Math 140
Lecture 24

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Outline

1. (5.5) The Substitution Rule
   - Symmetry

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   - Symmetry
2. (6.1) More About Areas

Example (Substitution Rule, more factors)

Evaluate $\int 3x^5\sqrt{1+x^3}\mathrm{d}x$.

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Let $u =$

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Example (Substitution Rule, more factors)

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Let $u = 1 + x^3$.

Then $\mathrm{d}u =$

Example (Substitution Rule, more factors)

Evaluate $\displaystyle\int 3x^5\sqrt{1+x^3}\mathrm{d}x.$

$$\text{Let } u = 1 + x^3.$$

$$\text{Then } \mathrm{d}u = 3x^2\mathrm{d}x.$$

Example (Substitution Rule, more factors)

Evaluate $\int 3x^5\sqrt{1+x^3}\mathrm{d}x = \int 3x^2 x^3 \sqrt{1+x^3}\mathrm{d}x.$

Let $u = 1 + x^3.$

Then $\mathrm{d}u = 3x^2\mathrm{d}x.$

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$$\text{Let } u = 1 + x^3.$$

$$\text{Then } \mathrm{d}u = 3x^2\mathrm{d}x.$$

$$x^3 = u - 1.$$

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Evaluate $\int 3x^5\sqrt{1+x^3}dx = \int 3x^2x^3\sqrt{1+x^3}dx.$

$$\text{Let } u = 1 + x^3.$$

$$\text{Then } du = 3x^2dx.$$

$$x^3 = u - 1.$$

$$\begin{aligned}\int 3x^2x^3\sqrt{1+x^3}dx &= \int (u-1)\sqrt{u}\,du \\ &= \int (u^{3/2} - u^{1/2})du\end{aligned}$$

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$$= \int (u^{3/2} - u^{1/2})du$$
$$= \left(\frac{u^{5/2}}{5/2} - \qquad \right)$$

Example (Substitution Rule, more factors)

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Example (Substitution Rule, more factors)

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$$\int 3x^2x^3\sqrt{1+x^3}\,\mathrm{d}x = \int (u-1)\sqrt{u}\,\mathrm{d}u$$

$$= \int (u^{3/2}-u^{1/2})\mathrm{d}u$$

$$= \left(\frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2}\right) + C$$

Example (Substitution Rule, more factors)

$$\text{Evaluate } \int 3x^5\sqrt{1+x^3}\mathrm{d}x = \int 3x^2x^3\sqrt{1+x^3}\mathrm{d}x.$$

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$$x^3 = u-1.$$

$$\begin{aligned}\int 3x^2x^3\sqrt{1+x^3}\mathrm{d}x &= \int (u-1)\sqrt{u}\,\mathrm{d}u \\ &= \int (u^{3/2}-u^{1/2})\mathrm{d}u \\ &= \left(\frac{u^{5/2}}{5/2}-\frac{u^{3/2}}{3/2}\right)+C \\ &= \frac{2}{5}(1+x^3)^{5/2}-\frac{2}{3}(1+x^3)^{3/2}+C.\end{aligned}$$

Symmetry

Theorem (Integrals of Symmetric Functions)

Suppose f is continuous on $[-a, a]$.

1. *If f is even (that is, $f(-x) = f(x)$), then* $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$.
2. *If f is odd (that is, $f(-x) = -f(x)$), then* $\int_{-a}^{a} f(x)\,dx = 0$.

Example (Example 9, p. 381)

Since $f(x) = x^6 + 1$ satisfies $f(-x) = f(x)$, it is even, and so

$$\int_{-2}^{2} (x^6 + 1)\,\mathrm{d}x =$$

Example (Example 9, p. 381)

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$$\int_{-2}^{2} (x^6 + 1)\,\mathrm{d}x = 2\int_{0}^{2} (x^6 + 1)\,\mathrm{d}x$$
$$= 2\left[\frac{1}{7}x^7 + x\right]_0^2$$

Example (Example 9, p. 381)

Since $f(x) = x^6 + 1$ satisfies $f(-x) = f(x)$, it is even, and so

$$\begin{aligned}\int_{-2}^{2} (x^6 + 1)\,dx &= 2\int_0^2 (x^6 + 1)\,dx \\ &= 2\left[\frac{1}{7}x^7 + x\right]_0^2 \\ &= 2\left(\frac{128}{7} + 2\right)\end{aligned}$$

Example (Example 9, p. 381)

Since $f(x) = x^6 + 1$ satisfies $f(-x) = f(x)$, it is even, and so

$$\begin{aligned}\int_{-2}^{2} (x^6 + 1)\, dx &= 2\int_{0}^{2} (x^6 + 1)\, dx \\ &= 2\left[\frac{1}{7}x^7 + x\right]_0^2 \\ &= 2\left(\frac{128}{7} + 2\right) \\ &= \frac{284}{7}.\end{aligned}$$

Example (Example 10, p. 381)

Since $f(x) = (\tan x)/(1 + x^2 + x^4)$ satisfies $f(-x) = -f(x)$, it is odd, and so

$$\int_{-1}^{1} \frac{\tan x}{1 + x^2 + x^4} \mathrm{d}x =$$

Example (Example 10, p. 381)

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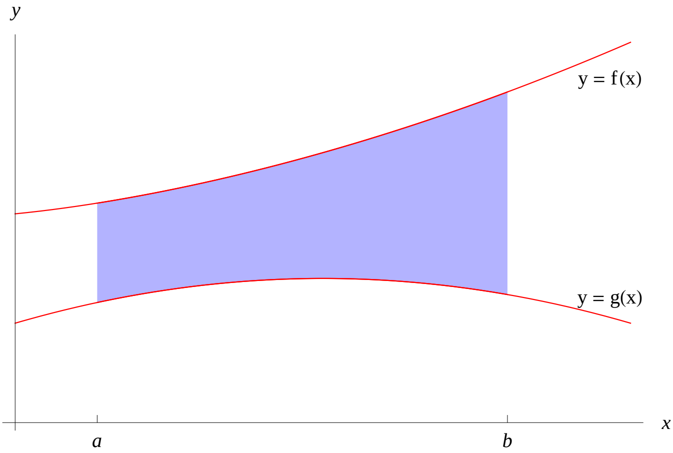
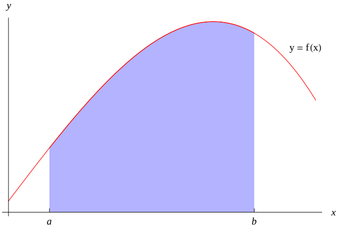
More About Areas

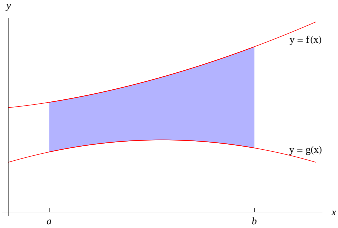
Suppose two curves, $y = f(x)$ and $y = g(x)$, are given.
How do we find the area bounded by those curves between the endpoints $x = a$ and $x = b$?



