Homework 8

Problem 1: Phase Plane Analysis in MATLAB

Write your own code for performing phase plane analysis. Hand in the code.

Problem 2: Reactor Model

The model describes a reactor where A is converted to B. The mathematical description of the model consists of two differential equations, which represent the dynamic component (C_A) as well as temperature (T) balance:

$$\frac{dC_A}{dt} = \frac{q}{V} \left(C_{A,f} - C_A \right) - k_0 \exp\left(-\frac{E}{RT} \right) C_A \tag{1}$$

$$\frac{dT}{dt} = \frac{q}{V} \left(T_F - T \right) - \frac{\Delta H}{\rho C_p} k_0 \exp \left(-\frac{E}{RT} \right) C_A + \frac{UA}{V \rho C_p} \left(T_C - T \right)$$
(2)

The parameters for the process as well as the nominal operating conditions are given in the following table:

Variable	Value	Variable	Value
q	$0.1667E-2 \text{ m}^3 \text{ s}^{-1}$	E/R	8750 K
$C_{A,f}$	1000 mol m ⁻³	k_0	1.2 E9 s ⁻¹
T_f	350 K	UA	8.333E2 J (s K) ⁻¹
V	0.1 m ³	$T_C^{nominal}$	300 K
ρ	1000 kg m ⁻³	$C_A^{nominal}$??? mol m ⁻³
C_P	239 J (kg K) ⁻¹	$T^{nominal}$??? K
ΔН	-5E4 J mol ⁻¹		

- a) For the nominal value of the input, $T_C^{nominal}$, determine the equilibrium point(s) of the system.
- b) Simulate the system for slight perturbations around the equilibrium points (by choosing initial conditions which are slightly different from the equilibrium points). Plot a few representative cases (if possible combine several simulations in the same plot).
- c) Perform phase plane analysis of the system. Make sure that your analysis includes the equilibrium points.

Problem 3: Phase Plane Analysis

For each of the following systems:

- 1) Mathematically determine the equilibrium points.
- 2) Plot the phase plane to include some of the equilibrium points.

a)
$$\frac{d^2x}{dt^2} + (x^2 - 1)\frac{dx}{dt} + x = 0$$
, with $x_1 = x$, $x_2 = dx/dt$

b)
$$\frac{dx}{dt} = \sin(x+y)$$
$$\frac{dy}{dt} = \cos(xy)$$