Review problems for May 9 Exam
Math 140

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The exam will be closed book, no calculators allowed. Try to solve all theoretical problems without using the lectures/textbook. If you get stuck, read the lectures/textbook, but close the textbook/lectures when going back to the problem. Finally, compare what you wrote with the lectures/textbook.

For problem answer key, look for updates on piazza.com.

Problem 1 *Find the maxima and minima of the function in the interval.*

1. $f(x) = \frac{1-x}{2+x+x^2}$, $x \in [-2, 4]$.

   answer: $\min = f(3) = -\frac{1}{7}$, $\max = f(-1) = 1$.

2. $f(x) = \frac{2-x}{3+x+x^2}$, $x \in [-2, 6]$.

   answer: $\min = f(5) = -\frac{1}{11}$, $\max = f(-1) = 1$.

Problem 2

1. $f(x) = x(x-1)^2(x-3)$, $x \in (-\infty, \infty)$.
2. $f(x) = x(x-2)^2(x-3)$, $x \in (-\infty, \infty)$.

For the above examples do the following.

- *Find the zeroes of the function.*

  answer: $0, 1, 3$ *and answer:* $0, 2, 3$.

- *Find the intervals where the function is increasing and decreasing.*

  answer $(-\infty, \frac{13-\sqrt{73}}{8})$-*decreasing* $(\frac{13-\sqrt{73}}{8}, 2)$-*increasing,* $(2, \frac{13+\sqrt{73}}{8})$-*decreasing,* $(\frac{13+\sqrt{73}}{8}, \infty)$-*increasing.*

  answer $(-\infty, \frac{11-\sqrt{73}}{8})$-*decreasing* $(\frac{11-\sqrt{73}}{8}, 1)$-*increasing,* $(1, \frac{11+\sqrt{73}}{8})$-*decreasing,* $(\frac{11+\sqrt{73}}{8}, \infty)$-*increasing.*

- *Find the values of x for which the function has local minima and maxima. If they exist, find the absolute minima and maxima of the function.*

  answer: local minima at $x = \frac{11 \pm \sqrt{73}}{8}$, *local maximum at* 1,

  absolute minimum $f(\frac{11-\sqrt{73}}{8}) = -\frac{827}{512} - \frac{73}{512}\sqrt{73}$, *has no absolute maximum.*

  answer: local minima at $x = \frac{13 \pm \sqrt{73}}{8}$, *local maximum at* 2,

  absolute minimum $f(\frac{13-\sqrt{73}}{8}) = -\frac{827}{512} - \frac{73}{512}\sqrt{73}$, *has no absolute maximum.*

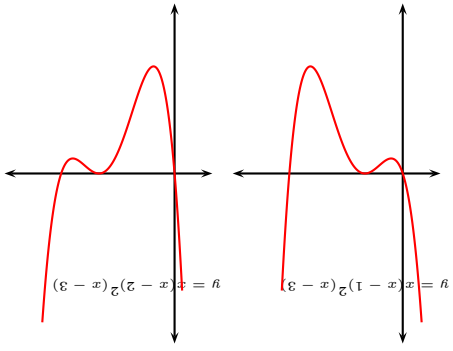
- *Find the intervals of concavity (up and down) of the function. Find the inflection points of the function.*

  answer: $\left(-\infty, \frac{21-\sqrt{57}}{12}\right)$-*concave up,* $\left(\frac{21-\sqrt{57}}{12}, \frac{21+\sqrt{57}}{12}\right)$-*concave down,* $\left(\frac{21+\sqrt{57}}{12}\right)$-*concave up. Inflection points at* $\frac{21 \pm \sqrt{57}}{12}$.

  answer: $\left(-\infty, \frac{15-\sqrt{57}}{12}\right)$-*concave up,* $\left(\frac{15-\sqrt{57}}{12}, \frac{15+\sqrt{57}}{12}\right)$-*concave down,* $\left(\frac{15+\sqrt{57}}{12}\right)$-*concave up. Inflection points at* $\frac{15 \pm \sqrt{57}}{12}$.

- *Plot the function roughly.*

  answer:



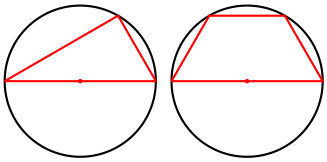
Problem 3

1. *Find the largest possible area of a triangle inscribed in circle of radius r such that one of the sides of the triangle is a diameter of the circle.*

   answer: r^2.

2. *Find the largest possible area of a trapezoid inscribed in circle of radius r such that one of the bases of the trapezoid is a diameter of the circle.*

   answer: $\frac{3\sqrt{3}}{2}r^2$.



Problem 4 *Evaluate the definite integral*

1. $\int_1^8 \frac{t^2-t^{2/3}+2}{t^{5/3}}dt$.

2. $\int_1^{32} \frac{t^2-t^{-2/5}-2}{t^{3/5}}dt$.

Problem 5 *Evaluate the indefinite integral*

1. $\int(\sin(3x+\pi/3)+e^{-x/3})\ dx.$
2. $\int(\cot(4x+\pi/4)+e^{-x/4})\ dx.$

Problem 6 *Evaluate the indefinite integral*

1. $\int \tan x\ dx.$

   answer: $-\ln|\sin x|+C.$

2. $\int \cot x\ dx.$

   answer: $\ln|\cos x|+C.$

Problem 7 *Evaluate the definite integral*

1. $\int_1^2 \frac{x}{(1-3x)(1+3x)}\ dx.$

   answer: $\frac{1}{18}\ln\frac{8}{35}.$

2. $\int_0^1 \frac{x}{(2-x)(2+x)}\ dx.$

   answer: $\frac{1}{2}\ln\frac{4}{3}.$

Problem 8

1. *State and prove the Mean Value Theorem.*
2. *Define Riemann sum.*
3. *Define definite integral.*
4. *State the Fundamental Theorem of Calculus (both parts).*
5. *State the substitution rule.*