The Area Under a Curve



The Area Between Two Curves



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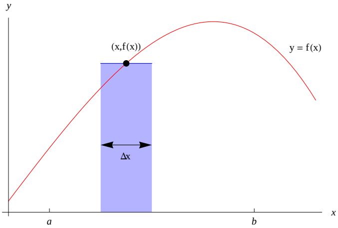
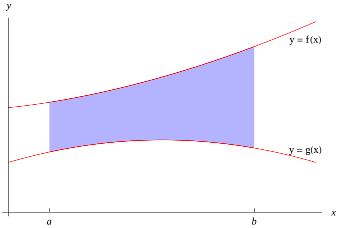


rectangle area $=$ height$\cdot$width

The Area Between Two Curves



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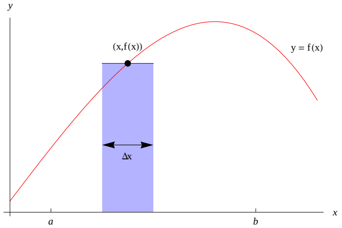

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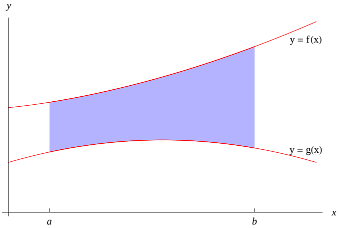
The Area Between Two Curves



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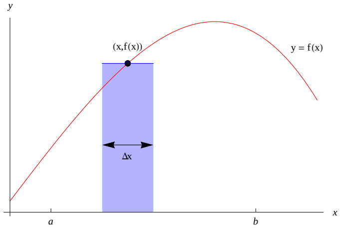
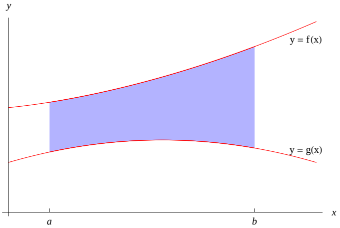


rectangle area $=$ height$\cdot\Delta x$

The Area Between Two Curves



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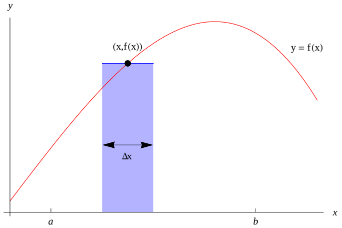

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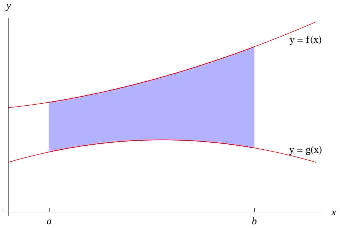

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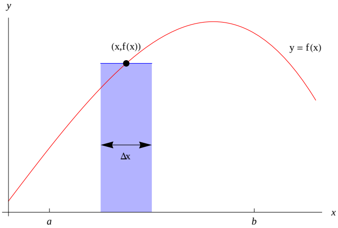
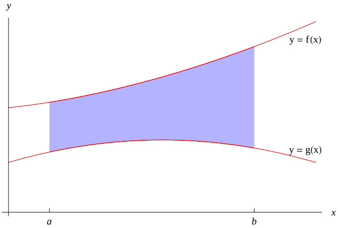


rectangle area $= f(x) \cdot \Delta x$

The Area Between Two Curves



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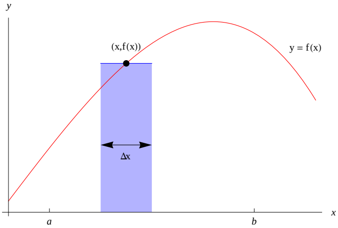

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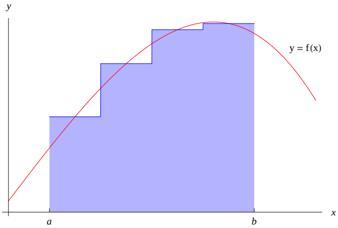
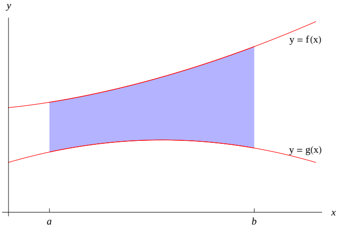


rectangles $= n = 4$

A = $\sum_{i=1}^{4} f(x_i)\Delta x$

The Area Between Two Curves



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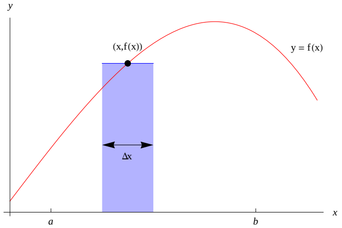
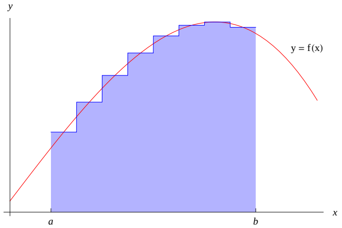


