MX-Quadtrees

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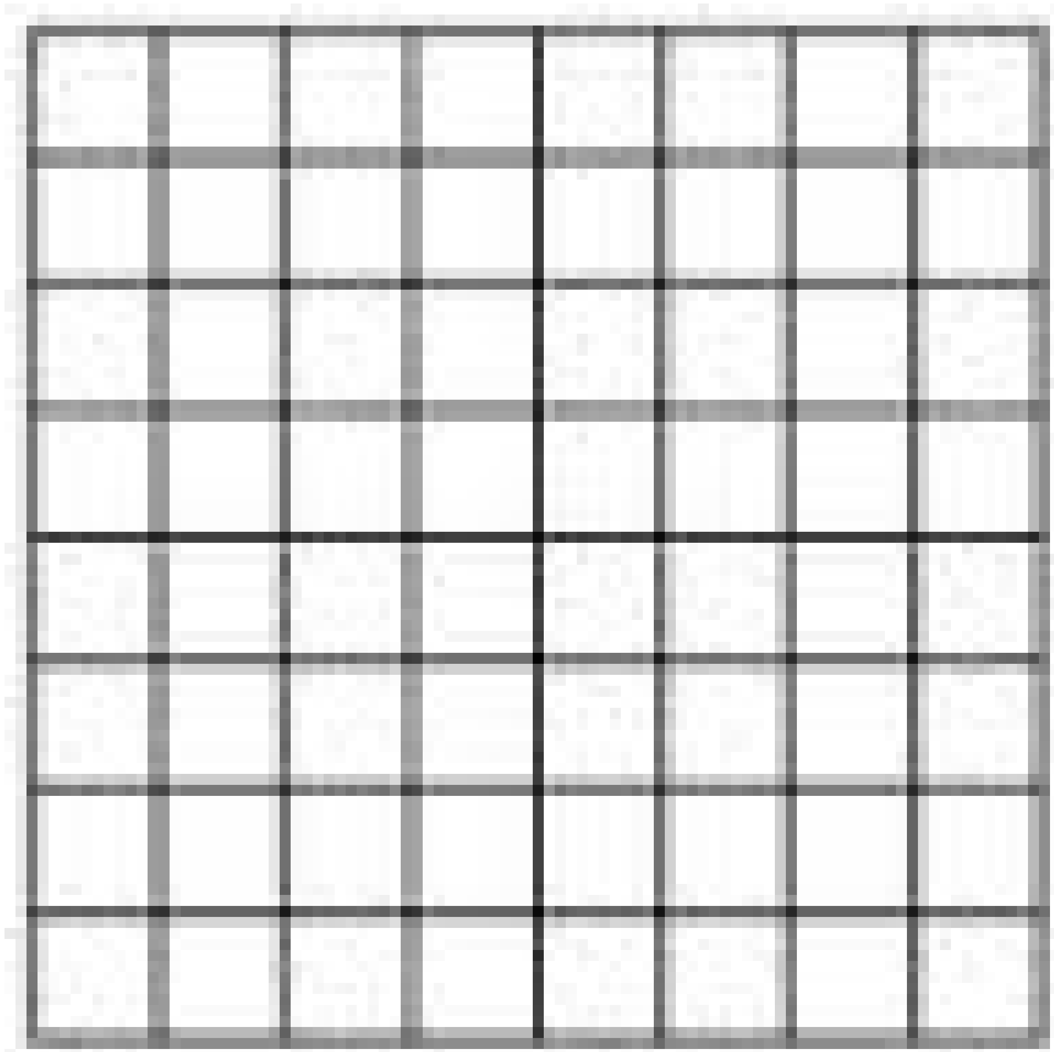
MX-Quadtrees

- Unlike point quadtrees, MX-quadtree nodes <u>always</u> split regions into equal-sized subregions.
- MX-quadtrees always assume that
  - The overall region is a $2^n \times 2^n$ region for some n
  - All coordinates x,y are <u>integers</u> ranging from 0 up to (and including) $2^n - 1$.
- All data is stored in leaf nodes.

Definition of an MX-Quadtree

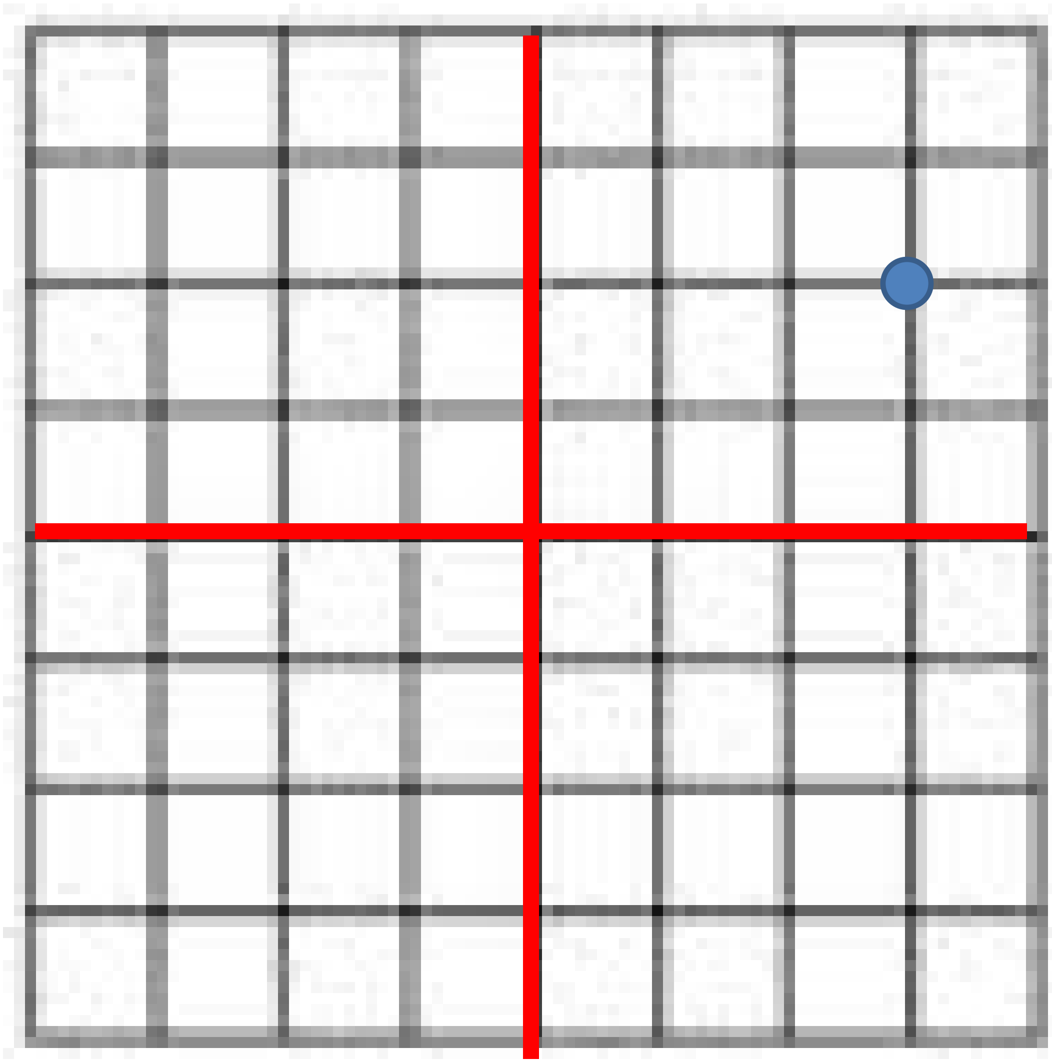
- MX quadtrees are used to represent $2^n \times 2^n$ region for some n.
- An MX-quadtree is a tree of arity 4 where each node implicitly represents a region and has (at least) the following fields.
- The root represents the entire $2^n \times 2^n$ region.
- Each node has upto 4 child links NW,SW,SE,NE.
- Region(N) is split into four *equal* pieces by drawing a horizontal and vertical line through the center of N, resulting in 4 quadrants (regions) corresponding to N's children.

Example: Insert (7,6)

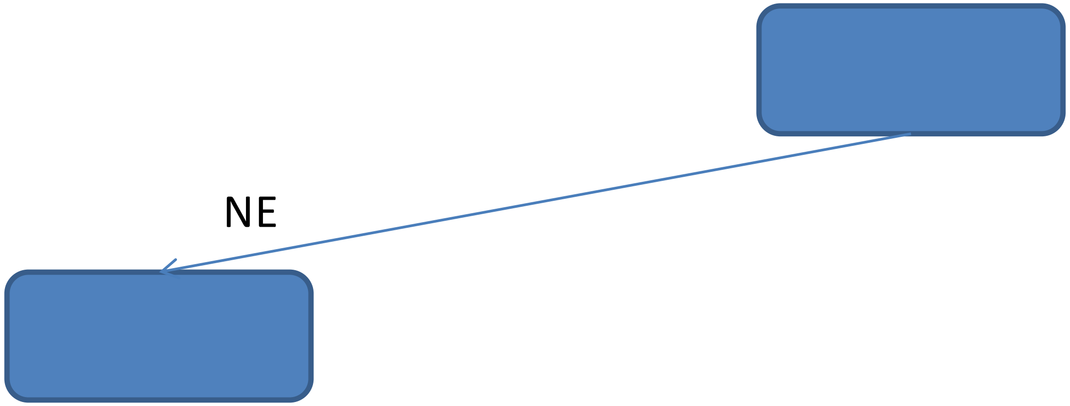


Insert (7,6)

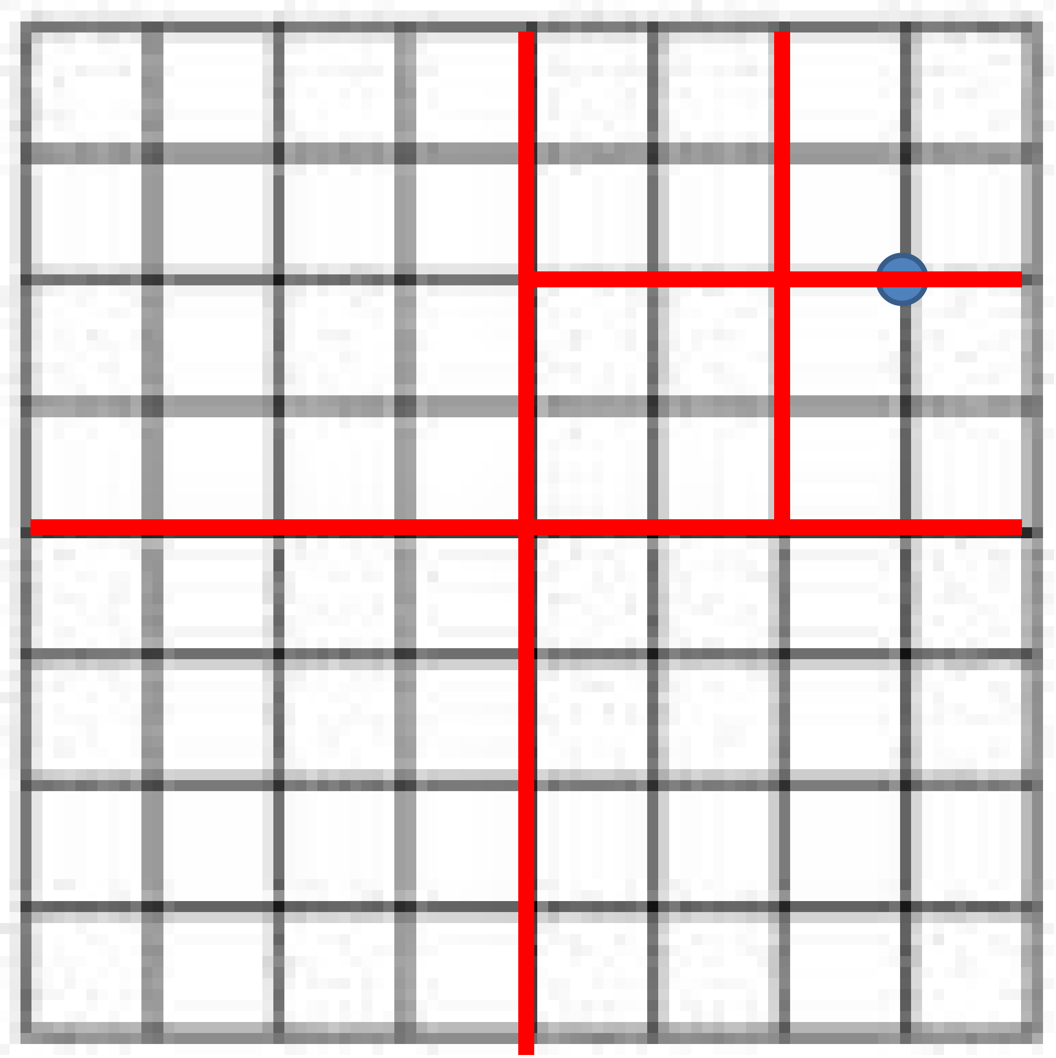
Example: Insert (7,6)



