrm{d}t$

$$= \int_1^9 (2 + t^{1/2} - t^{-2}) \mathrm{d}t = \left[\int (2 + t^{1/2} - t^{-2}) \mathrm{d}t \right]_1^9$$

$$= \left[\int 2 \mathrm{d}t + \int t^{1/2} \mathrm{d}t - \int t^{-2} \mathrm{d}t \right]_1^9$$

$$= \left[2t + \frac{t^{3/2}}{3/2} - \qquad \right]_1^9$$

Example

Evaluate: $\displaystyle\int_1^9 \frac{2t^2 + t^2\sqrt{t} - 1}{t^2} dt$

$$= \int_1^9 (2 + t^{1/2} - t^{-2}) dt = \left[\int (2 + t^{1/2} - t^{-2}) dt \right]_1^9$$

$$= \left[\int 2 dt + \int t^{1/2} dt - \int t^{-2} dt \right]_1^9$$

$$= \left[2t + \frac{t^{3/2}}{3/2} - \qquad \right]_1^9$$

Example

Evaluate: $$\int_1^9 \frac{2t^2 + t^2\sqrt{t} - 1}{t^2} \mathrm{d}t$$

$$= \int_1^9 (2 + t^{1/2} - t^{-2}) \mathrm{d}t = \left[\int (2 + t^{1/2} - t^{-2}) \mathrm{d}t \right]_1^9$$

$$= \left[\int 2 \mathrm{d}t + \int t^{1/2} \mathrm{d}t - \int t^{-2} \mathrm{d}t \right]_1^9$$

$$= \left[2t + \frac{t^{3/2}}{3/2} - \frac{t^{-1}}{-1} \right]_1^9$$

Example

Evaluate: $$\int_1^9 \frac{2t^2 + t^2\sqrt{t} - 1}{t^2} \mathrm{d}t$$

$$= \int_1^9 (2 + t^{1/2} - t^{-2}) \mathrm{d}t = \left[\int (2 + t^{1/2} - t^{-2}) \mathrm{d}t \right]_1^9$$

$$= \left[\int 2 \mathrm{d}t + \int t^{1/2} \mathrm{d}t - \int t^{-2} \mathrm{d}t \right]_1^9$$

$$= \left[2t + \frac{t^{3/2}}{3/2} - \frac{t^{-1}}{-1} \right]_1^9 = \left[2t + \frac{2}{3} t^{3/2} + \frac{1}{t} \right]_1^9$$

Example

Evaluate: $\displaystyle\int_1^9 \frac{2t^2 + t^2\sqrt{t} - 1}{t^2} \mathrm{d}t$

$$= \int_1^9 (2 + t^{1/2} - t^{-2})\mathrm{d}t = \left[\int (2 + t^{1/2} - t^{-2})\mathrm{d}t\right]_1^9$$

$$= \left[\int 2\mathrm{d}t + \int t^{1/2}\mathrm{d}t - \int t^{-2}\mathrm{d}t\right]_1^9$$

$$= \left[2t + \frac{t^{3/2}}{3/2} - \frac{t^{-1}}{-1}\right]_1^9 = \left[2t + \frac{2}{3}t^{3/2} + \frac{1}{t}\right]_1^9$$

$$= \left(2 \cdot 9 + \frac{2}{3} \cdot 9^{3/2} + \frac{1}{9}\right) - \left(2 \cdot 1 + \frac{2}{3} \cdot 1^{3/2} + \frac{1}{1}\right)$$

Example

Evaluate: $$\int_1^9 \frac{2t^2 + t^2\sqrt{t} - 1}{t^2} \mathrm{d}t$$

$$= \int_1^9 (2 + t^{1/2} - t^{-2})\mathrm{d}t = \left[\int (2 + t^{1/2} - t^{-2})\mathrm{d}t\right]_1^9$$

$$= \left[\int 2\mathrm{d}t + \int t^{1/2}\mathrm{d}t - \int t^{-2}\mathrm{d}t\right]_1^9$$

$$= \left[2t + \frac{t^{3/2}}{3/2} - \frac{t^{-1}}{-1}\right]_1^9 = \left[2t + \frac{2}{3}t^{3/2} + \frac{1}{t}\right]_1^9$$

$$= \left(2 \cdot 9 + \frac{2}{3} \cdot 9^{3/2} + \frac{1}{9}\right) - \left(2 \cdot 1 + \frac{2}{3} \cdot 1^{3/2} + \frac{1}{1}\right)$$

$$= 18 + 18 + \frac{1}{9} - 2 - \frac{2}{3} - 1$$

Example

Evaluate: $\displaystyle\int_1^9 \frac{2t^2 + t^2\sqrt{t} - 1}{t^2} \mathrm{d}t$

$$= \int_1^9 (2 + t^{1/2} - t^{-2})\mathrm{d}t = \left[\int (2 + t^{1/2} - t^{-2})\mathrm{d}t\right]_1^9$$