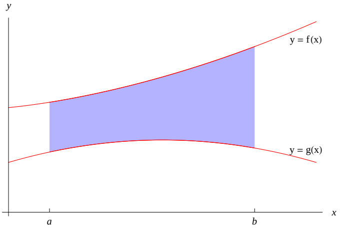
rectangle area $= f(x) \cdot \Delta x$



rectangles $= n =$ 8

$$A = \sum_{i=1}^{8} f(x_i)\Delta x$$

The Area Between Two Curves



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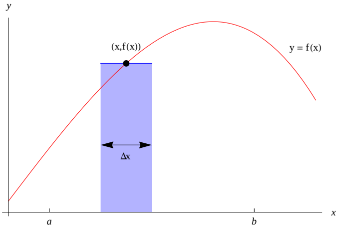
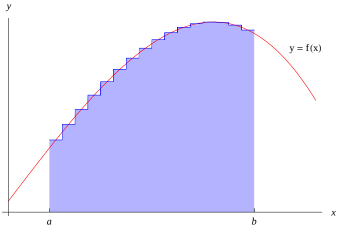


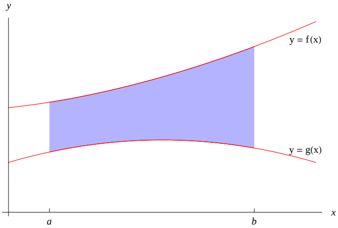
rectangle area $= f(x) \cdot \Delta x$



rectangles $= n =$ 16

$A = \sum_{i=1}^{16} f(x_i)\Delta x$

The Area Between Two Curves



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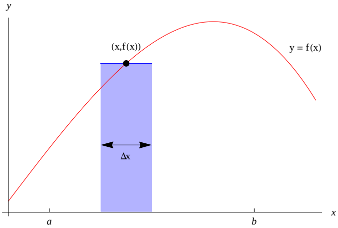
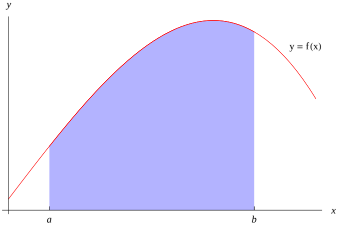


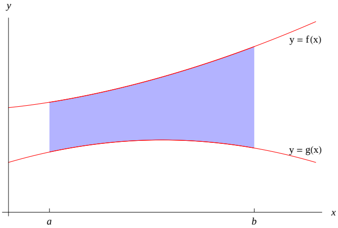
rectangle area $= f(x) \cdot \Delta x$



rectangles $= n \to \infty$

A = $\lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x$

The Area Between Two Curves



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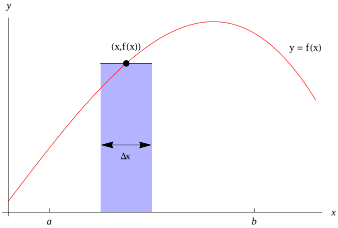
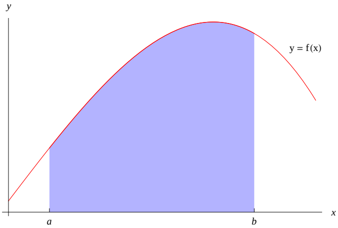


rectangle area $= f(x) \cdot \Delta x$



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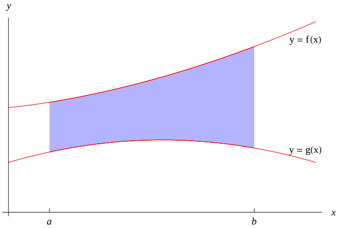
The Area Between Two Curves



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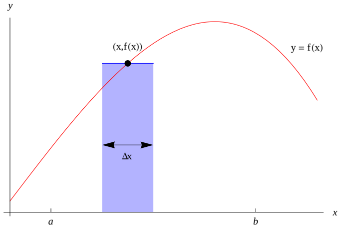
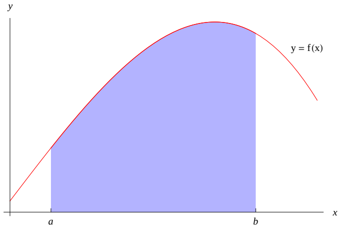


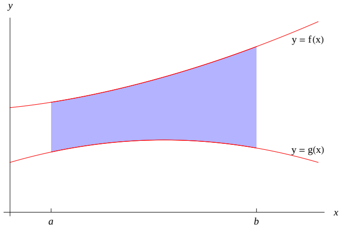
rectangle area $= f(x) \cdot \Delta x$



rectangles $= n \to \infty$

$$A = \int_a^b f(x)dx$$

The Area Between Two Curves



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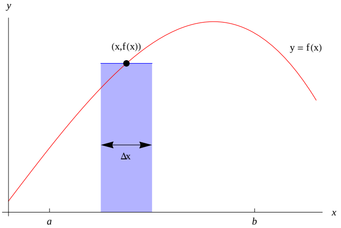
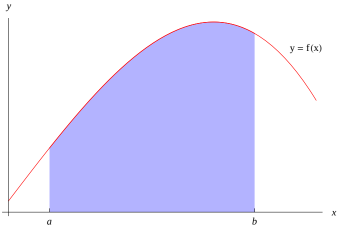


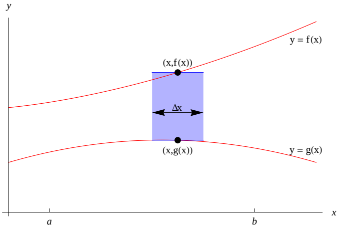
rectangle area $= f(x) \cdot \Delta x$



rectangles $= n \to \infty$

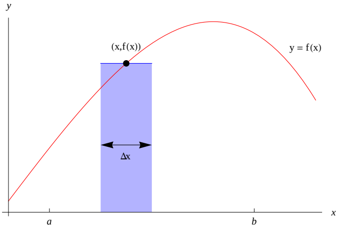
$$A = \int_a^b f(x) dx$$

The Area Between Two Curves



rectangle area = height·width

The Area Under a Curve



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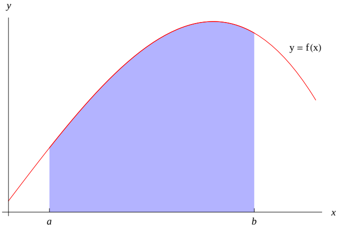

rectangles $= n \to \infty$

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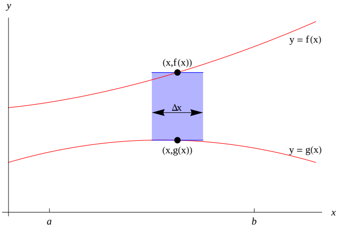
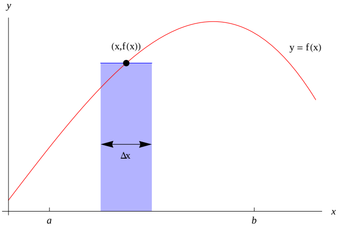
The Area Between Two Curves