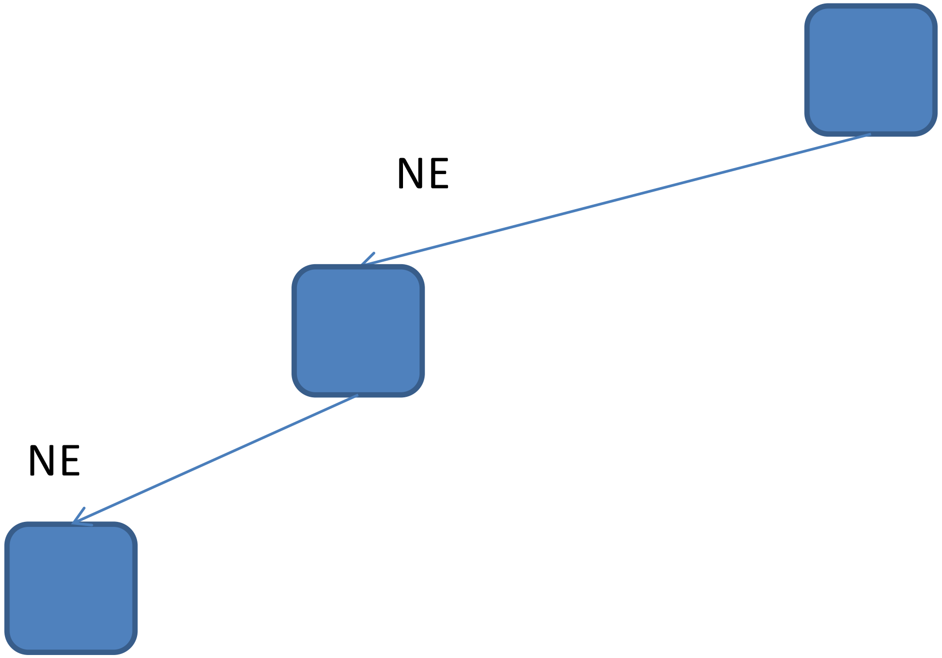
Insert (7,6)



Example: Insert (7,6)



Insert (7,6)



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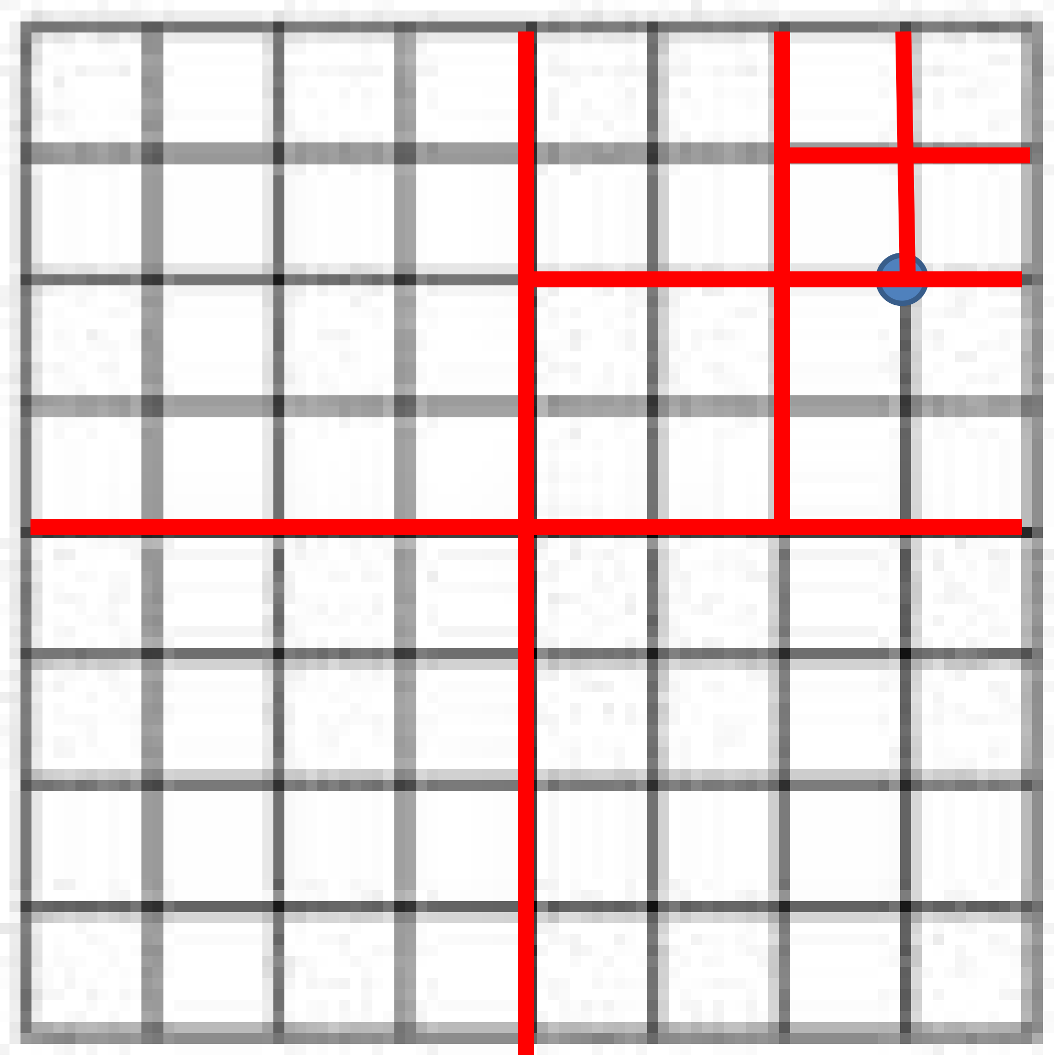
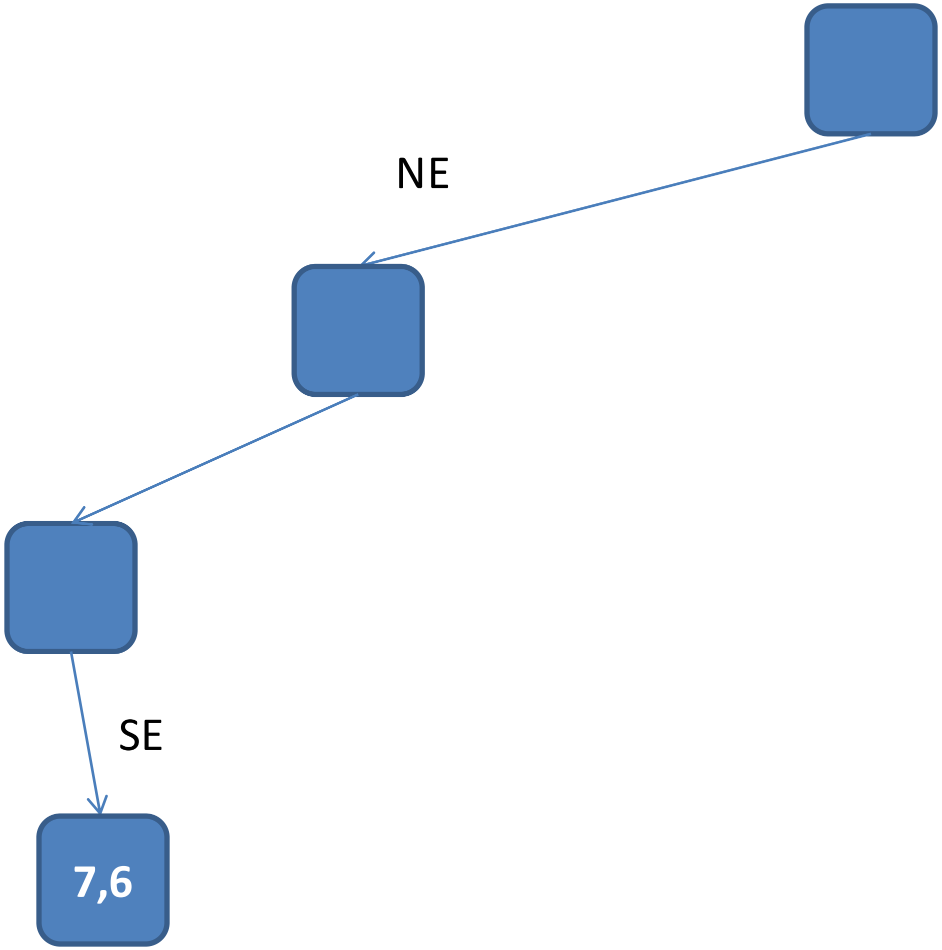
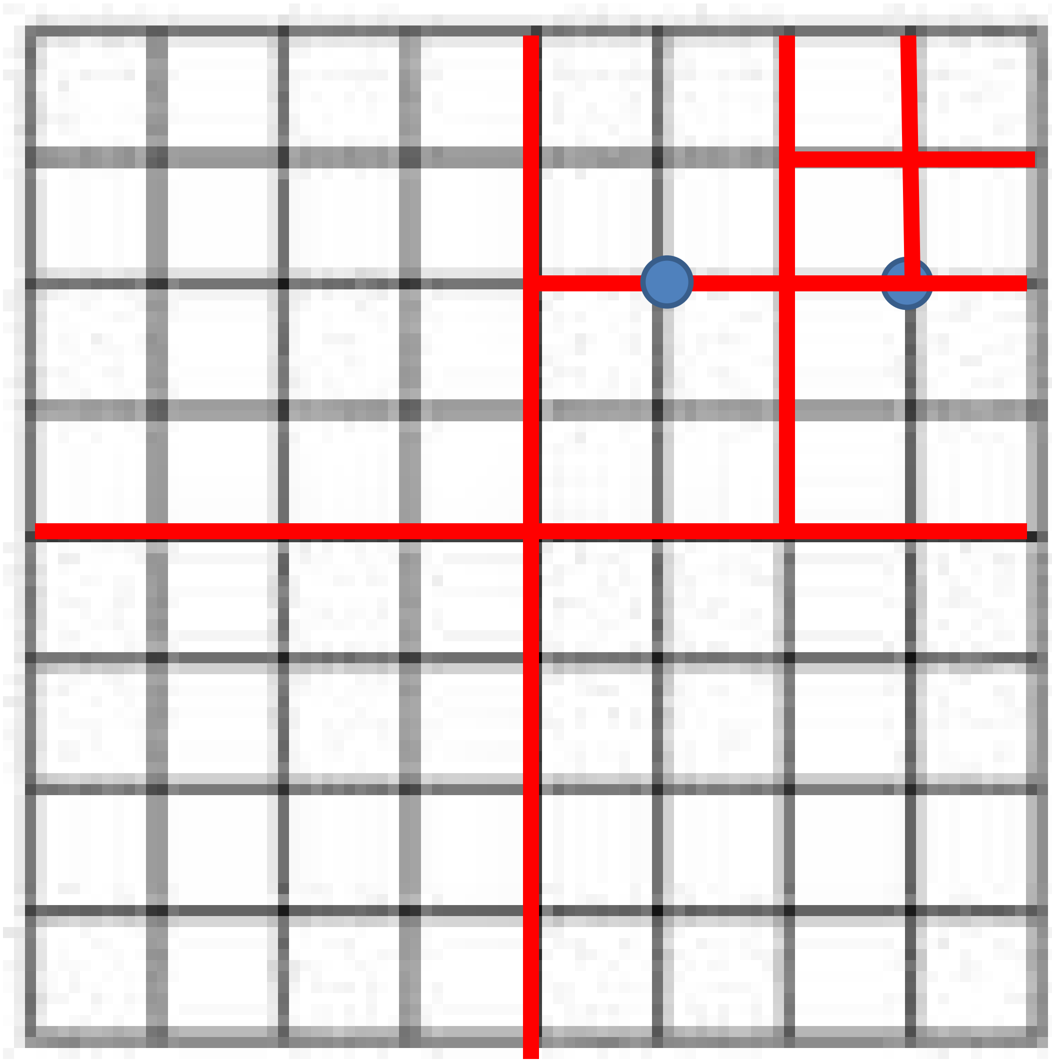


