Math 140

Lecture 25

Greg Maloney

with modifications by T. Milev

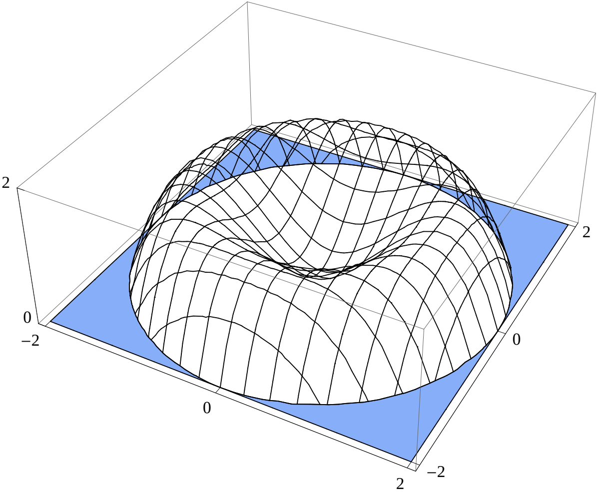
University of Massachusetts Boston

May 14, 2013

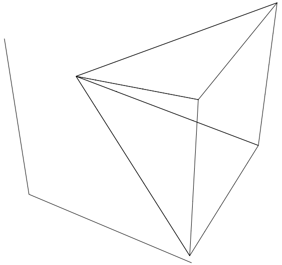
Outline

1 Volumes

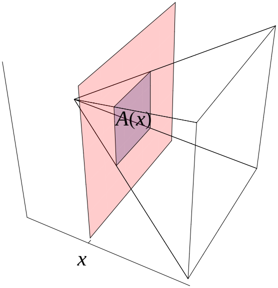
Volumes



We can use integration to find the volumes of certain solids.



- How do we find the volume of a solid S?



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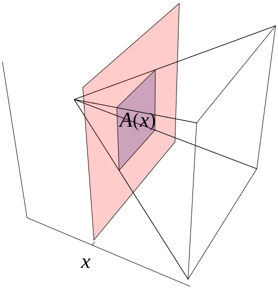
- Let $A(x)$ be the area of this cross-section.



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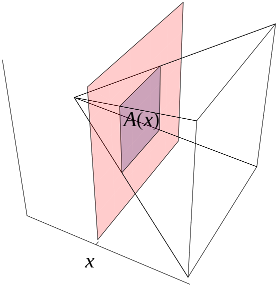

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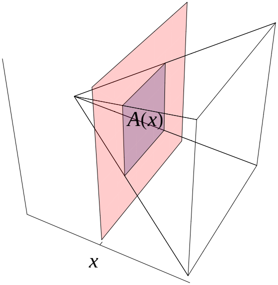

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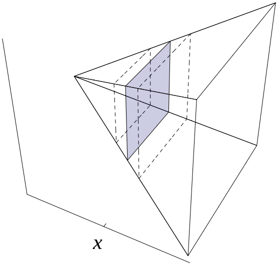

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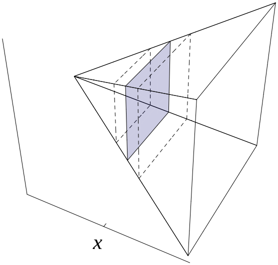

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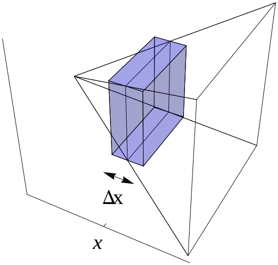

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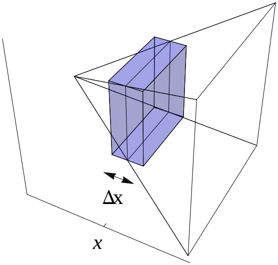

Approx. volume of slab:
$A(x^*)\Delta x$

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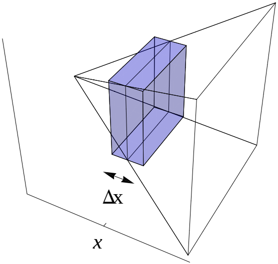