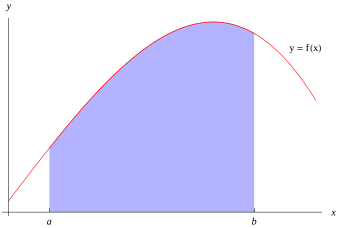


rectangle area $=$ height·width

The Area Under a Curve



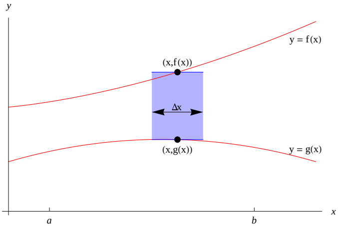
rectangle area $= f(x) \cdot \Delta x$



rectangles $= n \to \infty$

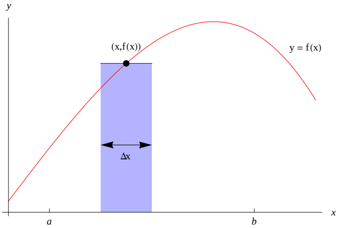
$$A = \int_a^b f(x)dx$$

The Area Between Two Curves

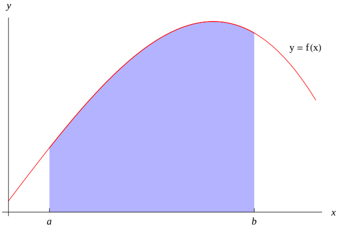


rectangle area $= \text{height} \cdot \Delta x$

The Area Under a Curve



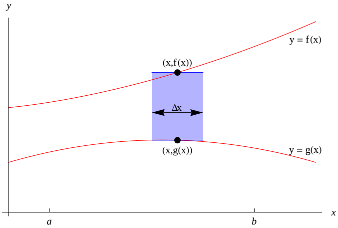
rectangle area $= f(x) \cdot \Delta x$



\# rectangles $= n \to \infty$

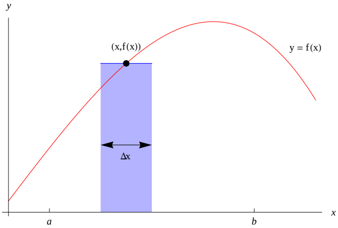
$$A = \int_a^b f(x) dx$$

The Area Between Two Curves

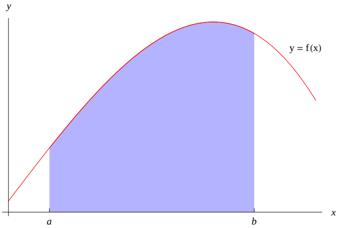


rectangle area $=$ height$\cdot \Delta x$

The Area Under a Curve



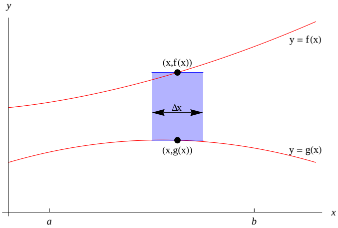
rectangle area $= f(x) \cdot \Delta x$



rectangles $= n \to \infty$

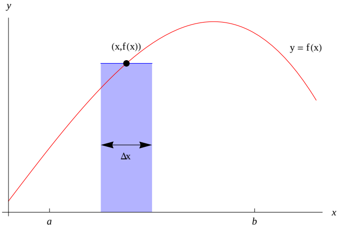
$$A = \int_a^b f(x)dx$$

The Area Between Two Curves

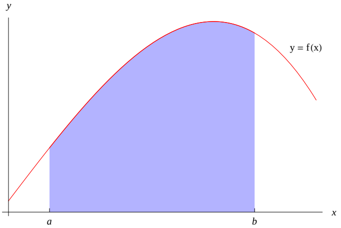


rectangle area $= (f(x) - g(x)) \cdot \Delta x$

The Area Under a Curve



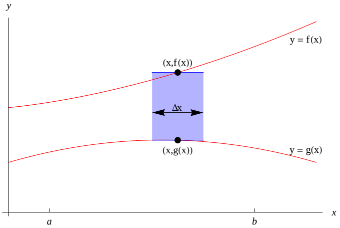
rectangle area $= f(x) \cdot \Delta x$



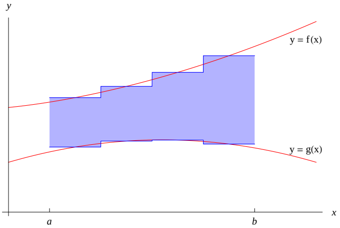
rectangles $= n \to \infty$

A = $\int_a^b f(x)dx$

The Area Between Two Curves



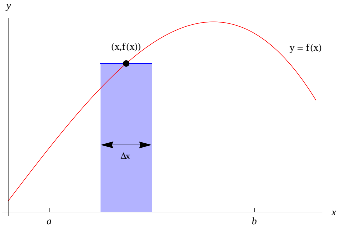
rectangle area $= (f(x) - g(x)) \cdot \Delta x$