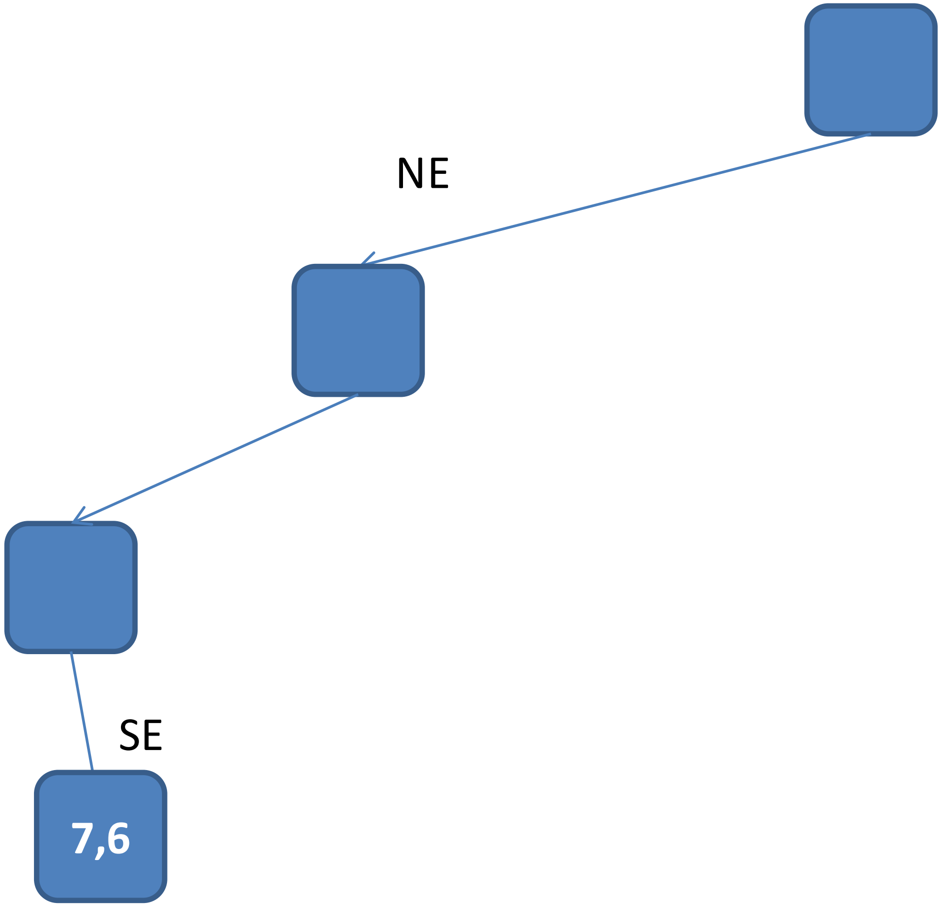
Insert (7,6)



Example: Insert (5,6)

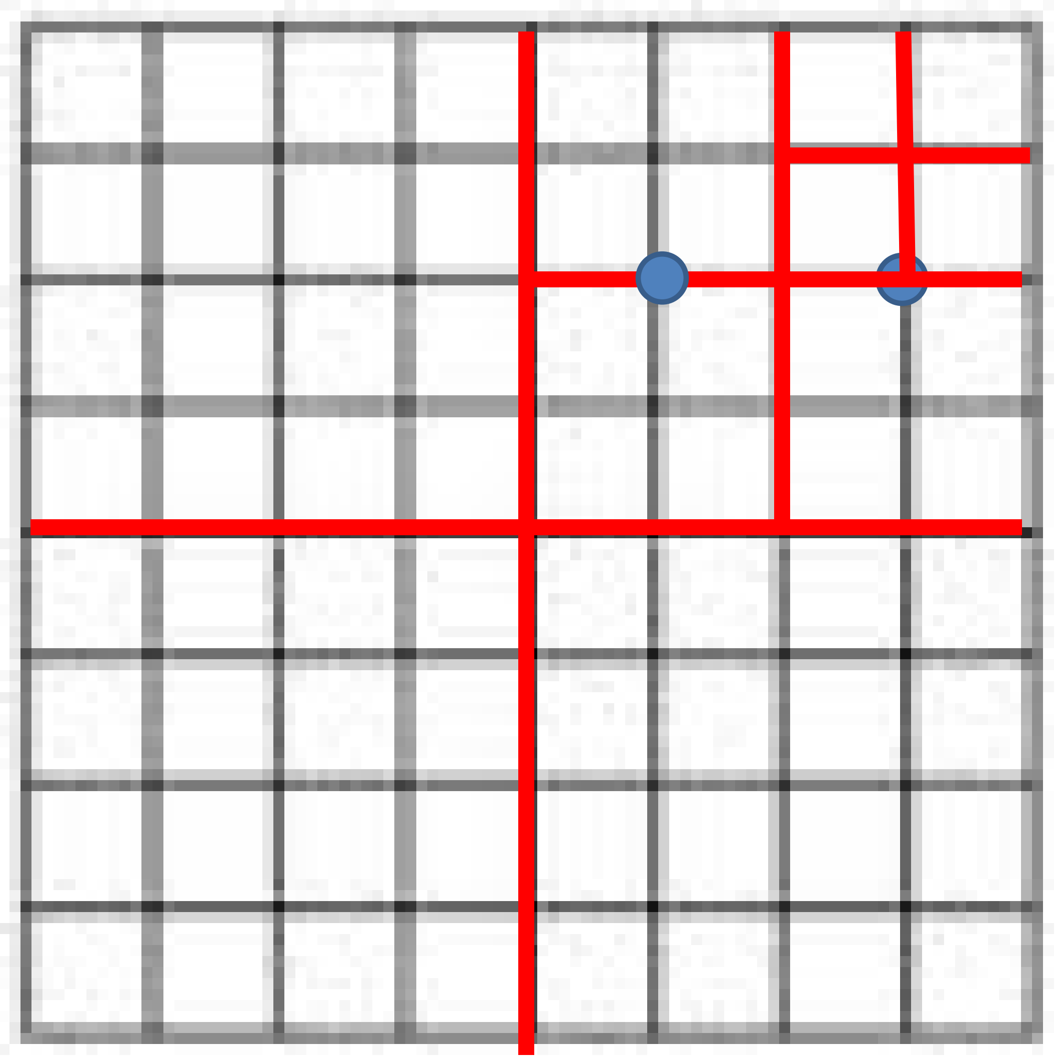


Insert (5,6)

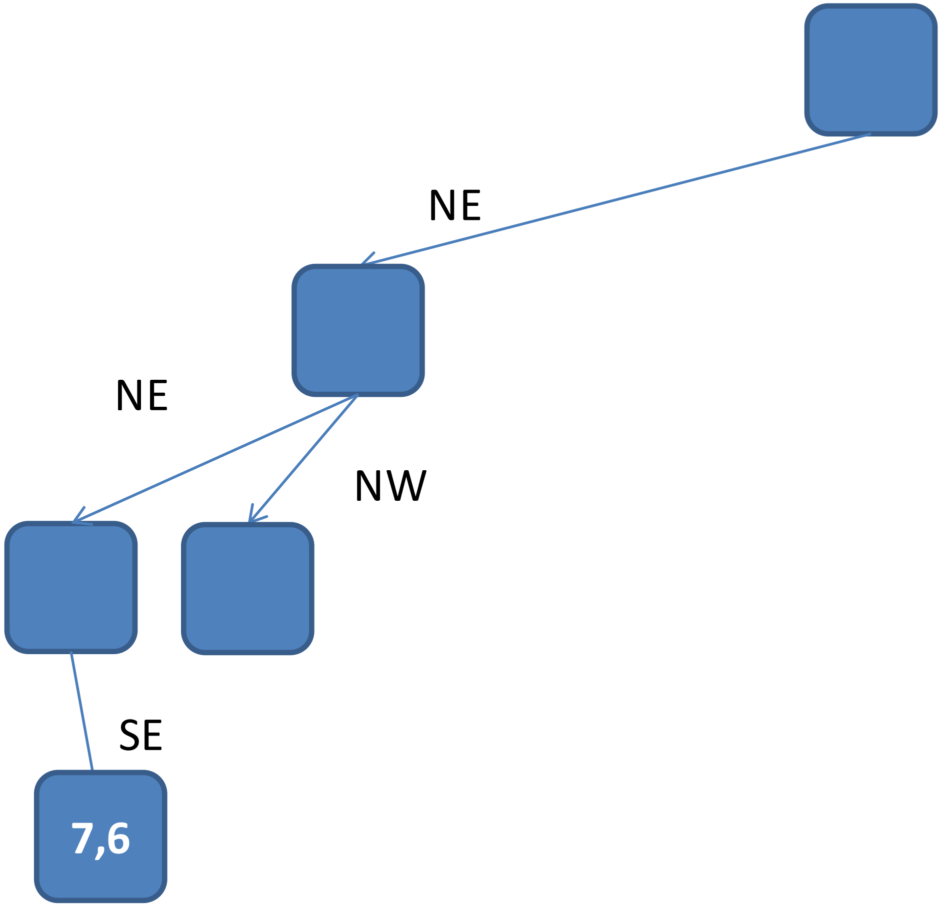


(5,6) is in the NE quadrant of the region associated with the root.

Example: Insert (5,6)

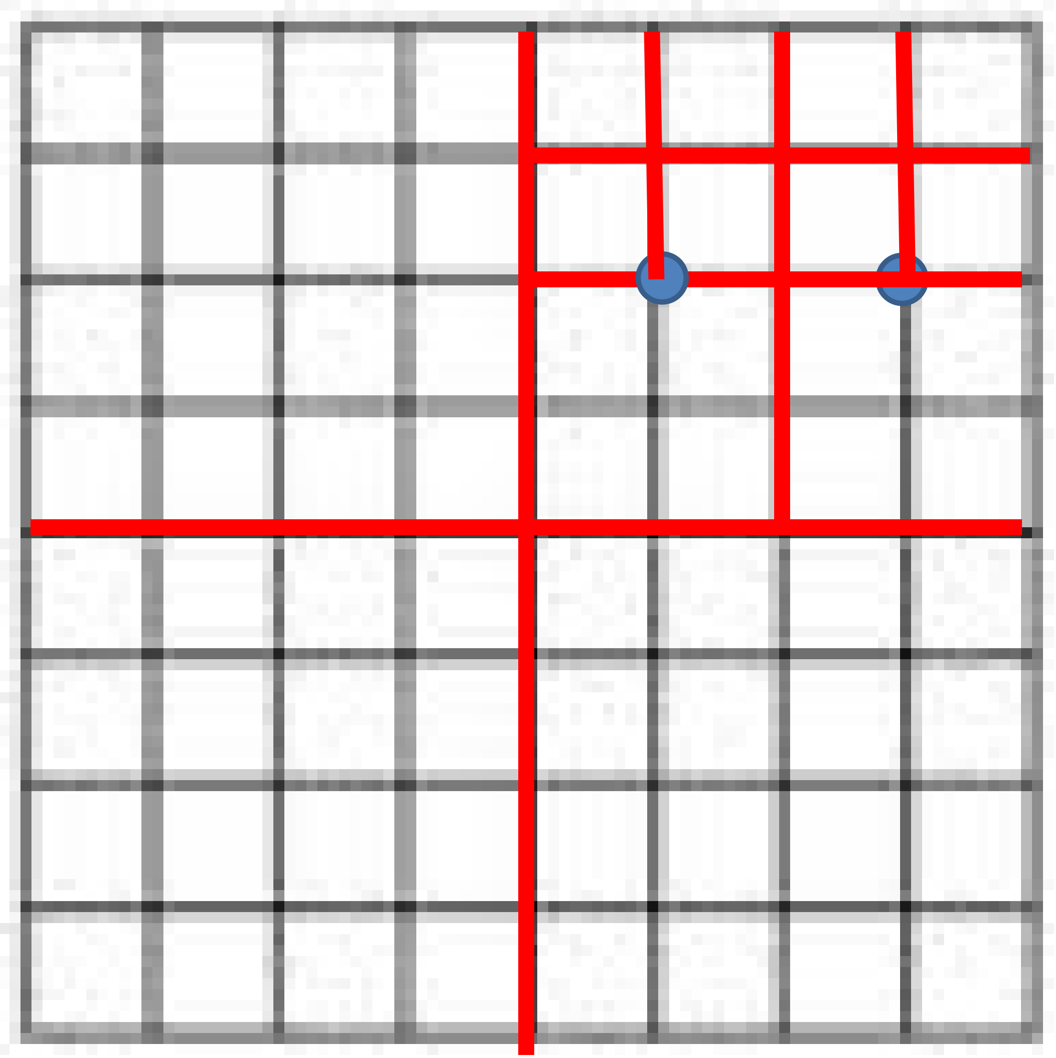


Insert (5,6)