$$= \left[\int 2\mathrm{d}t + \int t^{1/2}\mathrm{d}t - \int t^{-2}\mathrm{d}t\right]_1^9$$

$$= \left[2t + \frac{t^{3/2}}{3/2} - \frac{t^{-1}}{-1}\right]_1^9 = \left[2t + \frac{2}{3}t^{3/2} + \frac{1}{t}\right]_1^9$$

$$= \left(2 \cdot 9 + \frac{2}{3} \cdot 9^{3/2} + \frac{1}{9}\right) - \left(2 \cdot 1 + \frac{2}{3} \cdot 1^{3/2} + \frac{1}{1}\right)$$

$$= 18 + 18 + \frac{1}{9} - 2 - \frac{2}{3} - 1 = \frac{292}{9}.$$

- The Evaluation Theorem says that, if f is continuous on $[a, b]$, then
$$\int_a^b f(x)\mathrm{d}x = F(b) - F(a),$$
where $F(x)$ is an antiderivative of $f(x)$.
- This means $F' = f$, so
$$\int_a^b F'(x)\mathrm{d}x = F(b) - F(a),$$

- The Evaluation Theorem says that, if f is continuous on $[a, b]$, then
$$\int_a^b f(x)\mathrm{d}x = F(b) - F(a),$$
where $F(x)$ is an antiderivative of $f(x)$.
- This means $F' = f$, so
$$\int_a^b F'(x)\mathrm{d}x = F(b) - F(a),$$
- $F'(x)$ is the rate of change of $y = F(x)$ with respect to x.

- The Evaluation Theorem says that, if f is continuous on $[a, b]$, then
$$\int_a^b f(x)\mathrm{d}x = F(b) - F(a),$$
where $F(x)$ is an antiderivative of $f(x)$.
- This means $F' = f$, so
$$\int_a^b F'(x)\mathrm{d}x = F(b) - F(a),$$
- $F'(x)$ is the rate of change of $y = F(x)$ with respect to x.
- $F(b) - F(a)$ is the net change in y as x changes from a to b.

- The Evaluation Theorem says that, if f is continuous on $[a, b]$, then
$$\int_a^b f(x)\mathrm{d}x = F(b) - F(a),$$
where $F(x)$ is an antiderivative of $f(x)$.
- This means $F' = f$, so
$$\int_a^b F'(x)\mathrm{d}x = F(b) - F(a),$$
- $F'(x)$ is the rate of change of $y = F(x)$ with respect to x.
- $F(b) - F(a)$ is the net change in y as x changes from a to b.

Theorem (The Net Change Theorem)

The integral of the rate of change is the net change:
$$\int_a^b F'(x)dx = F(b) - F(a).$$

- If an object moves along a straight line with position function $s(t)$, then its velocity is $v(t) = s'(t)$.
- In this case, the Net Change Theorem says
$$\int_{t_1}^{t_2} v(t)\mathrm{d}t = s(t_2) - s(t_1).$$

- If an object moves along a straight line with position function $s(t)$, then its velocity is $v(t) = s'(t)$.
- In this case, the Net Change Theorem says

$$\int_{t_1}^{t_2} v(t)\mathrm{d}t = s(t_2) - s(t_1).$$

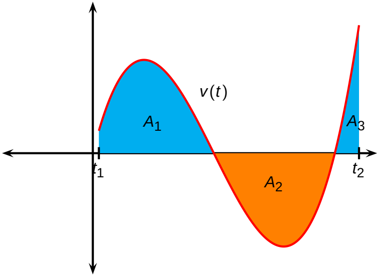
- This is the displacement, or net change of position.

- If an object moves along a straight line with position function $s(t)$, then its velocity is $v(t) = s'(t)$.
- In this case, the Net Change Theorem says
$$\int_{t_1}^{t_2} v(t)\mathrm{d}t = s(t_2) - s(t_1).$$
- This is the displacement, or net change of position.
- If we want to calculate the distance the object travels, we have to consider separately the intervals where $v(t) \geq 0$ (object moves to the right) and $v(t) \leq 0$ (object moves to the left).

- If an object moves along a straight line with position function $s(t)$, then its velocity is $v(t) = s'(t)$.
- In this case, the Net Change Theorem says

$$\int_{t_1}^{t_2} v(t)dt = s(t_2) - s(t_1).$$

- This is the displacement, or net change of position.
- If we want to calculate the distance the object travels, we have to consider separately the intervals where $v(t) \geq 0$ (object moves to the right) and $v(t) \leq 0$ (object moves to the left).



$$\text{displacement} = \int_{t_1}^{t_2} v(t)dt = A_1 - A_2 + A_3$$

$$\text{distance} = \int_{t_1}^{t_2} |v(t)|dt = A_1 + A_2 + A_3$$

Example

A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

1. Find the displacement of the particle during the time period $1 \leq t \leq 4$.
2. Find the distance traveled during this time period.

