Homework #6

Problem 1: Types of ODEs

Determine for each of the ODEs: a) Order, b) linear or nonlinear, c) homogeneous or non-homogeneous

1. $7\frac{dy}{dt} + y = 5$
2. $5^2 \frac{d^2 y}{dt^2} + \frac{dy}{dt} + 3y = 0$
3. $\frac{d^2 y}{dt^2} + 5\frac{dy}{dt} + 5y + 2 = 0$
4. $-2\frac{d^2 y}{dt^2} + 15\frac{dy}{dt} + 25y = \sin(t)$
5. $y^{0.33} \frac{dy}{dt} + y = 5$
6. $\left(\frac{dy}{dt}\right)^2 + y + 1 = 0$
7. $\frac{d^4 y}{dt^4} + t^3 \frac{dy}{dt} + y = 0$
8. $y\frac{dy}{dt} + y = 0$

Problem 2: Canonical Form

Transform $\frac{d^4 y}{dt^4} + 3\frac{dy}{dt} + y = 0$ into canonical form (i.e., turn a higher order ODE into a system of first order ODEs).

Problem 3: Canonical Form (the opposite of problem 2 ☺)

Transform $\frac{dx}{dt} = Ax$ into one ODE of higher order.

Note: $A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix}$

Problem 4: Numerical Solution of ODE (Initial Value Problem)

Consider the following ODE:

$\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y = 0$ with initial conditions y(0) = 5, y'(0) = -1.

a) Discretize the equation using a forward difference scheme with step size of magnitude h. Write the equation and the initial conditions in discretized form.
b) Solve the discretized equation for h = 0.1 from t = 0 until t = 2. Present all intermediate steps (y_0, y_1, y_2, etc.). Using MATLAB is strongly encouraged.
c) Plot the numerical solution of the ODE and its analytical solution in the same plot.

Problem 5: Numerical Solution of ODE (Boundary Value Problem)

Consider the following ODE:

$\frac{d^2y}{dt^2} - 5\frac{dy}{dt} + 6y = 0$ with boundary conditions y(0) = 5, y(1.5) = 1.

a) Discretize the equation using a forward difference scheme with step size of magnitude h. Write the equation and the boundary conditions in discretized form.
b) Solve the discretized equation for h = 0.1 from t = 0 until t = 1.5. Present all intermediate steps (y_0, y_1, y_2, etc.). Using MATLAB is strongly encouraged.
c) Plot the numerical solution of the ODE.

Problem 6: Solution of ODE in MATLAB

You are given the following reaction scheme:

$$E + S \underset{k_{-1}}{\overset{k_1}{\rightleftarrows}} ES \overset{k_2}{\rightarrow} E + P$$

With initial conditions S(0) = 1.0, E(0) = 0.1, ES(0) = 0, P(0) = 0, and rate constants k_1 = 0.1, k_{-1} = 0.1, and k_2 = 0.3.

Assume mass action kinetics. Model the system and simulate it from time t = 0 until 100 using ODE45 in MATLAB.