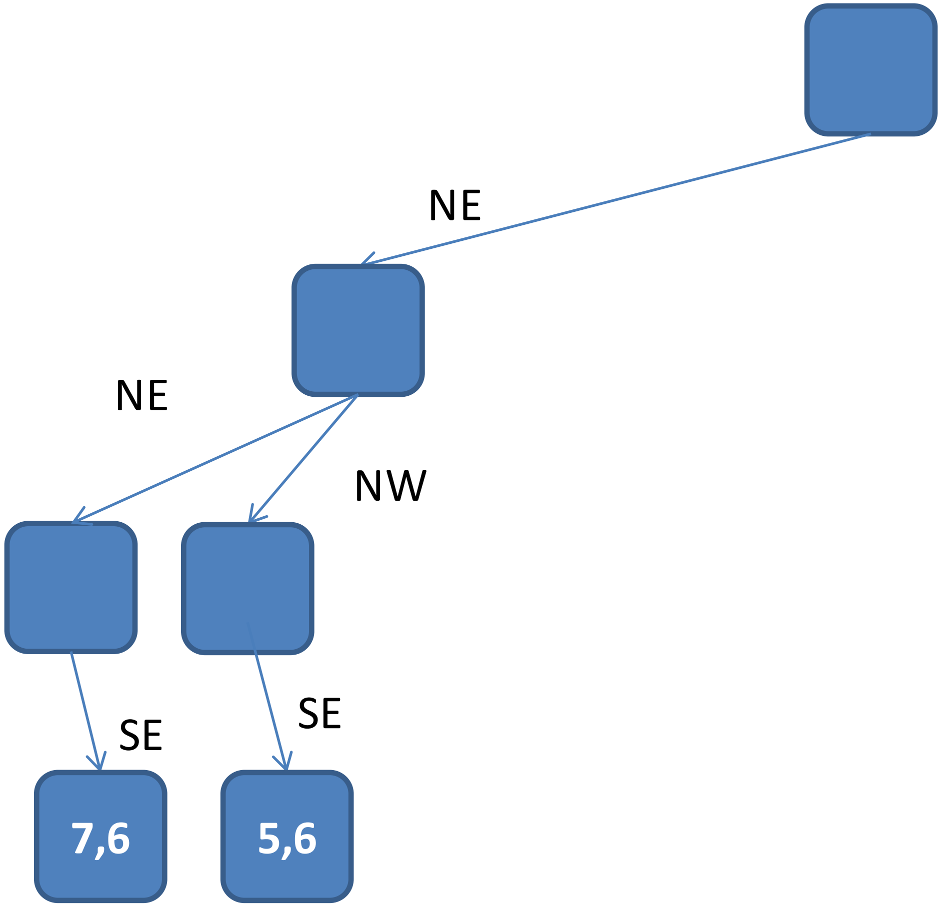


(5,6) is in the NW quadrant of the region associated with the root's NE child.

Example: Insert (5,6)

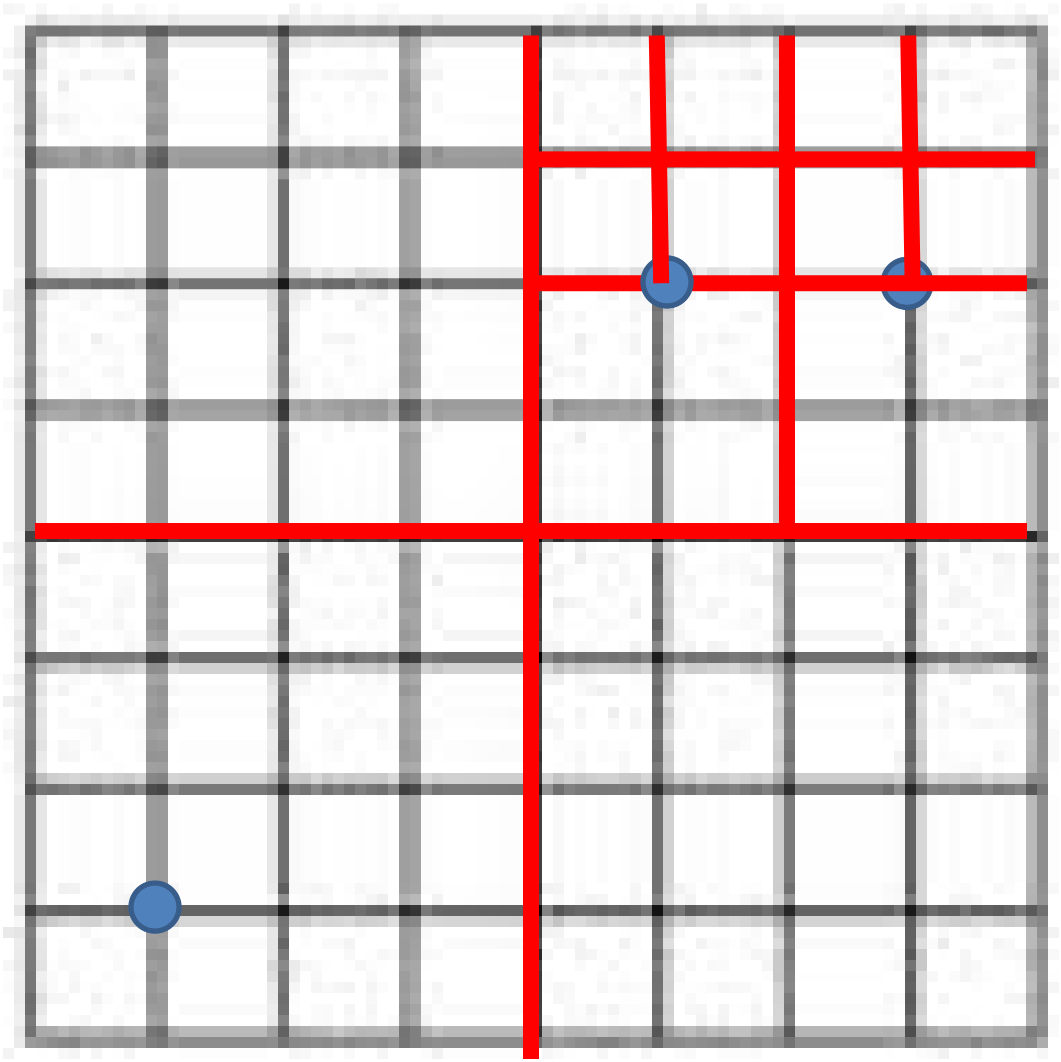


Insert (5,6)

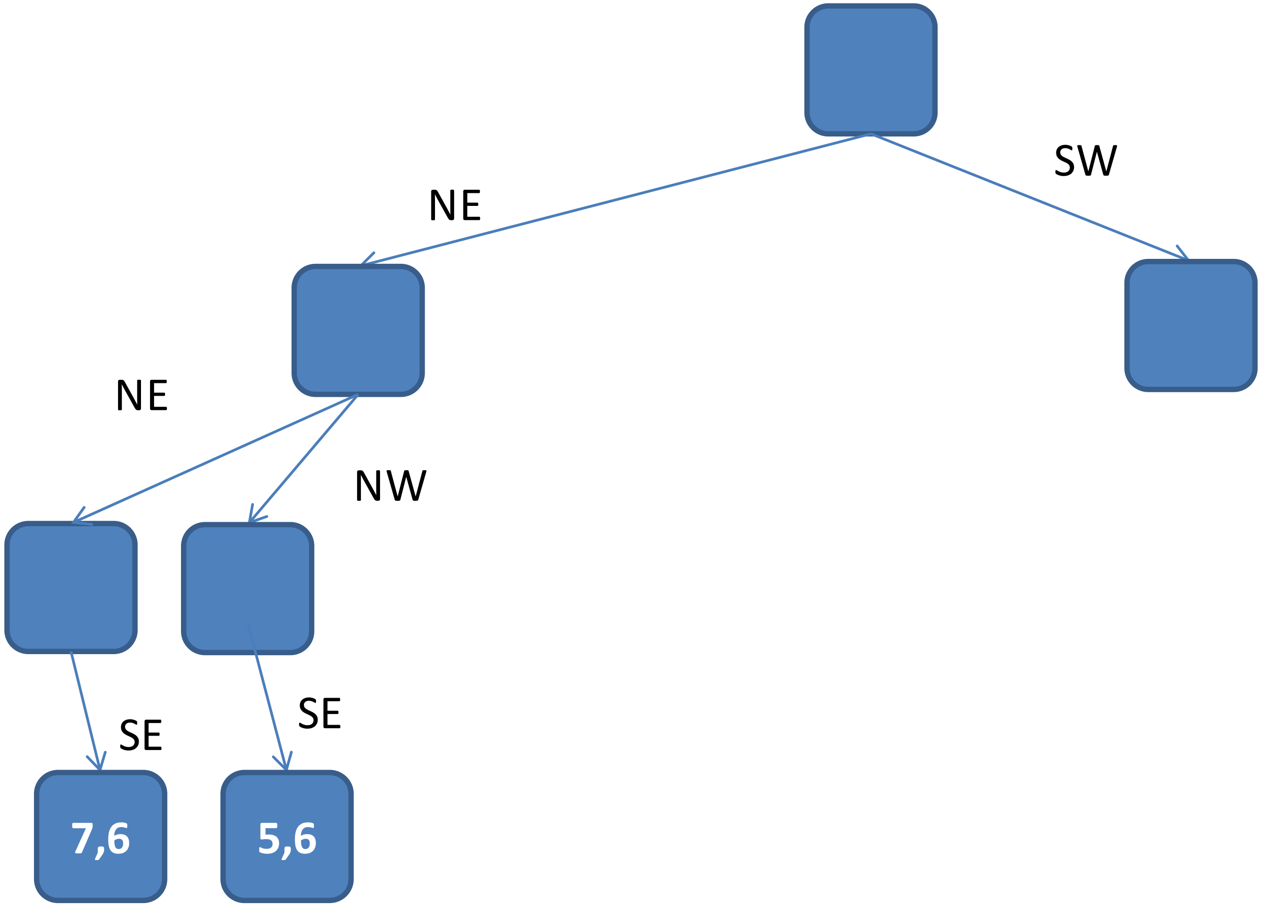


We then branch to the SE

Example: Insert (1,1)

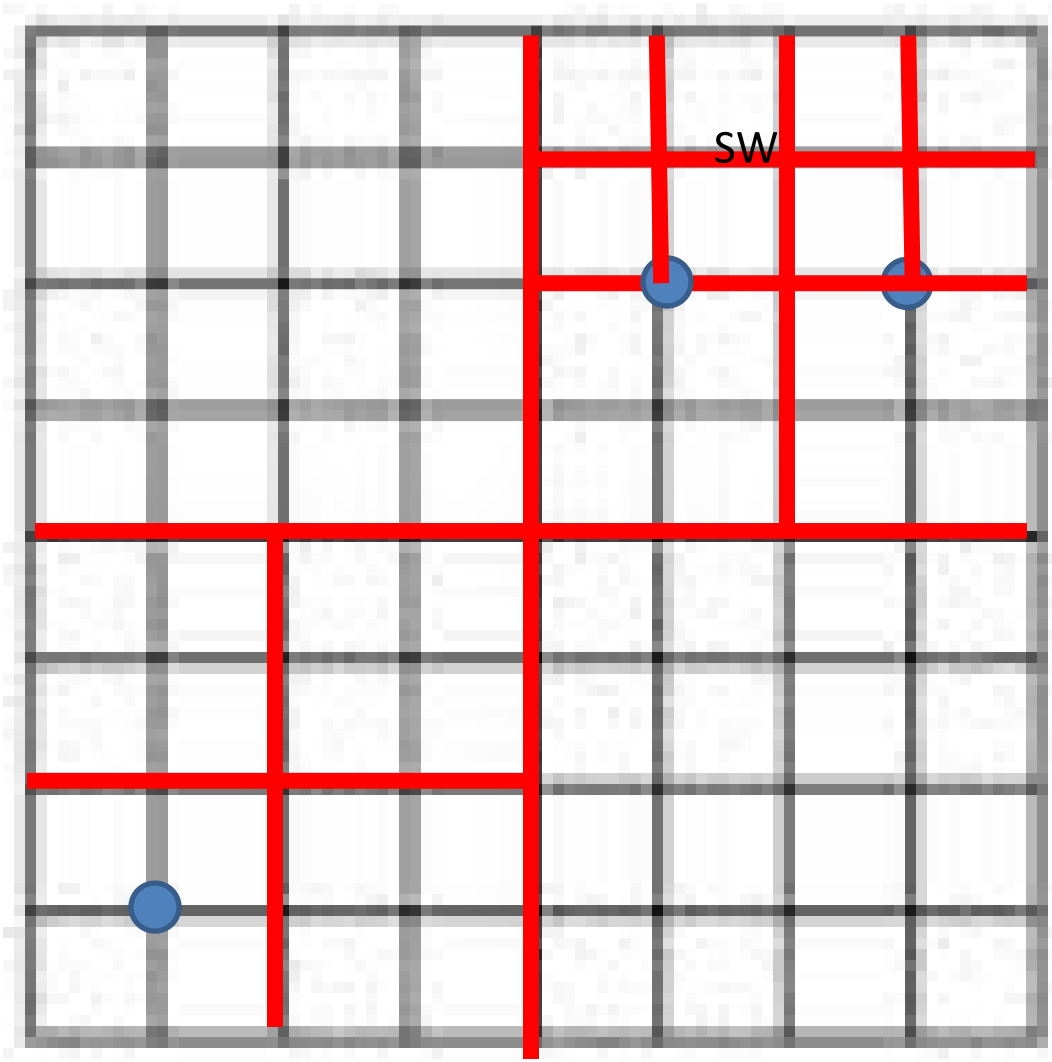


Insert (1,1)