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Approx. volume of S:

$$V \approx \sum_{i=1}^{n} A(x_i^*)\Delta x$$

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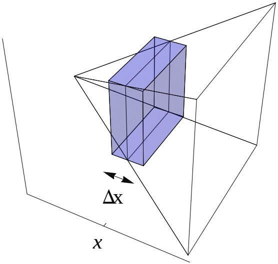

Approx. volume of slab:

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Approx. volume of S:

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Exact volume of S:

$$V = \lim_{n\to\infty} \sum_{i=1}^{n} A(x_i^*)\Delta x$$

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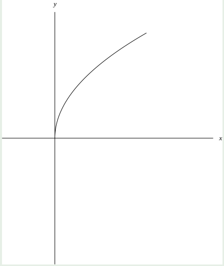
Definition (Volume)

Let S be a solid that lies between $x = a$ and $x = b$. If the cross-sectional area of S in the plane P_x is a continuous function $A(x)$, then the volume of S is

$$V = \lim_{n\to\infty} \sum_{i=1}^{n} A(x_i^*)\Delta x = \int_a^b A(x)\mathrm{d}x$$

Example (Example 2, p. 440)

Find the volume of the solid obtained by rotating about the x-axis the region under the curve $y = \sqrt{x}$ from 0 to 1.



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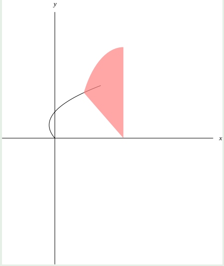

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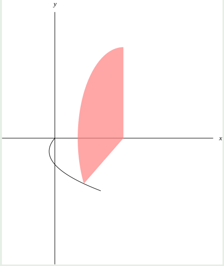

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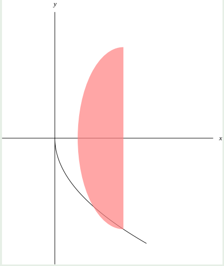

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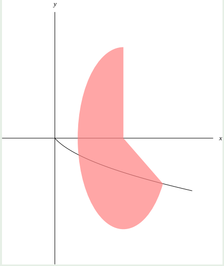

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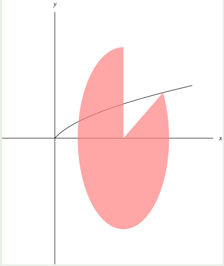

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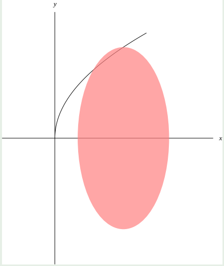

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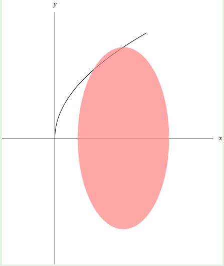
- The cross-sections of this solid are all circles.



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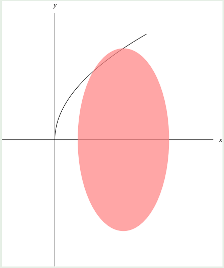

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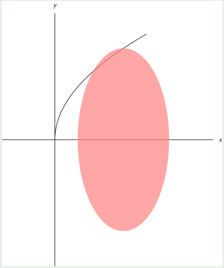

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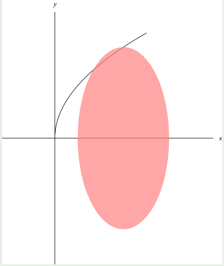

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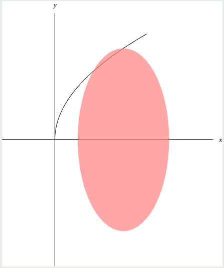

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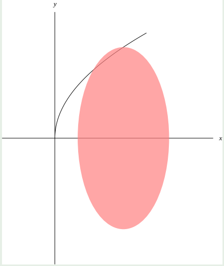

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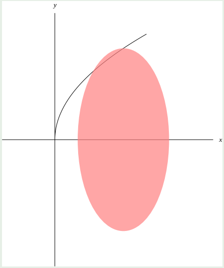

$$V = \int_0^1 A(x)\,dx$$

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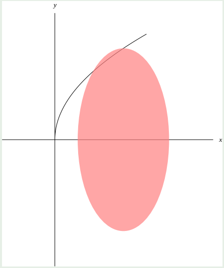

