Review problems
Final Math 140

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Problem 1 *Define horizontal and vertical asymptote.*

Problem 2 *Compute the horizontal and vertical asymptotes of*

1. $f(x) = \dfrac{x^2 + x + 1}{-x^2 - 3x + 4}$ — *answer: horizontal asymptote:* $y = -1$, *vertical asymptotes* $x = 1, x = -4$.
2. $f(x) = \dfrac{x^2 + x + 1}{x^2 - 2x - 3}$ — *answer: horizontal asymptote:* $y = 1$, *vertical asymptotes* $x = -1, x = 3$.

Problem 3 *Define derivative.*

Problem 4 *Find the limit*

1. $\lim\limits_{h \to 0} \dfrac{\sin^2(x+h) - \sin^2(x)}{h}$. — *answer:* $2 \sin x \cos x$.
2. $\lim\limits_{h \to 0} \dfrac{\frac{1}{(x+h)^3} - \frac{1}{x^3}}{h}$, *where* $x \neq 0$. — *answer:* $\frac{-3}{x^4}$.

Problem 5 *Find the derivative.*

1. x^{x^x}. — *answer:* $x^{x^x+x-1} + x^{x^x+x} \ln x + x^{x^x+x-1} (\ln x)^2$.
2. $\sqrt[3]{\ln x}$ — *answer:* $\frac{(\ln x)^{-2/3}}{3x}$.

Problem 6 *Consider the curve given by the equation* $x^3 + x^2y^2 + 2y^3 = 4$.

- *Use implicit differentiation to express* $\frac{dy}{dx}$ *via* y *and* x *on the curve.*
- *Find the equation of the tangent line to the curve at the point* $(x, y) = (1, 1)$.

Problem 7 *State the fundamental theorem of Calculus (both parts).*

Problem 8 *Compute the derivative.*

1. $\displaystyle\int_{\sqrt{x}}^{x^2} e^t \, dt$.
2. $\displaystyle\int_{\ln x}^{-x} e^t \, dt$.

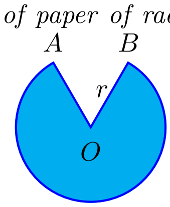
Problem 9 *Compute the integral*

1. $\displaystyle\int_{-1}^{1} \frac{x^3}{1 + x^2} dx$
2. $\displaystyle\int_{-2}^{2} \frac{x^5}{1 + x^2} dx$

Problem 10 *Find intervals of increasing, decreasing, maxima, minima (local and absolute if those exist), concavity, inflection points. Finally, plot the function roughly by hand. Compare your answer with the output of a graphic calculator (you may use an online graphic calculator as well).*

1. $f(x) = x^4 - 3x^3 + 2$.
2. $f(x) = x^4 - 2x^3 - 3$.

Problem 11 *A cone-shaped drinking cup is made from a circular piece of paper of radius* r *by cutting out a sector and joining the edges* OA *and* OB. *Find the maximum capacity of such a cup.*



Problem 12 *The minute hand on a watch is 0.5cm long and the hour hand is 0.25cm long. How fast is the distance between the tips of the hands changing at 2 o'clock?*

Problem 13 *Define the linearization of a function at a point.*

Problem 14 *Explain the difference between definite integral and indefinite integral.*

Problem 15 *Define Riemann sum. Define definite integral. Give an example of a Riemann sum.*

Problem 16 *State Rolle's theorem. Use Rolle's theorem to prove that* $x^3 + x^2 + 5x - 4$ *has only one real root.*

Problem 17 *Consider the parabola* $y = 1 - x^2$.

1. *Find the area locked between the parabola and the* x *axis.*
2. *Find the volume of the solid of revolution obtained by rotating the parabola around the* x *axis.*