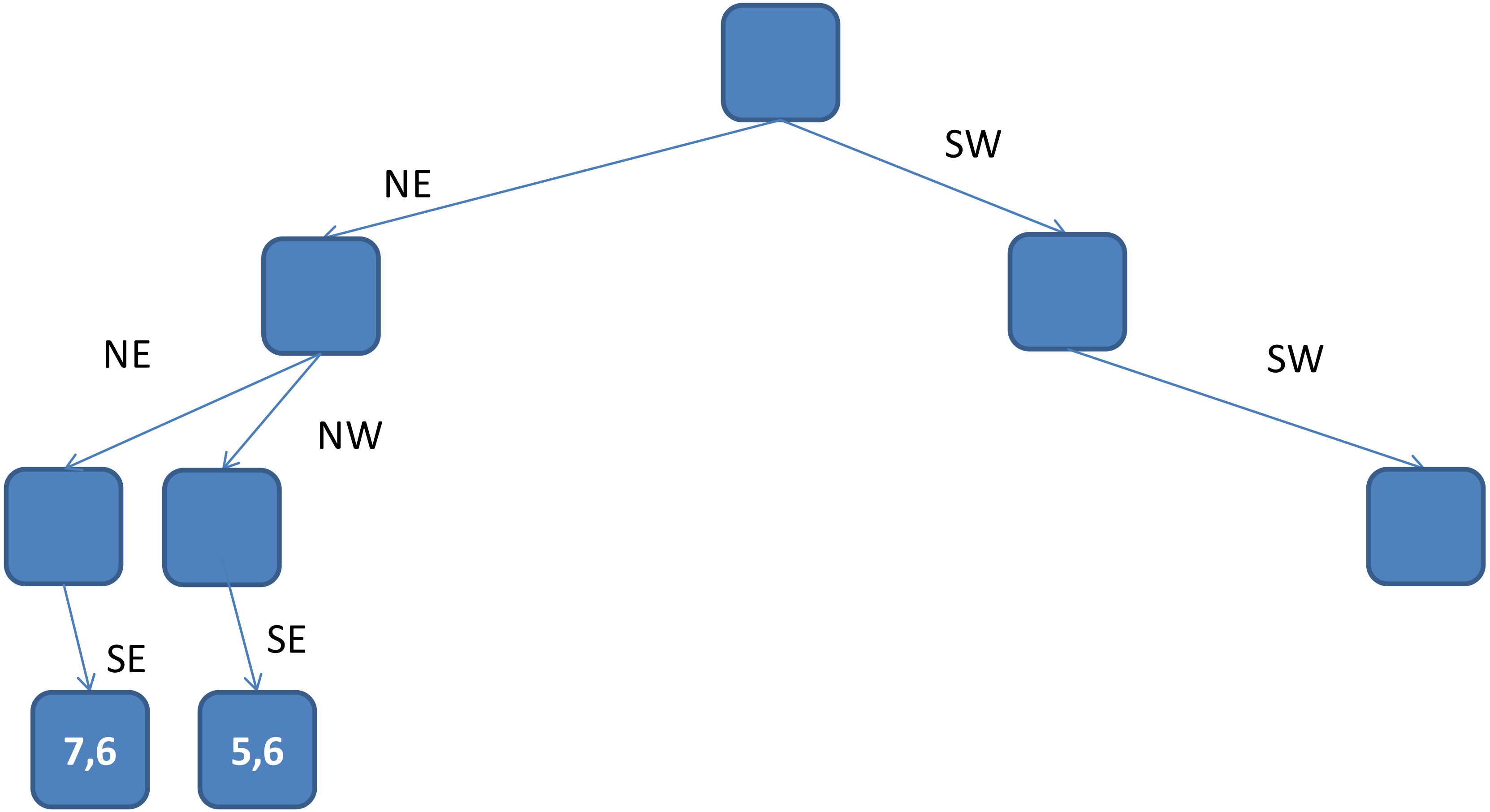


(1,1) is in the SW quadrant of the region associated with the rot.

Example: Insert (1,1)

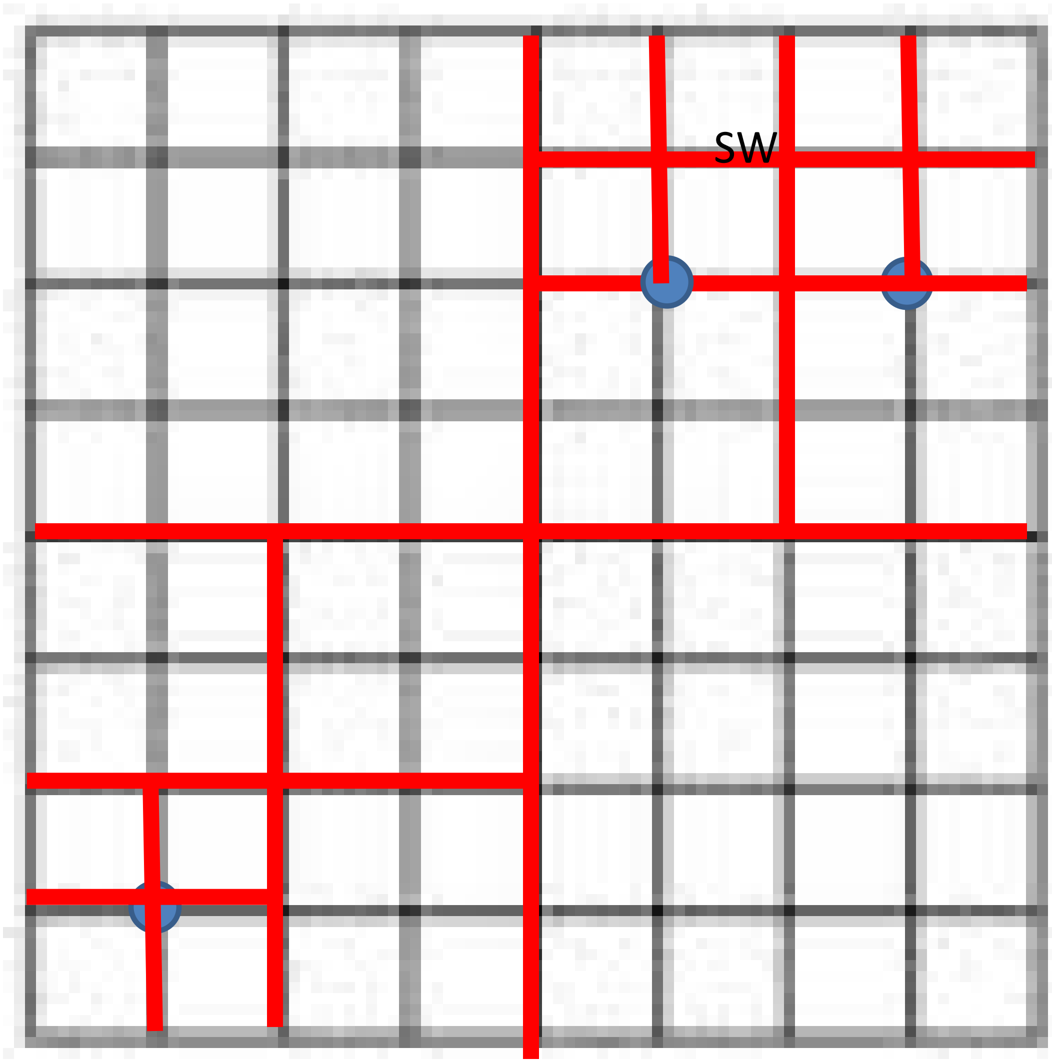


Insert (1,1)

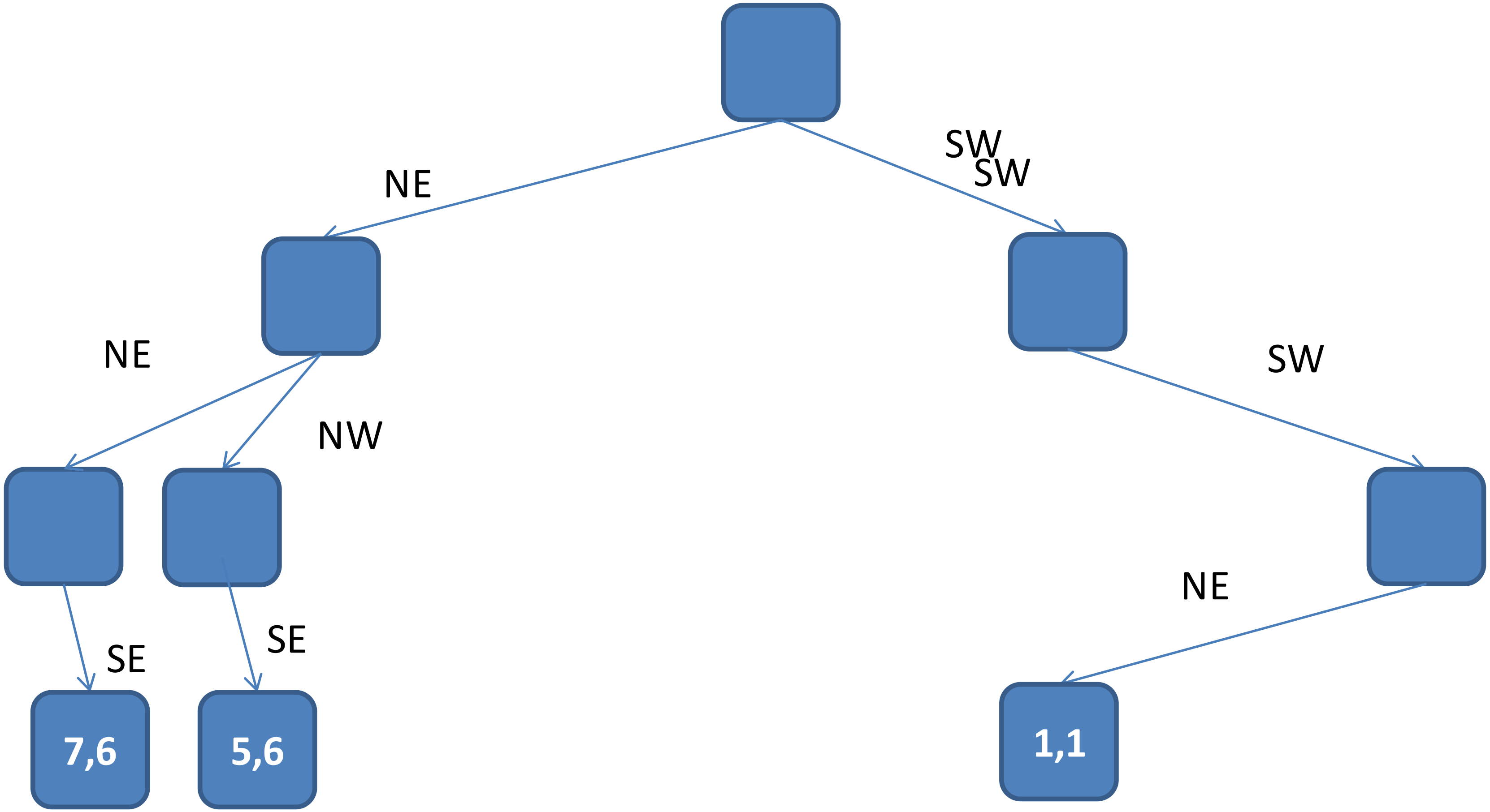


Need to continue branching SW.