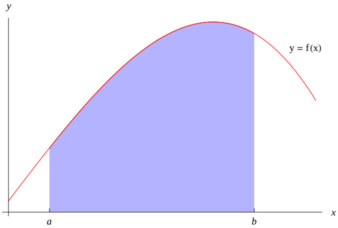
rectangles $= n = 4$

A = $\sum_{i=1}^{4} (f(x_i) - g(x_i))\Delta x$

The Area Under a Curve



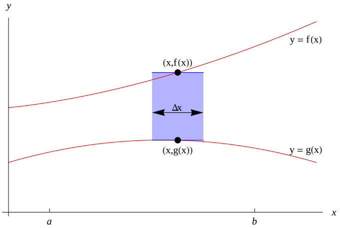
rectangle area $= f(x) \cdot \Delta x$



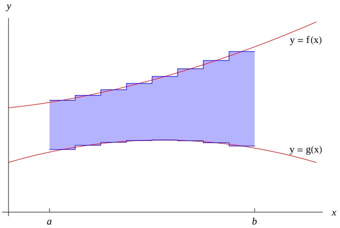
rectangles $= n \to \infty$

$$A = \int_a^b f(x)dx$$

The Area Between Two Curves



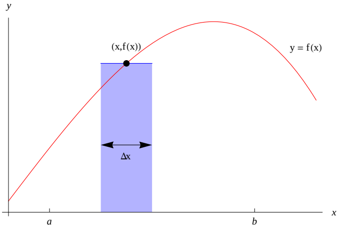
rectangle area $= (f(x) - g(x)) \cdot \Delta x$



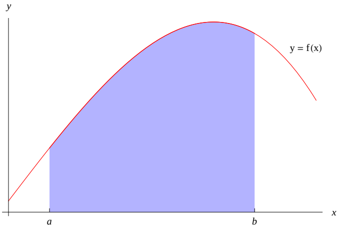
rectangles $= n = 8$

$$A = \sum_{i=1}^{8} (f(x_i) - g(x_i))\Delta x$$

The Area Under a Curve



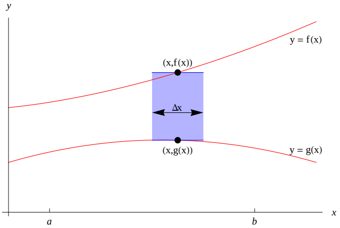
rectangle area $= f(x) \cdot \Delta x$



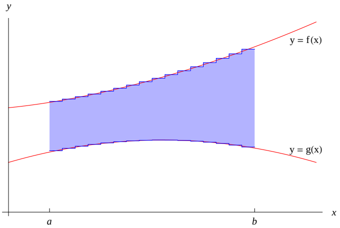
rectangles $= n \to \infty$

A = $\int_a^b f(x) dx$

The Area Between Two Curves



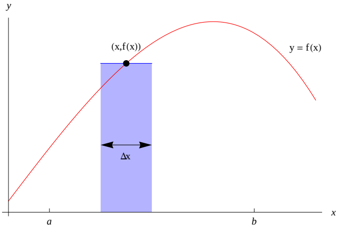
rectangle area $= (f(x) - g(x)) \cdot \Delta x$



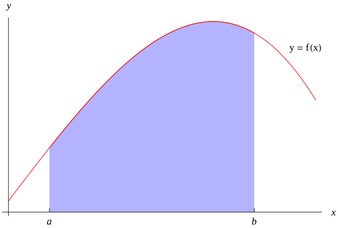
rectangles $= n = 16$

A = $\sum_{i=1}^{16} (f(x_i) - g(x_i)) \Delta x$

The Area Under a Curve



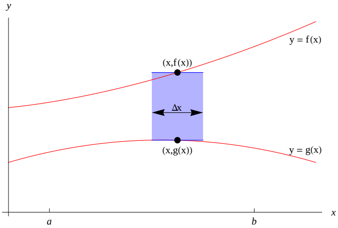
rectangle area $= f(x) \cdot \Delta x$



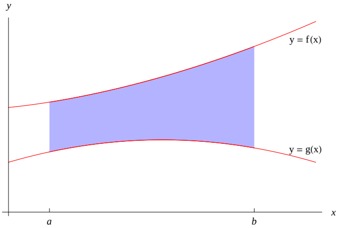
rectangles $= n \to \infty$

$$A = \int_a^b f(x) dx$$

The Area Between Two Curves



rectangle area $= (f(x) - g(x)) \cdot \Delta x$



rectangles $= n \to \infty$

$$A = \int_a^b [f(x) - g(x)] dx$$

Definition (The Area Between Two Curves)

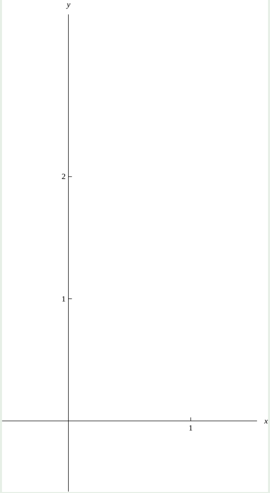
The area between two curves $y = f(x)$ and $y = g(x)$ bounded by the endpoints $x = a$ and $x = b$ is

$$\int_a^b |f(x) - g(x)| \mathrm{d}x.$$

Note that we use the absolute value, because in general we don't know which curve is above the other.

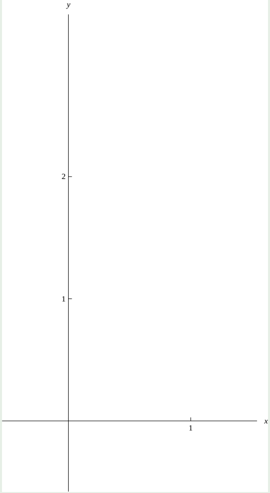
Example

Find the area of the region bounded above by $y = x^2 + 1$, bounded below by $y = x$, and bounded on its sides by $x = 0$ and $x = 1$.



Example

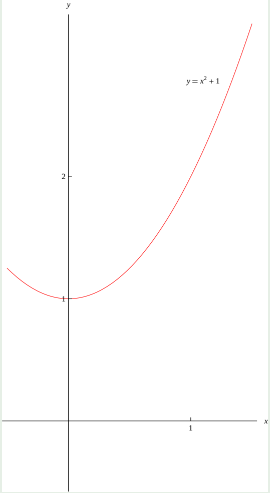
Find the area of the region bounded above by $y = x^2 + 1$, bounded below by $y = x$, and bounded on its sides by $x = 0$ and $x = 1$.



1. Graph the functions.

Example

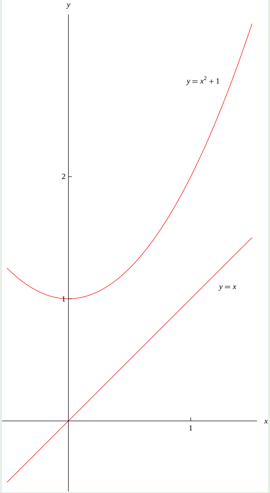
Find the area of the region bounded above by $y = x^2 + 1$, bounded below by $y = x$, and bounded on its sides by $x = 0$ and $x = 1$.