Example

A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

1. Find the displacement of the particle during the time period $1 \leq t \leq 4$.

The displacement is

$$s(4) - s(1) = \int_1^4 v(t)\mathrm{d}t$$

Example

A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

1. Find the displacement of the particle during the time period $1 \leq t \leq 4$.

The displacement is

$$s(4) - s(1) = \int_1^4 v(t)\mathrm{d}t = \int_1^4 (t^2 - t - 6)\mathrm{d}t$$

Example

A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

1. Find the displacement of the particle during the time period $1 \leq t \leq 4$.

The displacement is

$$s(4) - s(1) = \int_1^4 v(t) \mathrm{d}t = \int_1^4 (t^2 - t - 6) \mathrm{d}t$$

$$= \left[\quad - \quad - \quad \right]_1^4$$

Example

A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

1. Find the displacement of the particle during the time period $1 \leq t \leq 4$.

The displacement is

$$s(4) - s(1) = \int_1^4 v(t) \mathrm{d}t = \int_1^4 (t^2 - t - 6) \mathrm{d}t$$

$$= \left[\frac{t^3}{3} - \quad - \quad \right]_1^4$$

Example

A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

1. Find the displacement of the particle during the time period $1 \leq t \leq 4$.

The displacement is

$$s(4) - s(1) = \int_1^4 v(t)\mathrm{d}t = \int_1^4 (t^2 - t - 6)\mathrm{d}t$$
$$= \left[\frac{t^3}{3} - \quad - \quad \right]_1^4$$

Example

A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

1. Find the displacement of the particle during the time period $1 \leq t \leq 4$.

The displacement is

$$s(4) - s(1) = \int_1^4 v(t)\mathrm{d}t = \int_1^4 (t^2 - t - 6)\mathrm{d}t$$
$$= \left[\frac{t^3}{3} - \frac{t^2}{2} - \quad \right]_1^4$$

Example

A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

1. Find the displacement of the particle during the time period $1 \leq t \leq 4$.

The displacement is

$$s(4) - s(1) = \int_1^4 v(t)\mathrm{d}t = \int_1^4 (t^2 - t - 6)\mathrm{d}t$$
$$= \left[\frac{t^3}{3} - \frac{t^2}{2} - \quad \right]_1^4$$

Example

A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

1. Find the displacement of the particle during the time period $1 \leq t \leq 4$.

The displacement is

$$\begin{aligned} s(4) - s(1) &= \int_1^4 v(t)\mathrm{d}t = \int_1^4 (t^2 - t - 6)\mathrm{d}t \\ &= \left[\frac{t^3}{3} - \frac{t^2}{2} - 6t\right]_1^4 \end{aligned}$$

Example

A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

1. Find the displacement of the particle during the time period $1 \leq t \leq 4$.

The displacement is

$$\begin{aligned} s(4) - s(1) &= \int_1^4 v(t)\mathrm{d}t = \int_1^4 (t^2 - t - 6)\mathrm{d}t \\ &= \left[\frac{t^3}{3} - \frac{t^2}{2} - 6t\right]_1^4 \\ &= \left(\frac{4^3}{3} - \frac{4^2}{2} - 6\cdot 4\right) - \left(\frac{1^3}{3} - \frac{1^2}{2} - 6\cdot 1\right) \end{aligned}$$

Example

A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

1. Find the displacement of the particle during the time period $1 \leq t \leq 4$.

The displacement is

$$\begin{aligned} s(4) - s(1) &= \int_1^4 v(t)\mathrm{d}t = \int_1^4 (t^2 - t - 6)\mathrm{d}t \\ &= \left[\frac{t^3}{3} - \frac{t^2}{2} - 6t\right]_1^4 \\ &= \left(\frac{4^3}{3} - \frac{4^2}{2} - 6 \cdot 4\right) - \left(\frac{1^3}{3} - \frac{1^2}{2} - 6 \cdot 1\right) \\ &= -\frac{9}{2}. \end{aligned}$$

Example

A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

1. Find the displacement of the particle during the time period $1 \leq t \leq 4$.

The displacement is

$$\begin{aligned} s(4) - s(1) &= \int_1^4 v(t) \mathrm{d}t = \int_1^4 (t^2 - t - 6) \mathrm{d}t \\ &= \left[\frac{t^3}{3} - \frac{t^2}{2} - 6t \right]_1^4 \\ &= \left(\frac{4^3}{3} - \frac{4^2}{2} - 6 \cdot 4 \right) - \left(\frac{1^3}{3} - \frac{1^2}{2} - 6 \cdot 1 \right) \\ &= -\frac{9}{2}. \end{aligned}$$

Therefore the particle moves 4.5m to the left.