Example: Insert (1,1)



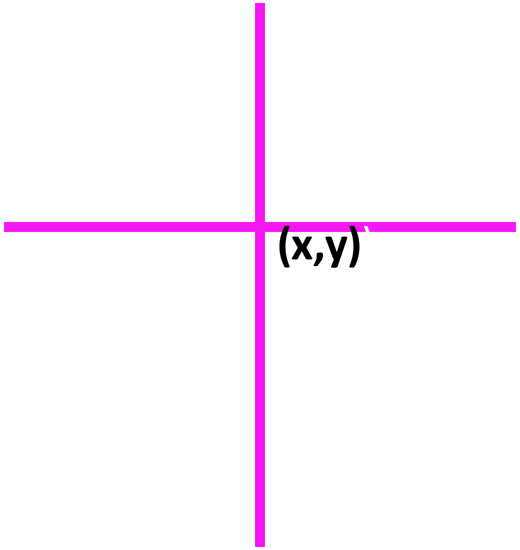
Insert (1,1)



Now branch NE.

Branching Convention

- If you are inserting point (x',y') and you are at a node labeled (x,y), branch to
  - NE if $x' \geq x$ & $y' \geq y$.
  - NW if $x' < x$ & $y' \geq y$.
  - SE if $x' \geq x$ & $y' < y$
  - SW if $x' < x$ & $y' < y$.



Intuitively, quadrants are closed on the left and bottom, open on the right and top.

In-class Exercise

- Insert (1,3)
- Insert (6,7)
- Insert (3,1)

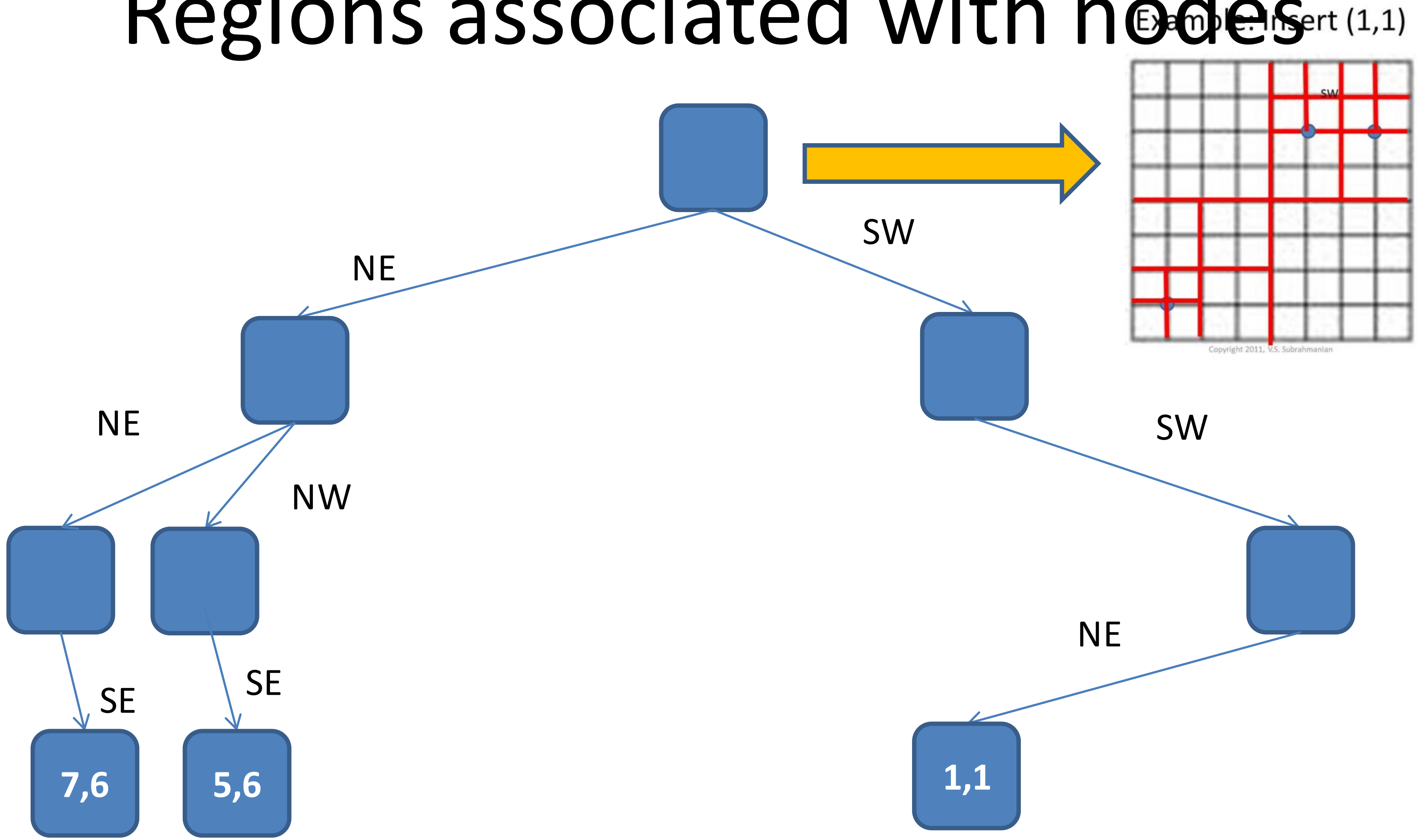
Points for Discussion

- Is the “shape” of an MX-quadtree affected by the order in which nodes are inserted?
- What is the worst-case complexity of searches for a given point in an MX-quadtree?

Regions associated with nodes

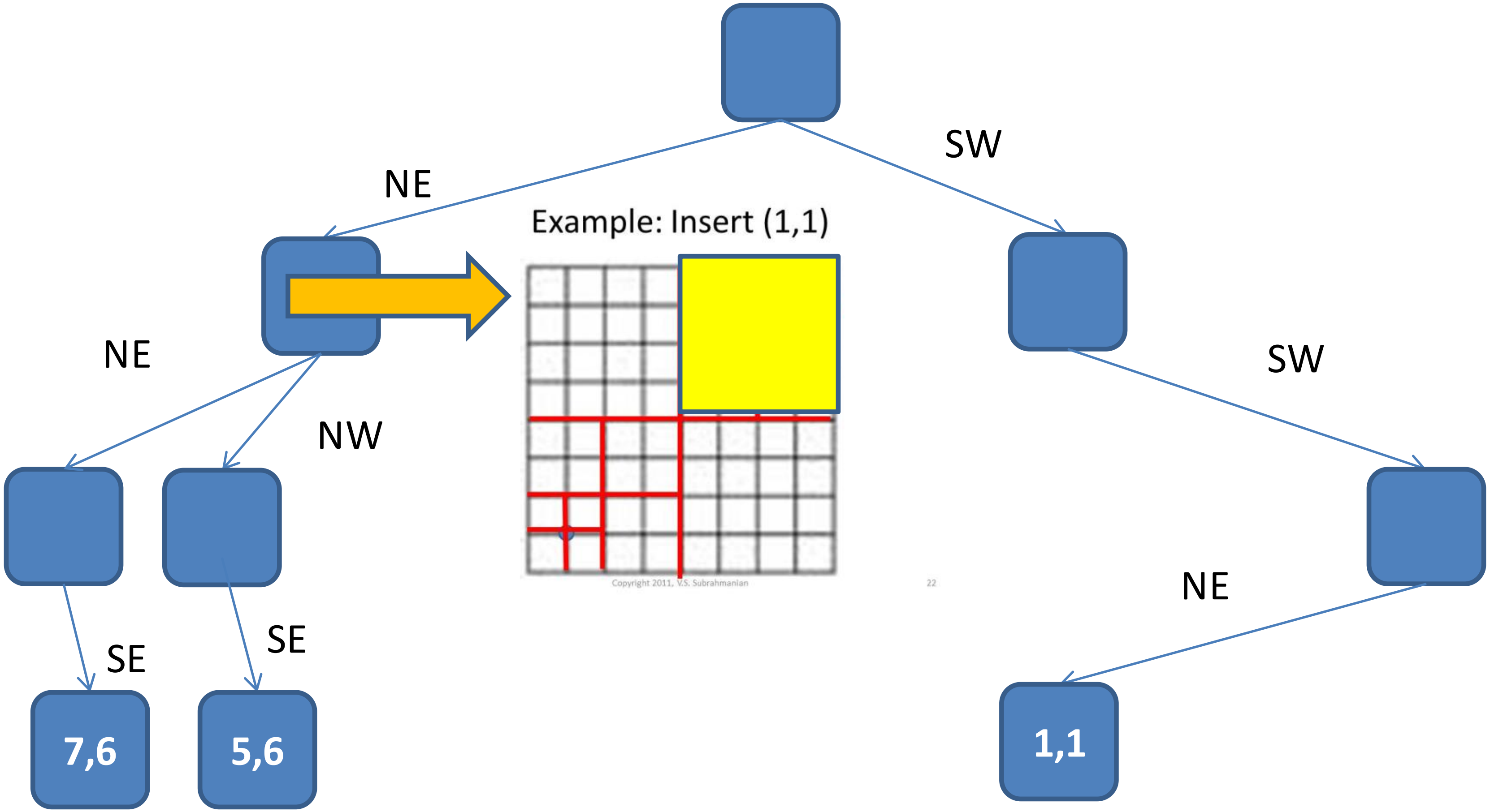
- Every node N in the MX-Quadtree is implicitly associated with a region, Reg(N).
- The root represents the entire 2^n x 2^n region.
- Regions are <u>always</u> split by drawing a vertical line and a horizontal line through the center point of a region.

Regions associated with nodes



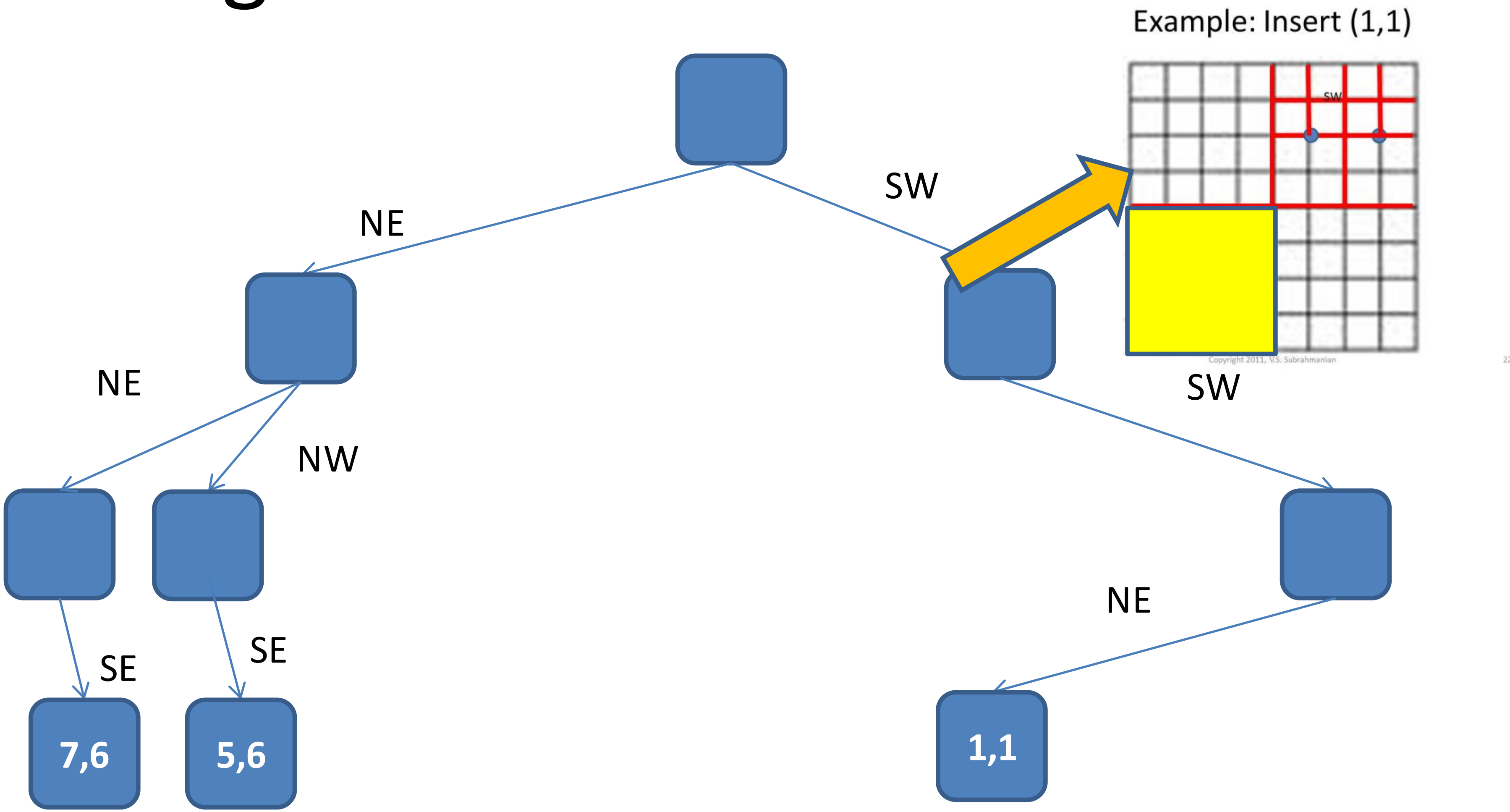
Now branch NE.