1. Graph the functions.

Example

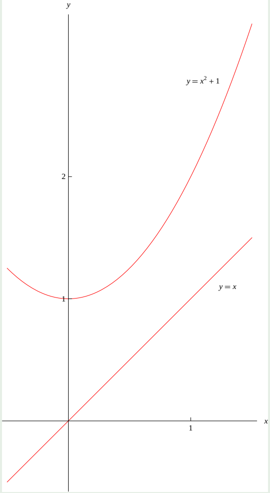
Find the area of the region bounded above by $y = x^2 + 1$, bounded below by $y = x$, and bounded on its sides by $x = 0$ and $x = 1$.



1. Graph the functions.

Example

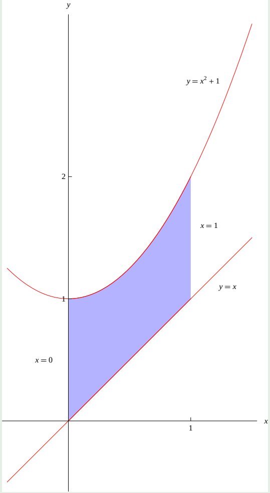
Find the area of the region bounded above by $y = x^2 + 1$, bounded below by $y = x$, and bounded on its sides by $x = 0$ and $x = 1$.



1. Graph the functions.
2. Identify the region.

Example

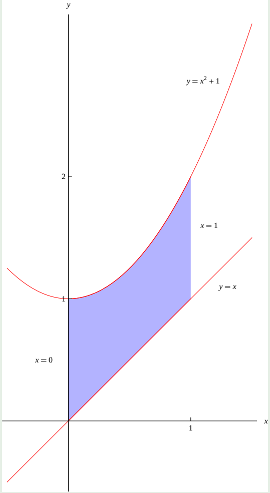
Find the area of the region bounded above by $y = x^2 + 1$, bounded below by $y = x$, and bounded on its sides by $x = 0$ and $x = 1$.



1. Graph the functions.
2. Identify the region.

Example

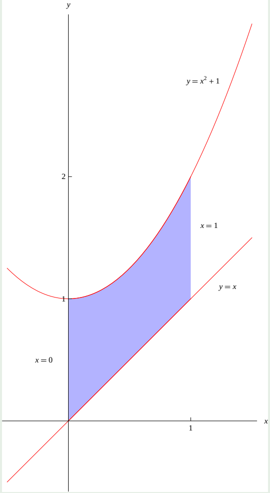
Find the area of the region bounded above by $y = x^2 + 1$, bounded below by $y = x$, and bounded on its sides by $x = 0$ and $x = 1$.



1. Graph the functions.
2. Identify the region.
3. Integrate.

Example

Find the area of the region bounded above by $y = x^2 + 1$, bounded below by $y = x$, and bounded on its sides by $x = 0$ and $x = 1$.

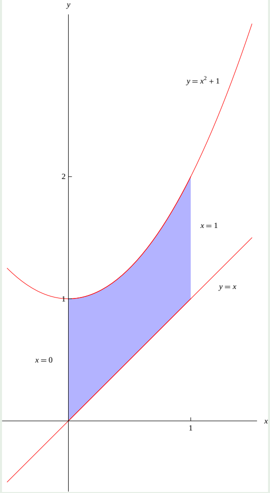


1. Graph the functions.
2. Identify the region.
3. Integrate.

$$A = \int_0^1 |(x^2 + 1) - x| dx$$
$$= \int_0^1 (x^2 - x + 1) dx$$

Example

Find the area of the region bounded above by $y = x^2 + 1$, bounded below by $y = x$, and bounded on its sides by $x = 0$ and $x = 1$.

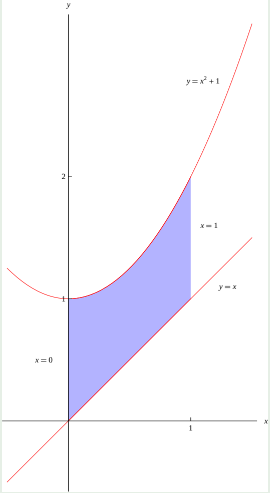


1. Graph the functions.
2. Identify the region.
3. Integrate.

$$A = \int_0^1 |(x^2+1) - x| \mathrm{d}x$$
$$= \int_0^1 (x^2 - x + 1) \mathrm{d}x$$
$$= \left[\frac{x^3}{3} - \frac{x^2}{2} + x\right]_0^1$$

Example

Find the area of the region bounded above by $y = x^2 + 1$, bounded below by $y = x$, and bounded on its sides by $x = 0$ and $x = 1$.

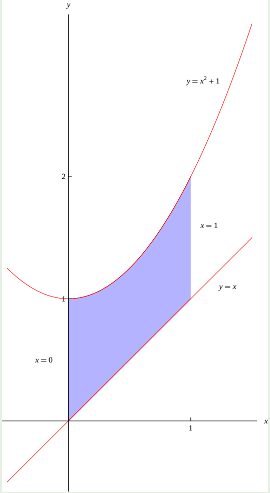


1. Graph the functions.
2. Identify the region.
3. Integrate.

$$\begin{aligned} A &= \int_0^1 |(x^2+1) - x| \mathrm{d}x \\ &= \int_0^1 (x^2 - x + 1) \mathrm{d}x \\ &= \left[\frac{x^3}{3} - \frac{x^2}{2} + x\right]_0^1 \\ &= \frac{1}{3} - \frac{1}{2} + 1 \end{aligned}$$

Example

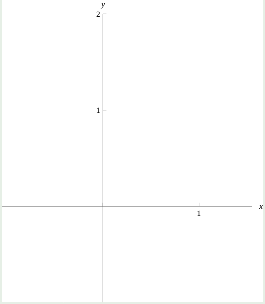
Find the area of the region bounded above by $y = x^2 + 1$, bounded below by $y = x$, and bounded on its sides by $x = 0$ and $x = 1$.