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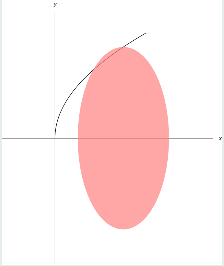

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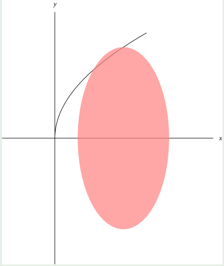

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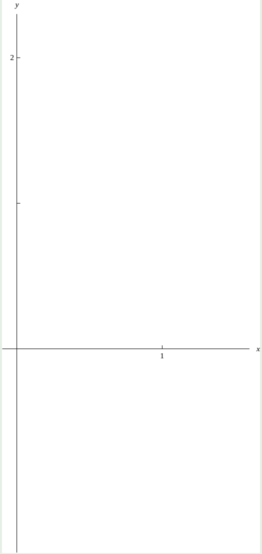
Example (Typical Cross-Section is a Washer)



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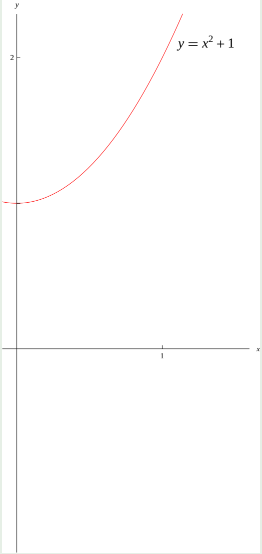

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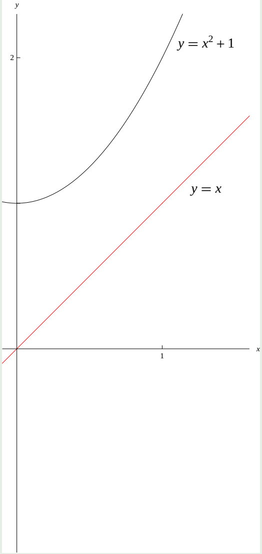

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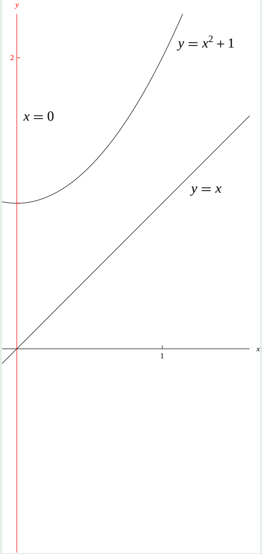

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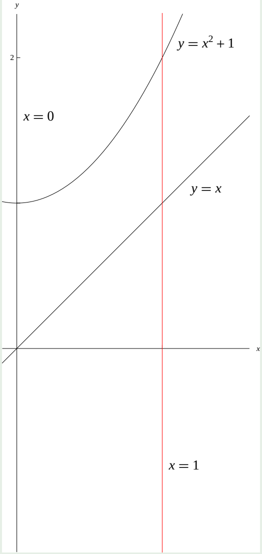

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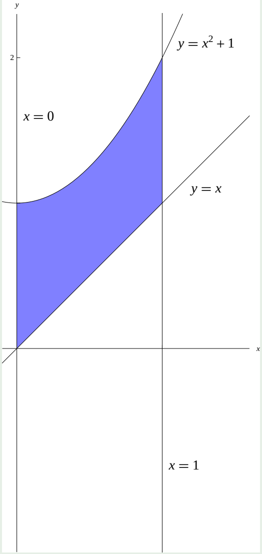

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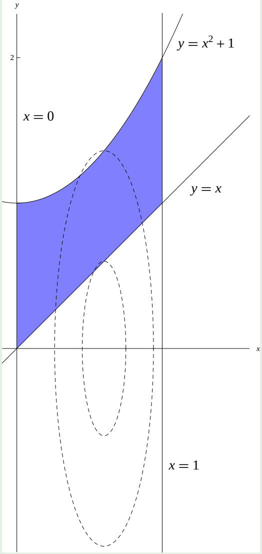