Regions associated with nodes



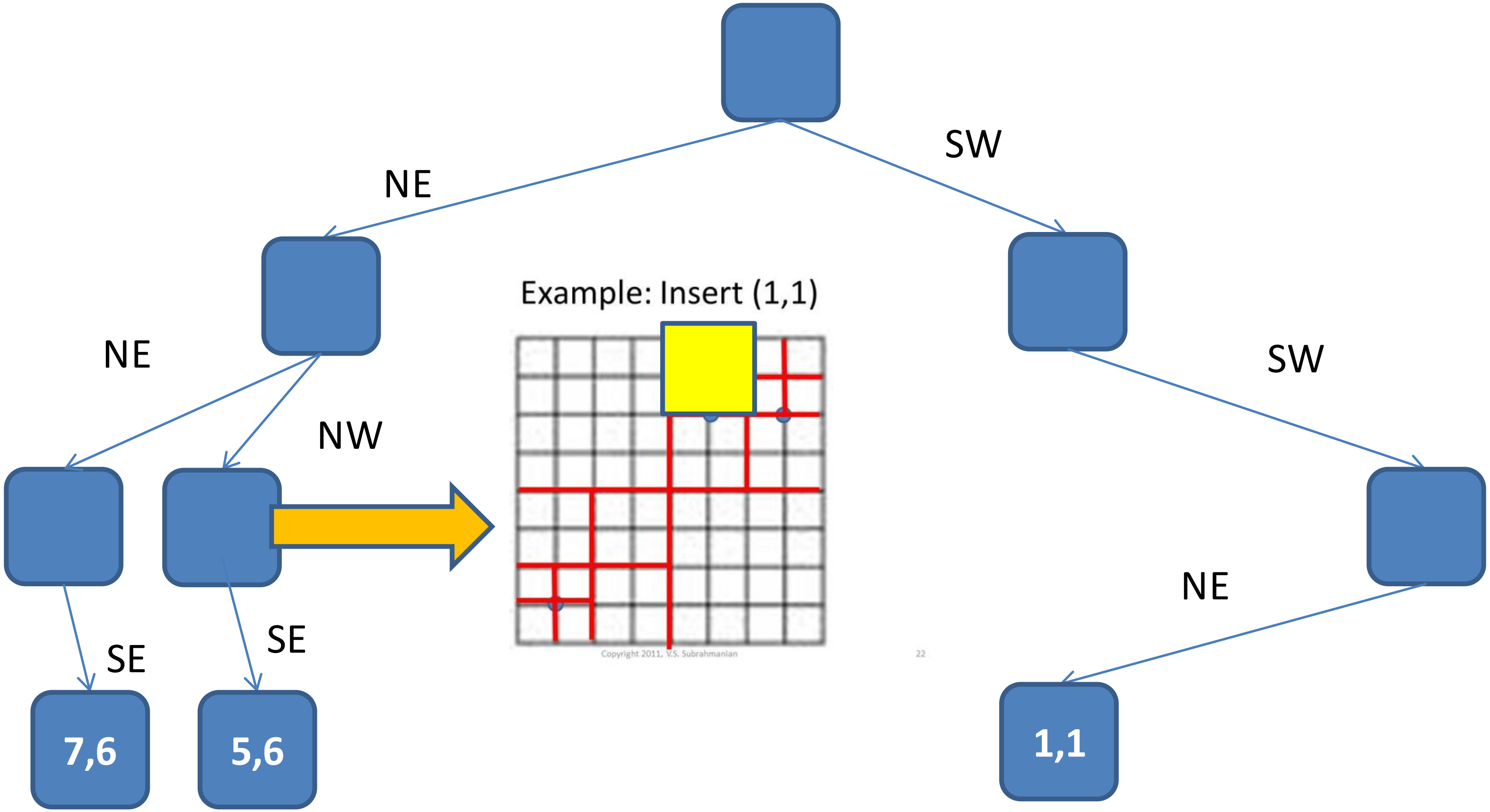
Now branch NE.

Regions associated with nodes



Now branch NE.

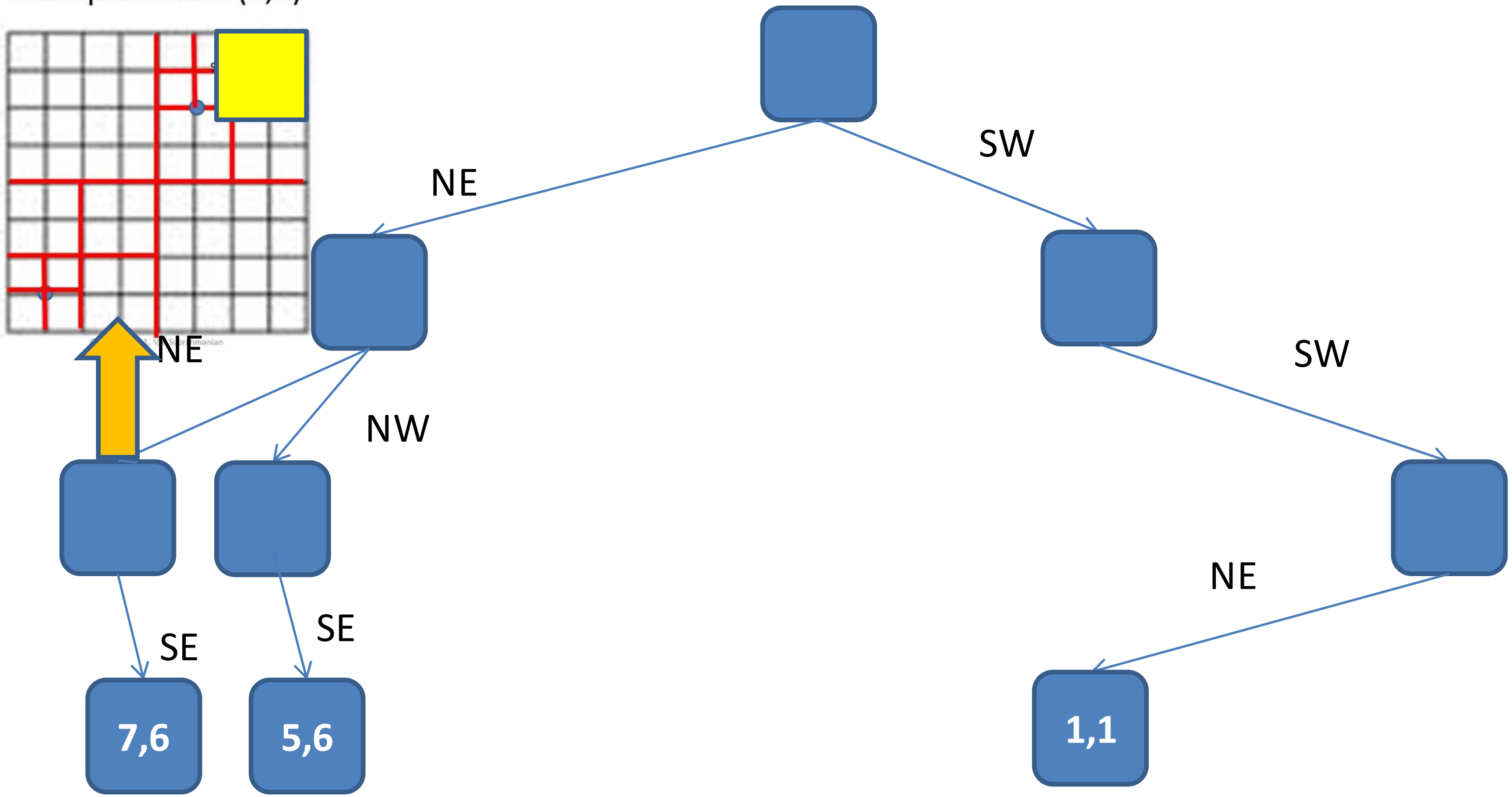
Regions associated with nodes



Now branch NE.

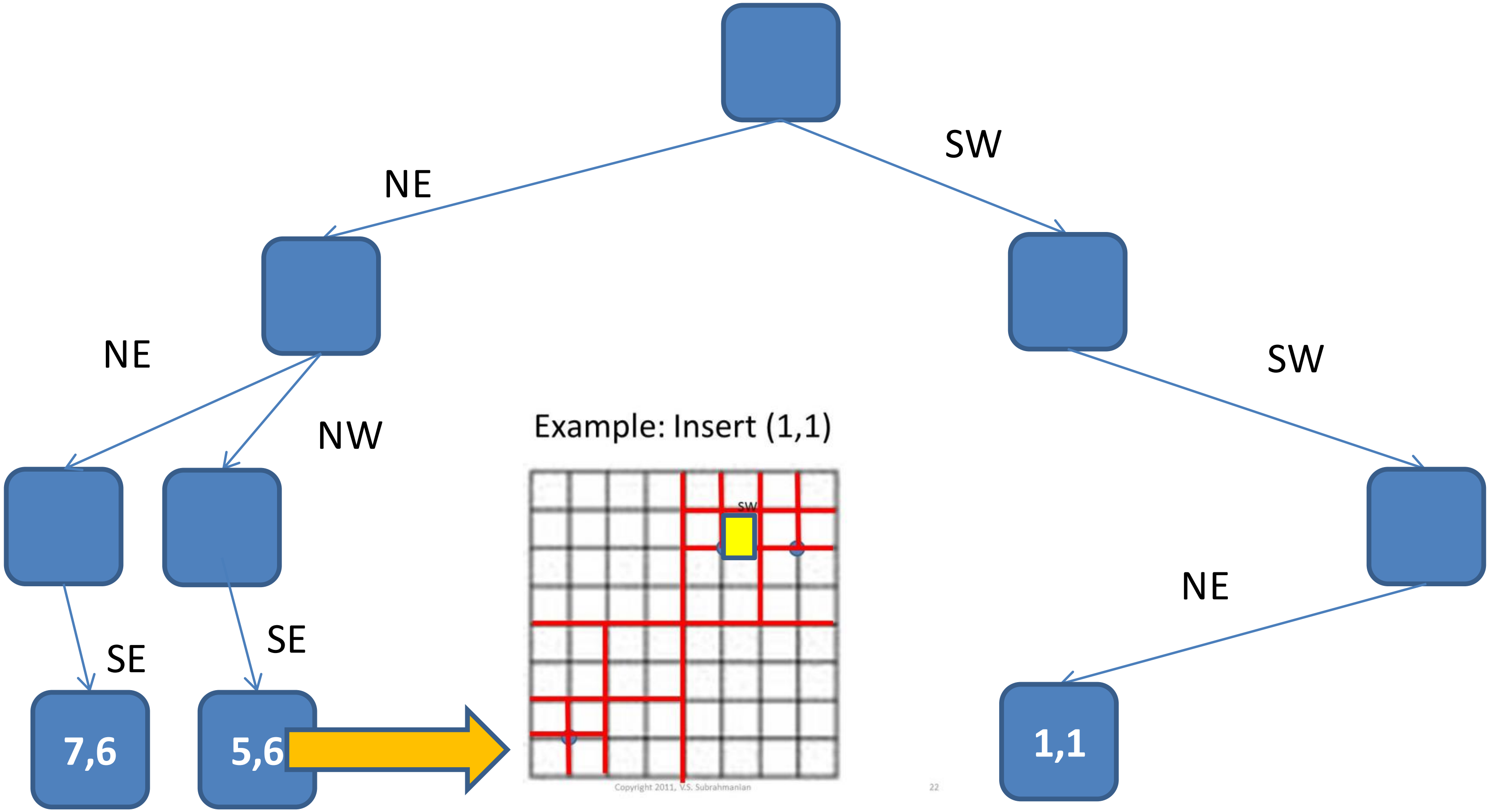
Regions associated with nodes

Example: Insert (1,1)



Now branch NE.