1. Graph the functions.
2. Identify the region.
3. Integrate.

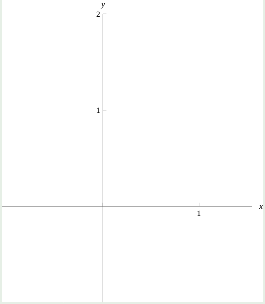
$$A = \int_0^1 |(x^2 + 1) - x| \mathrm{d}x$$
$$= \int_0^1 (x^2 - x + 1) \mathrm{d}x$$
$$= \left[\frac{x^3}{3} - \frac{x^2}{2} + x \right]_0^1$$
$$= \frac{1}{3} - \frac{1}{2} + 1 = \frac{5}{6}.$$

Example (Example 2, p. 433)



Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

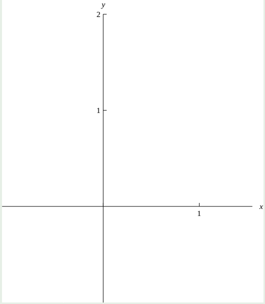
Example (Example 2, p. 433)



Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

1. Find the point of intersection.

Example (Example 2, p. 433)

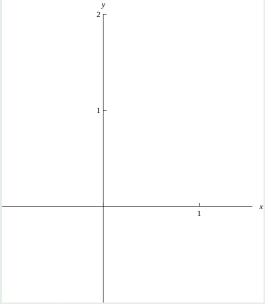


Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

$$x^2 = 2x - x^2$$

1. Find the point of intersection.

Example (Example 2, p. 433)



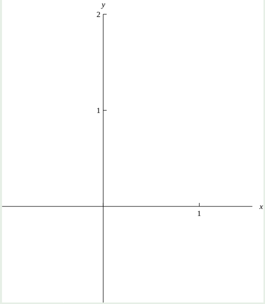
Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

$$x^2 = 2x - x^2$$

$$0 = 2x - 2x^2$$

1. Find the point of intersection.

Example (Example 2, p. 433)



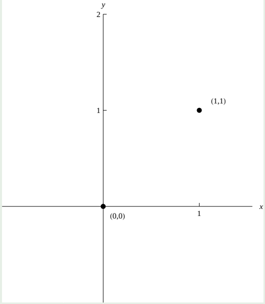
Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

$$x^2 = 2x - x^2$$

$$0 = 2x - 2x^2 = 2x(1 - x)$$

1. Find the point of intersection.

Example (Example 2, p. 433)



Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

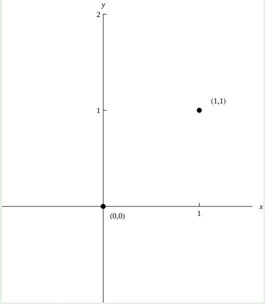
$$x^2 = 2x - x^2$$

$$0 = 2x - 2x^2 = 2x(1 - x)$$

$$x = 0 \text{ or } 1.$$

1. Find the point of intersection.

Example (Example 2, p. 433)

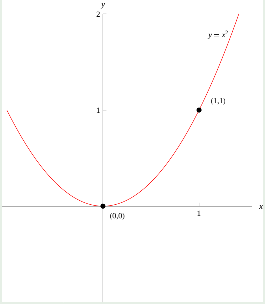


Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

$$
\begin{aligned}
x^2 &= 2x - x^2 \\
0 &= 2x - 2x^2 = 2x(1-x) \\
x &= 0 \text{ or } 1.
\end{aligned}
$$

1. Find the point of intersection.
2. Graph the functions.

Example (Example 2, p. 433)

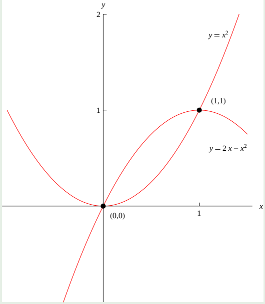


Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

$$x^2 = 2x - x^2$$
$$0 = 2x - 2x^2 = 2x(1 - x)$$
$$x = 0 \text{ or } 1.$$

1. Find the point of intersection.
2. Graph the functions.

Example (Example 2, p. 433)

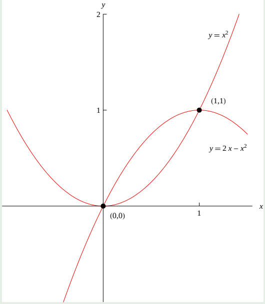


1. Find the point of intersection.
2. Graph the functions.

Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

$$x^2 = 2x - x^2$$
$$0 = 2x - 2x^2 = 2x(1 - x)$$
$$x = 0 \text{ or } 1.$$

Example (Example 2, p. 433)



1. Find the point of intersection.
2. Graph the functions.
3. Identify the region.

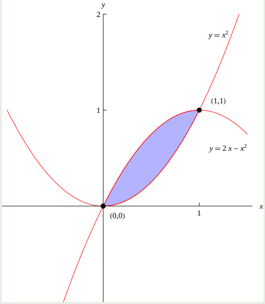
Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

$$x^2 = 2x - x^2$$

$$0 = 2x - 2x^2 = 2x(1 - x)$$

$$x = 0 \text{ or } 1.$$

Example (Example 2, p. 433)



Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

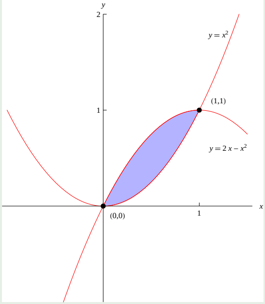
$$x^2 = 2x - x^2$$

$$0 = 2x - 2x^2 = 2x(1 - x)$$

$$x = 0 \text{ or } 1.$$

1. Find the point of intersection.
2. Graph the functions.
3. Identify the region.

Example (Example 2, p. 433)



Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

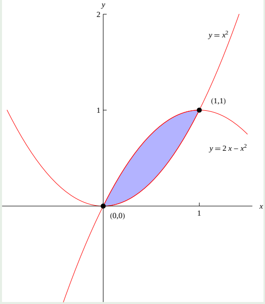
$$x^2 = 2x - x^2$$

$$0 = 2x - 2x^2 = 2x(1 - x)$$

$$x = 0 \text{ or } 1.$$

1. Find the point of intersection.
2. Graph the functions.
3. Identify the region.
4. Integrate.

Example (Example 2, p. 433)



