Homework #4

Problem 1: Derivation of central difference

Derive the central difference approximation of the first derivative of y_i with an error of magnitude h^2:

$y_i' = (y_{i+0.5} - y_{i-0.5})/h + O(h^2)$

Problem 2: Solution of ODE

Consider the following ODE:

$\frac{dy}{dt} = -y$ with initial condition y(0) = 5.

a) Discretize the equation using a forward difference scheme with step size of magnitude h. Write the equation and the initial condition in discretized form.
b) Solve the discretized equation for h = 0.1 from t = 0 until t = 2. Present all intermediate steps (y_0, y_1, y_2, etc.). Using MATLAB is strongly encouraged.
c) Plot the solution of the ODE (derived from analytically solving the equation) and the solution derived from part b) in the same plot.

Problem 3: Solution of nonlinear equation

The following empirical relationship is available for the light propagation through biological tissues:

$$\frac{A-1}{A+1} = -1.440n_{rel}^{-2} + 0.719n_{rel}^{-1} + 0.688 + 0.0636n_{rel}$$

Where A is the internal reflectance factor for a tissue, and n_{rel} is the ratio of the refractive indices of the tissue and the medium. When n_{rel} equals 1, A reduces to unity. Find the value of n_{rel} that results in a reflectance factor of 4. Use a method of your choice (from among the ones covered in this class). Illustrate your technique by showing intermediate steps.