Homework 9

Problem 1: Initial and Final Value Problem

Consider the following nonlinear ODE:

$$\frac{dy}{dt} = -y^3$$

a) For y(0) = 10, solve the ODE numerically using a forward finite difference approximation of the derivative. Simulate the system from 0 to 2 using h = 0.001.
b) For y(2) = 0.2, solve the ODE numerically using a backward finite difference approximation of the derivative. Simulate the system over the time span from 0 to 2 using h = 0.001.
c) Briefly comment on what would have happened to your system that you needed to solve if you had been asked to use a backward finite difference approximation for part a) or a forward finite difference approximation for part b)?

Problem 2: Numerical Solution of PDE

Numerically solve the following PDE

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

With initial/boundary conditions:

$$u(0,x) = \frac{2x}{1+x^2}, u(t,0) = 0, u(t,1) = 1$$

Use a forward finite difference approximation for the time derivative and central finite difference for the space derivative. Solve for t from 0 to 2 and x from 0 to 1. Use Δx=0.1 and Δt=0.2.

Plot u as a function of x and t.

Note: you will have to repeatedly solve this problem for each point in time and for each point in space not given by the boundary conditions.

Also note,

$$\frac{\partial u_{i,j}}{\partial t} \approx \frac{1}{\Delta t}\left(u_{i+1,j} - u_{i,j}\right), \frac{\partial^2 u_{i,j}}{\partial x^2} \approx \frac{1}{\Delta \mathrm{x}^2}\left(u_{i,j+1} - 2u_{i,j} + u_{i,j-1}\right)$$

Problem 3: Repeat Problem 2 but with Difference Step Sizes

Solve problem 2 again, but now use $\Delta x=0.2$ and $\Delta t=0.01$. What do you observe when comparing the solutions?

Give a brief explanation of what you are seeing.