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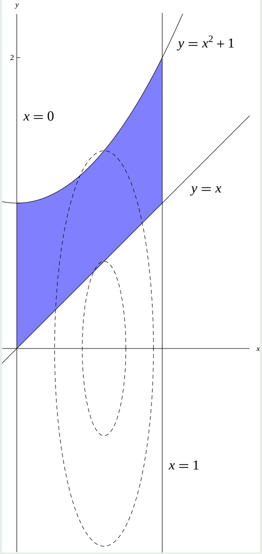

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Area of the outer circle:

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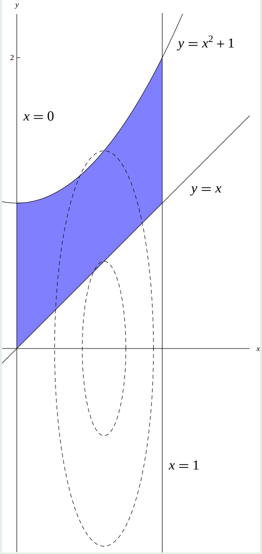

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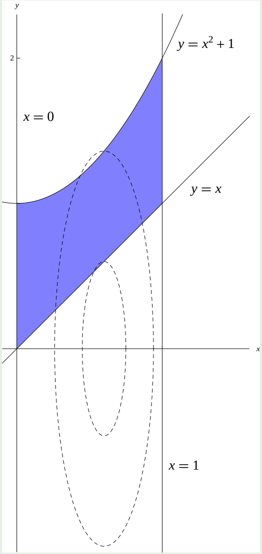

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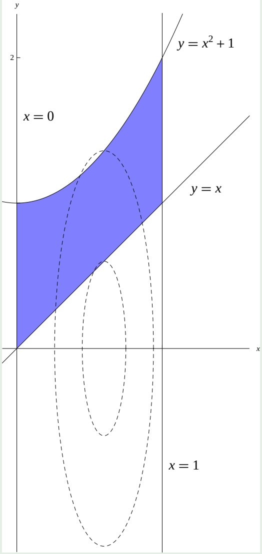

Find the volume of the solid obtained by rotating about the x-axis the region bounded by $y = x^2 + 1$, $y = x$, $x = 0$, and $x = 1$.

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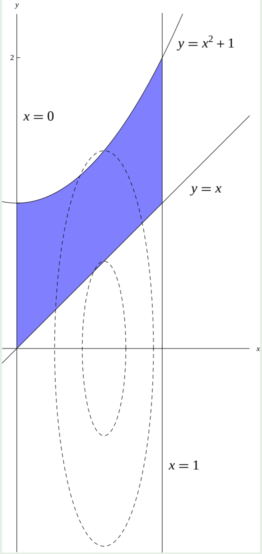

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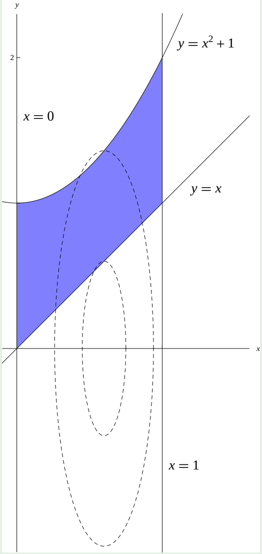

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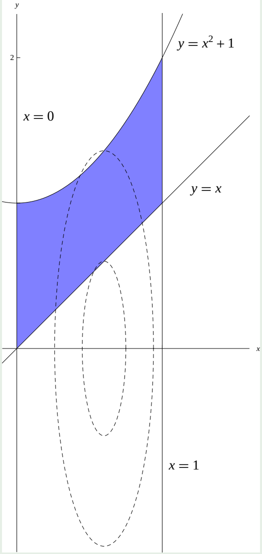

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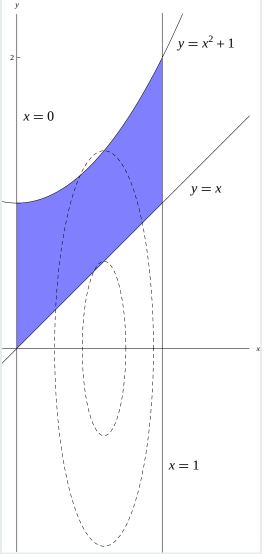