1. Find the point of intersection.
2. Graph the functions.
3. Identify the region.
4. Integrate.

Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

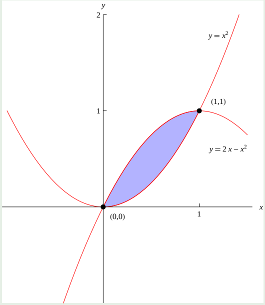
$$x^2 = 2x - x^2$$

$$0 = 2x - 2x^2 = 2x(1 - x)$$

$$x = 0 \text{ or } 1.$$

$$A = \int_0^1 (2x - 2x^2)\mathrm{d}x$$

Example (Example 2, p. 433)



1. Find the point of intersection.
2. Graph the functions.
3. Identify the region.
4. Integrate.

Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

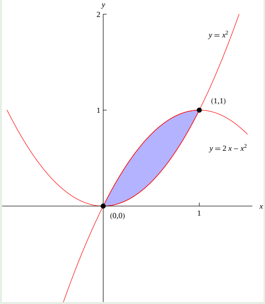
$$x^2 = 2x - x^2$$

$$0 = 2x - 2x^2 = 2x(1 - x)$$

$$x = 0 \text{ or } 1.$$

$$A = \int_0^1 (2x - 2x^2)\mathrm{d}x = 2\int_0^1 (x - x^2)\mathrm{d}x$$

Example (Example 2, p. 433)



1. Find the point of intersection.
2. Graph the functions.
3. Identify the region.
4. Integrate.

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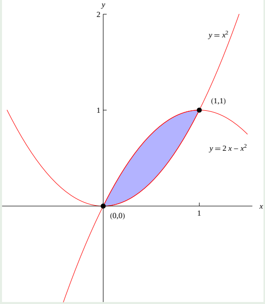

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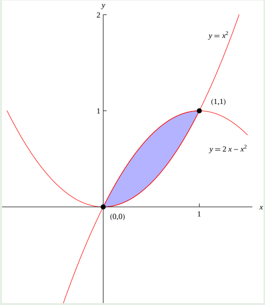
Example (Example 2, p. 433)



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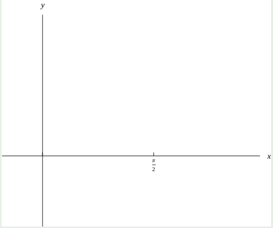
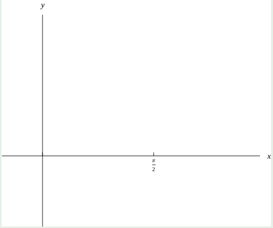
Example



Find the area of the region enclosed by the curves $y = \sin x$, $y = \cos x$, $x = 0$ and $x = \pi/2$.

Example



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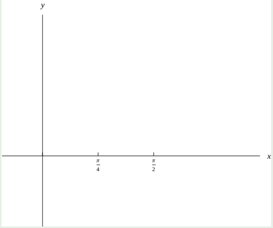

Find the area of the region enclosed by the curves $y = \sin x$, $y = \cos x$, $x = 0$ and $x = \pi/2$. The only point of intersection in the interval $[0, \pi/2]$ is $(\pi/4, 1/\sqrt{2})$.

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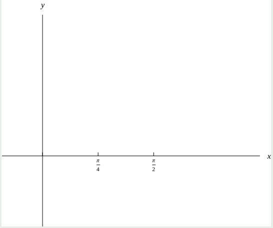

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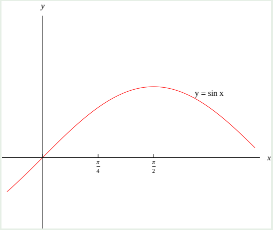

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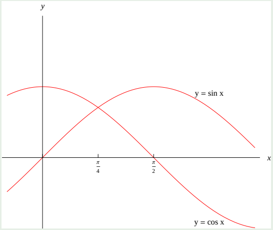

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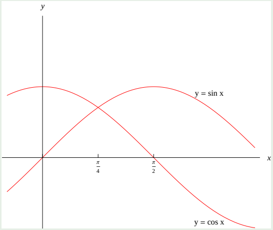

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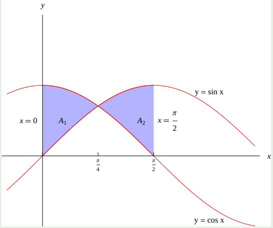

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$$A = A_1 + A_2$$

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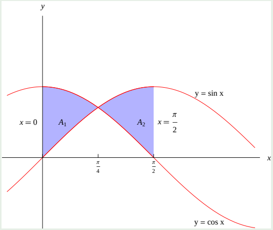
2. Graph the functions.
3. Identify the region.

Example



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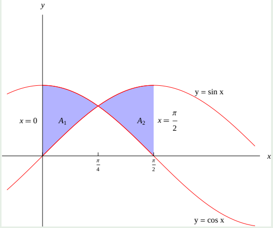
$$A = A_1 + A_2$$
$$= \int_0^{\pi/4} (\cos x - \sin x) dx$$
$$+ \int_{\pi/4}^{\pi/2} (\sin x - \cos x) dx$$

Example



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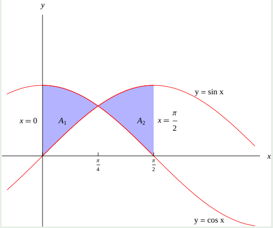
$$= [\sin x + \cos x]_0^{\pi/4} + [-\cos x - \sin x]_{\pi/4}^{\pi/2}$$

Example



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$$
\begin{aligned}
A &= A_1 + A_2 \\
&= \int_0^{\pi/4} (\cos x - \sin x)dx \\
&\quad + \int_{\pi/4}^{\pi/2} (\sin x - \cos x)dx \\
&= [\sin x + \cos x]_0^{\pi/4} + [-\cos x - \sin x]_{\pi/4}^{\pi/2} \\
&= 2\sqrt{2} - 2.
\end{aligned}
$$