Example

A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

2. Find the distance traveled during this time period.

$v(t) = t^2 - t - 6 =$

Example

A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

2. Find the distance traveled during this time period.

$v(t) = t^2 - t - 6 = (t-3)(t+2)$

Example

A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

2. Find the distance traveled during this time period.

$v(t) = t^2 - t - 6 = (t-3)(t+2)$ and so $v(t) \leq 0$ on the interval and $v(t) \geq 0$ on the interval

Example

A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

2. Find the distance traveled during this time period.

$v(t) = t^2 - t - 6 = (t-3)(t+2)$ and so $v(t) \leq 0$ on the interval $[1,3]$ and $v(t) \geq 0$ on the interval

Example

A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

2. Find the distance traveled during this time period.

$v(t) = t^2 - t - 6 = (t-3)(t+2)$ and so $v(t) \leq 0$ on the interval $[1, 3]$ and $v(t) \geq 0$ on the interval

Example

A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

2. Find the distance traveled during this time period.

$v(t) = t^2 - t - 6 = (t-3)(t+2)$ and so $v(t) \leq 0$ on the interval $[1,3]$ and $v(t) \geq 0$ on the interval $[3,4]$.

Example

A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

2. Find the distance traveled during this time period.

$v(t) = t^2 - t - 6 = (t-3)(t+2)$ and so $v(t) \leq 0$ on the interval $[1, 3]$ and $v(t) \geq 0$ on the interval $[3, 4]$.

The distance is

$$\int_1^4 |v(t)| \mathrm{d}t$$

Example

A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

2. Find the distance traveled during this time period.

$v(t) = t^2 - t - 6 = (t-3)(t+2)$ and so $v(t) \leq 0$ on the interval $[1, 3]$ and $v(t) \geq 0$ on the interval $[3, 4]$.

The distance is

$$\int_1^4 |v(t)|\mathrm{d}t = \int_1^3 [-v(t)]\mathrm{d}t + \int_3^4 v(t)\mathrm{d}t$$

Example

A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

2. Find the distance traveled during this time period.

$v(t) = t^2 - t - 6 = (t-3)(t+2)$ and so $v(t) \leq 0$ on the interval $[1, 3]$ and $v(t) \geq 0$ on the interval $[3, 4]$.
The distance is

$$\begin{aligned}\int_1^4 |v(t)| \mathrm{d}t &= \int_1^3 [-v(t)] \mathrm{d}t + \int_3^4 v(t) \mathrm{d}t \\ &= \int_1^3 (-t^2 + t + 6) \mathrm{d}t + \int_3^4 (t^2 - t - 6) \mathrm{d}t\end{aligned}$$

Example

A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

2. Find the distance traveled during this time period.

$v(t) = t^2 - t - 6 = (t-3)(t+2)$ and so $v(t) \leq 0$ on the interval $[1, 3]$ and $v(t) \geq 0$ on the interval $[3, 4]$.
The distance is

$$\begin{aligned}\int_1^4 |v(t)| \mathrm{d}t &= \int_1^3 [-v(t)] \mathrm{d}t + \int_3^4 v(t) \mathrm{d}t \\ &= \int_1^3 (-t^2 + t + 6) \mathrm{d}t + \int_3^4 (t^2 - t - 6) \mathrm{d}t \\ &= \left[-\frac{t^3}{3} + \frac{t^2}{2} + 6t \right]_1^3 + \left[\frac{t^3}{3} - \frac{t^2}{2} - 6t \right]_3^4\end{aligned}$$

Example

A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

2. Find the distance traveled during this time period.

$v(t) = t^2 - t - 6 = (t-3)(t+2)$ and so $v(t) \leq 0$ on the interval $[1, 3]$ and $v(t) \geq 0$ on the interval $[3, 4]$.

The distance is

$$
\begin{aligned}
\int_1^4 |v(t)| \mathrm{d}t &= \int_1^3 [-v(t)] \mathrm{d}t + \int_3^4 v(t) \mathrm{d}t \\
&= \int_1^3 (-t^2 + t + 6) \mathrm{d}t + \int_3^4 (t^2 - t - 6) \mathrm{d}t \\
&= \left[-\frac{t^3}{3} + \frac{t^2}{2} + 6t \right]_1^3 + \left[\frac{t^3}{3} - \frac{t^2}{2} - 6t \right]_3^4 \\
&= \frac{61}{6} \approx 10.17\mathrm{m}
\end{aligned}
$$