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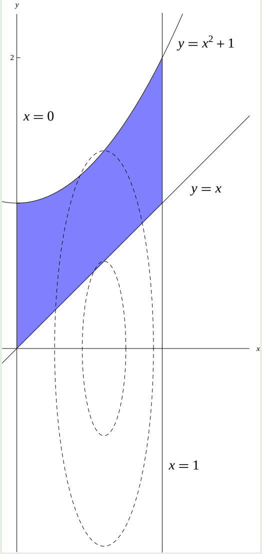

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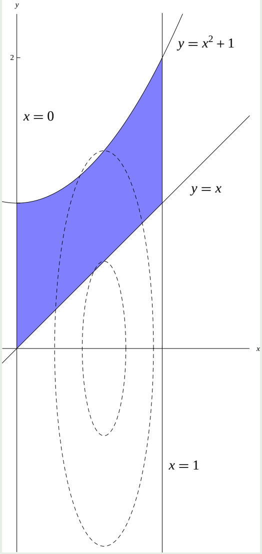

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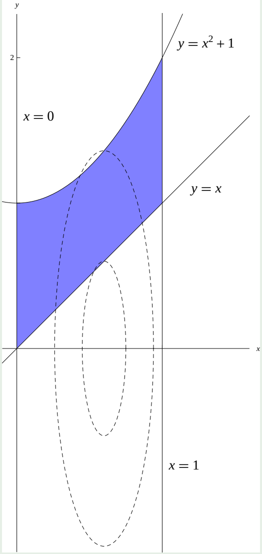

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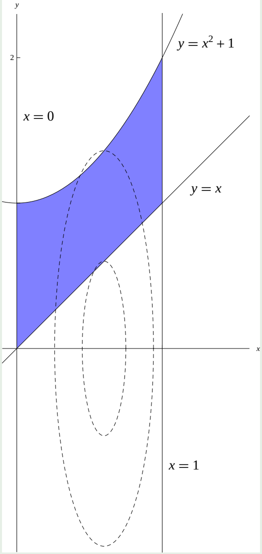

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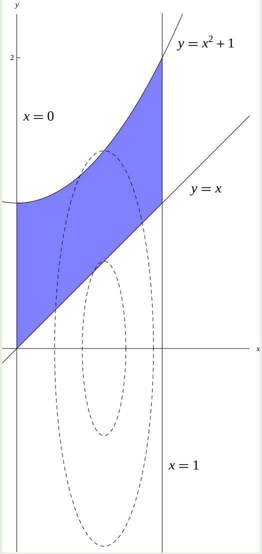

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&= \pi \left(\frac{1}{5} + \frac{1}{3} + 1\right)
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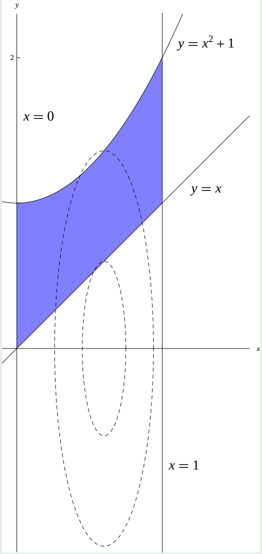

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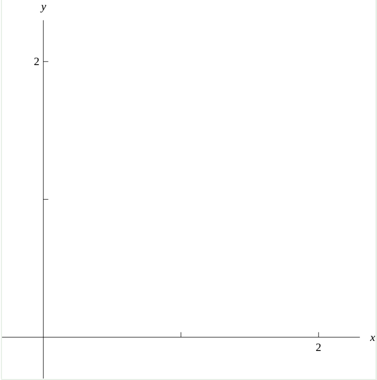
Example (Rotated About a Line Other Than the x-axis)

Find the volume of the solid obtained by rotating about the line $y = 1$ the region bounded by $y = -x^2 + 2x + 1$ and $y = 1$.



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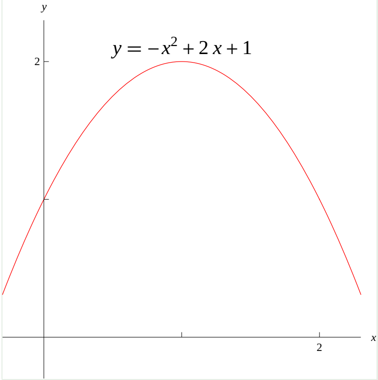

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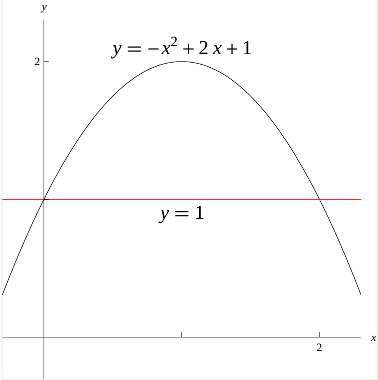

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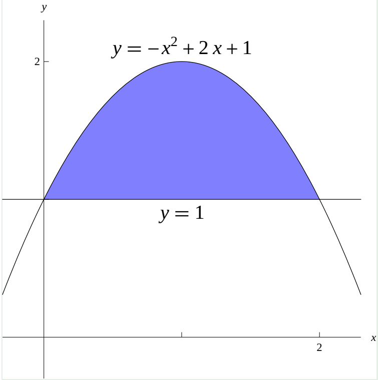

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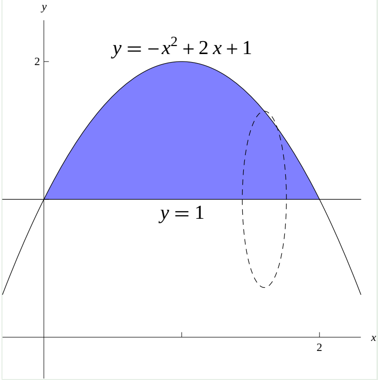

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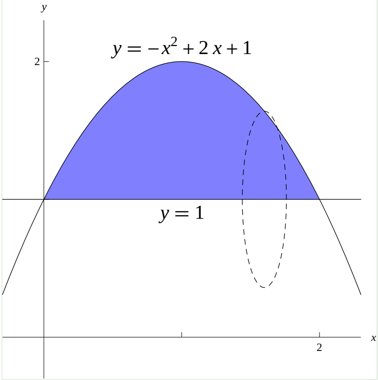

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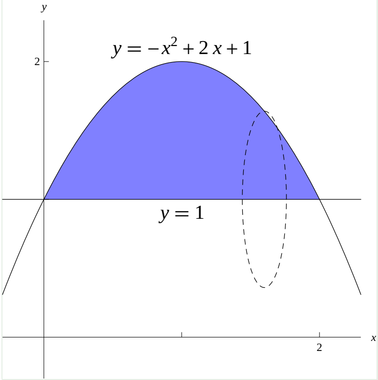

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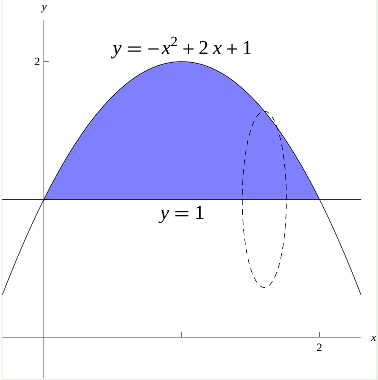

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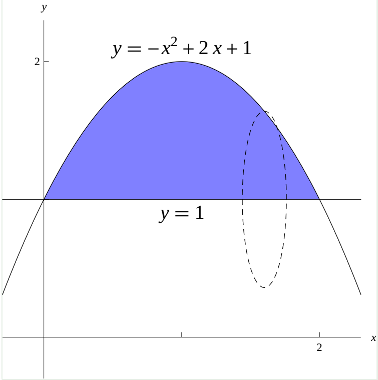

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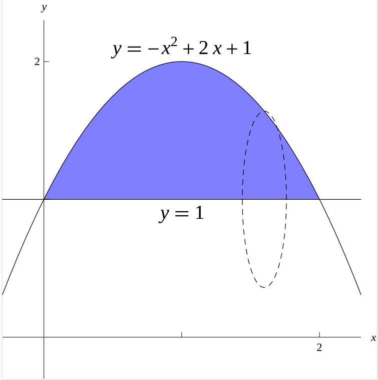

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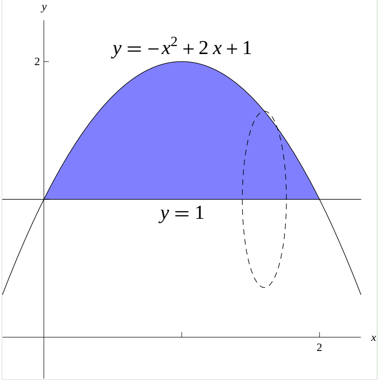

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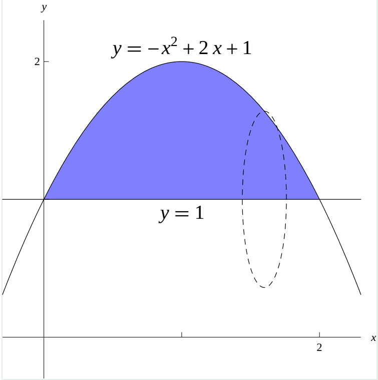

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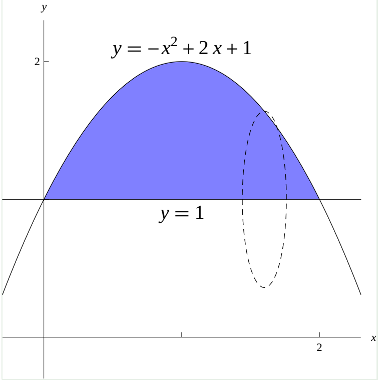

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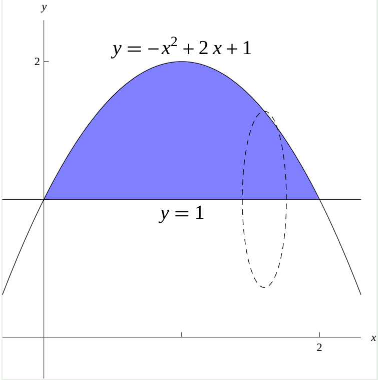

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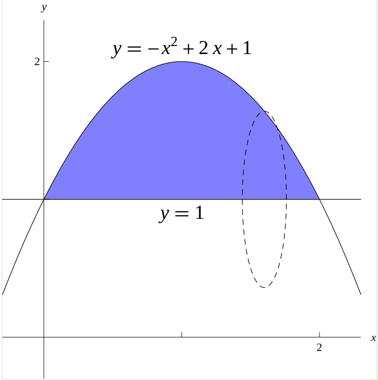

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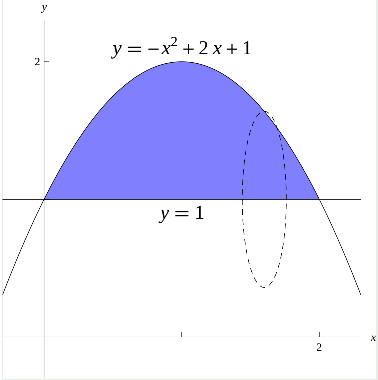

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&= \pi \int_0^2 \left(x^4 - 4x^3 + 4x^2 \right) \, \mathrm{d}x \\
&= \pi \left[\frac{x^5}{5} - x^4 + \frac{4x^3}{3} \right]_0^2 \\
&= \pi \left(\frac{32}{5} - 16 + \frac{32}{3} \right)
\end{aligned}
$$

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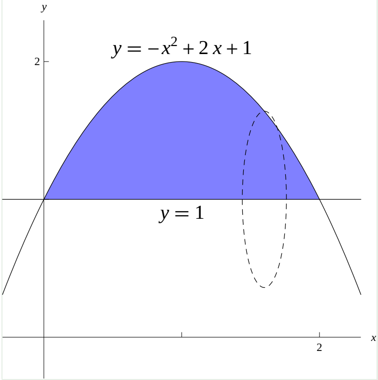

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$$= \pi \left(\frac{32}{5} - 16 + \frac{32}{3}\right) = \frac{16}{15}\pi$$