Regions associated with nodes



Now branch NE.

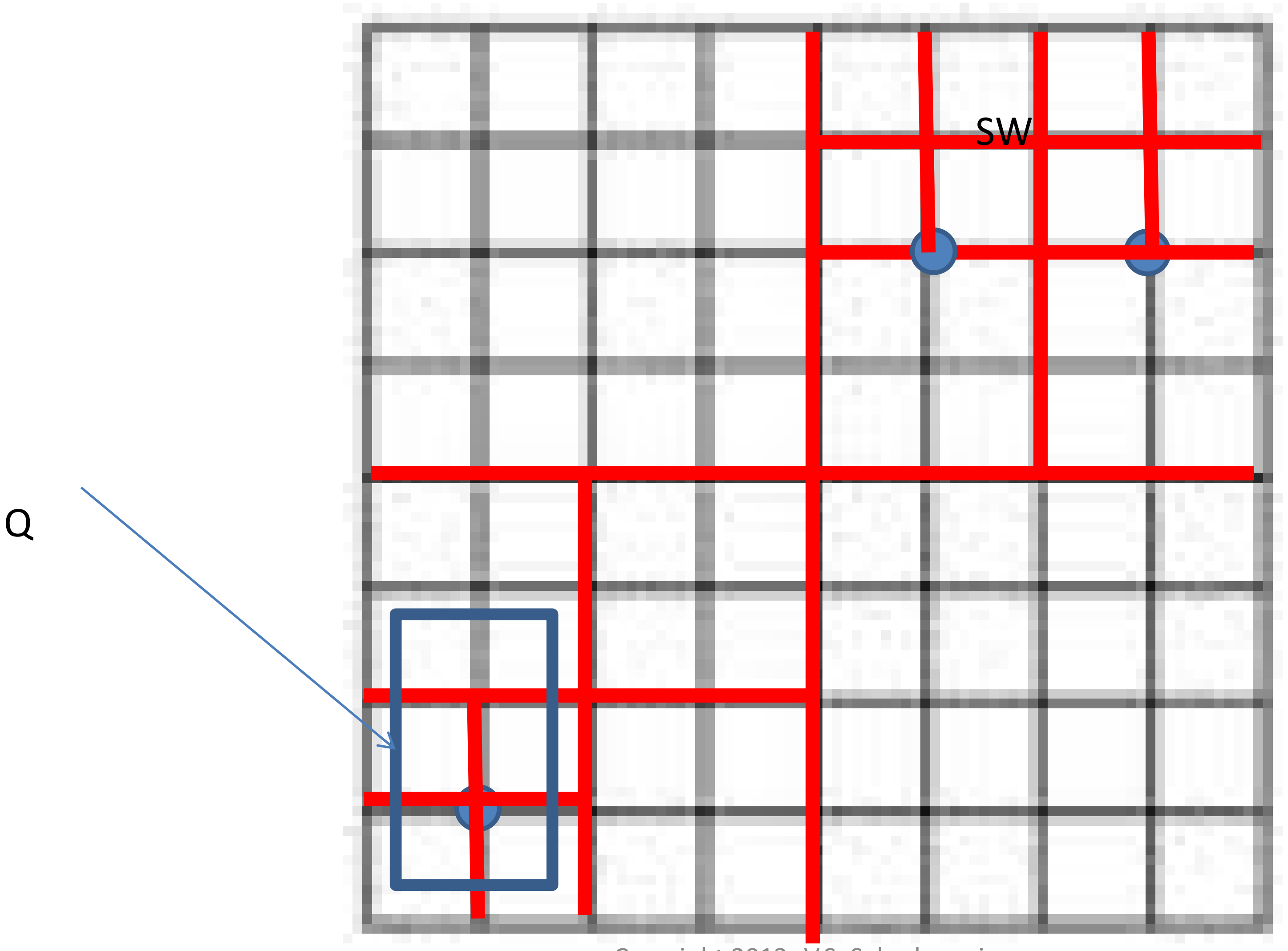
Range Searches

- INPUTS:
  - pointer T to the root of an MX-Quadtree
  - Region Q.
- Need to find all points in T that are in region Q

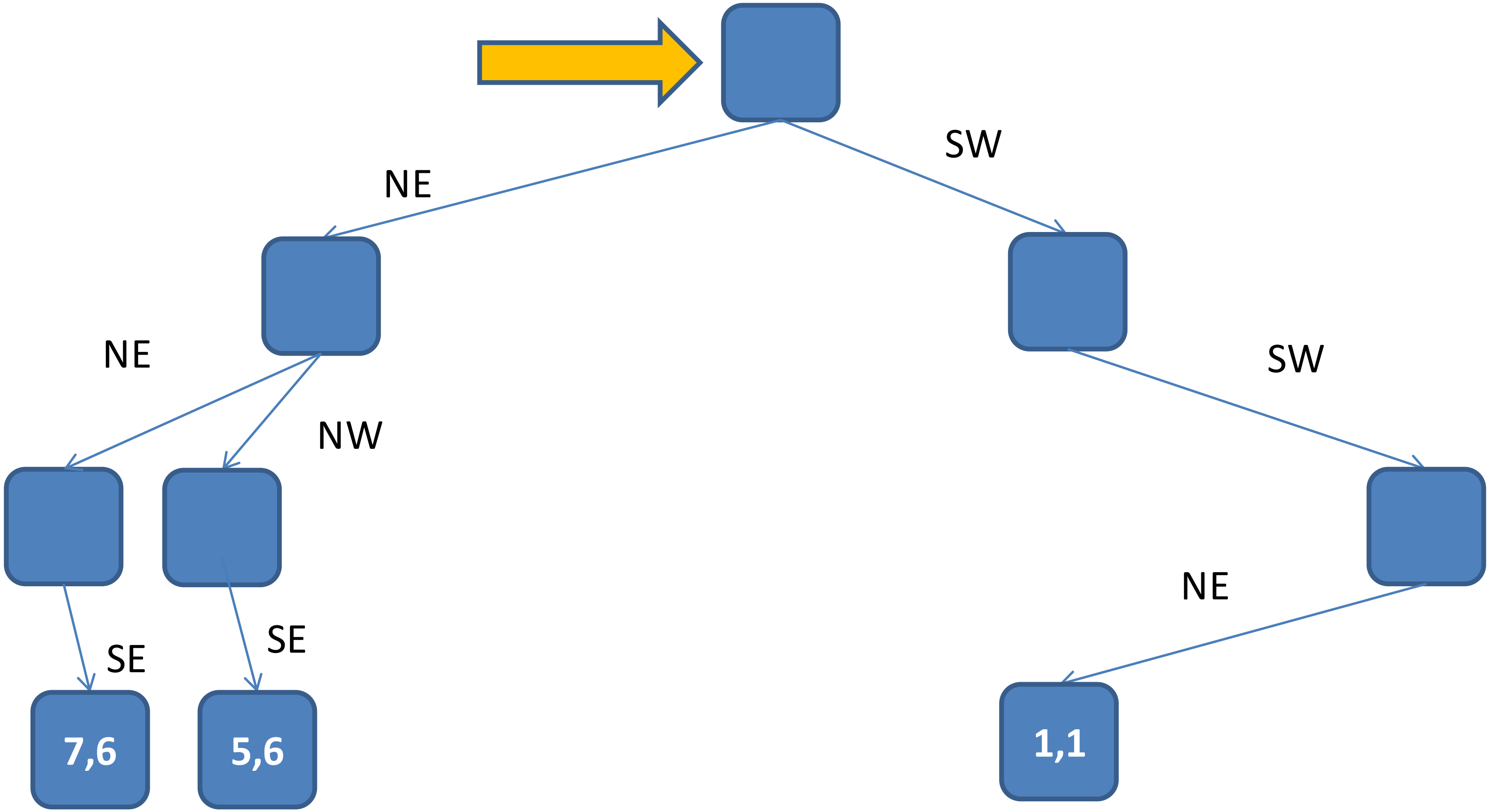
Visit(N)

- If N is a non-leaf node
  - If Reg(N) intersects Q, then recursively visit all of N’s children.
  - Otherwise PRUNE !
- Else (N is a leaf) check if N.Point in Q. If yes, insert N.Point into SOL.

Example Range Search

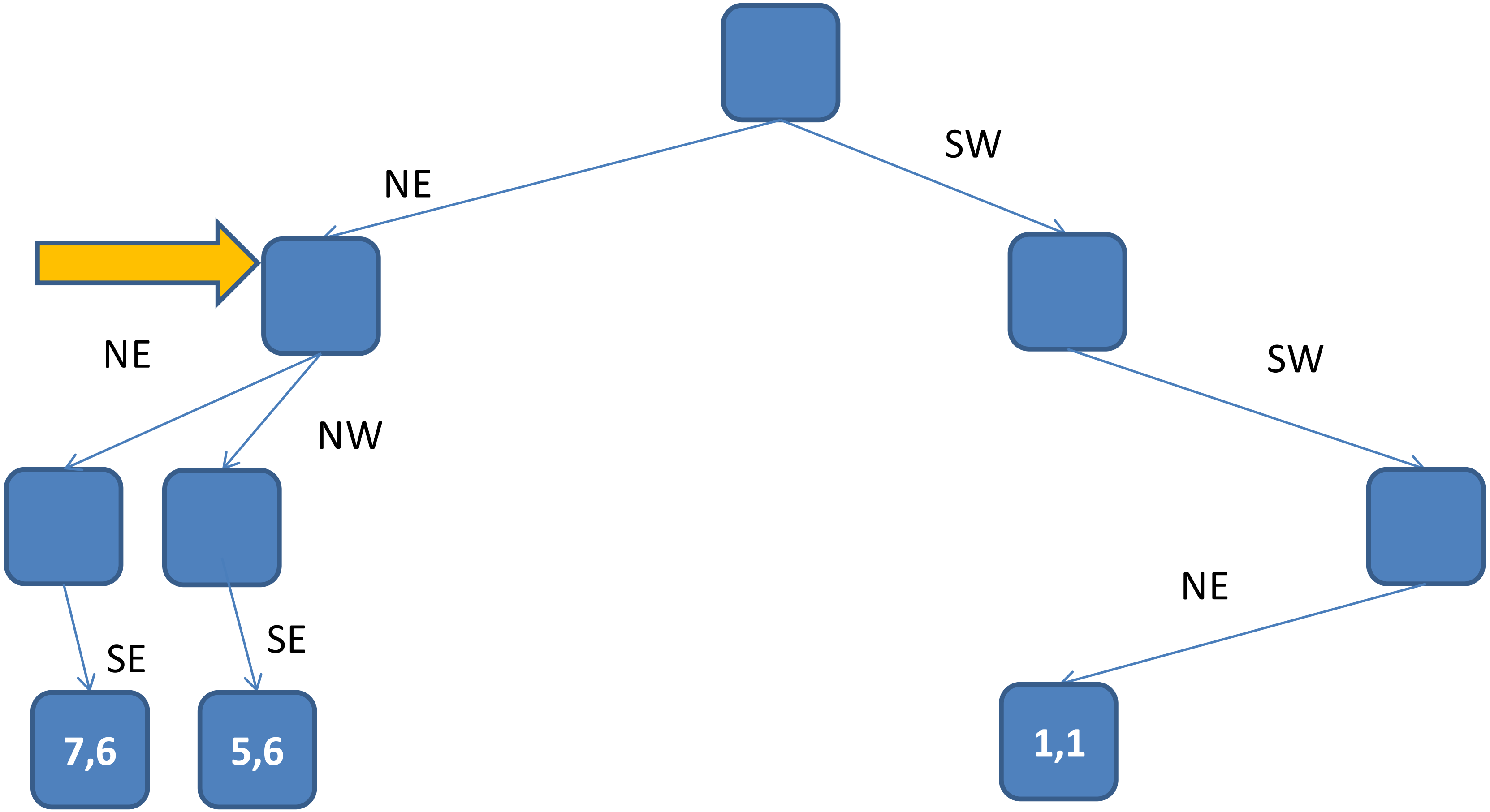


Example range search



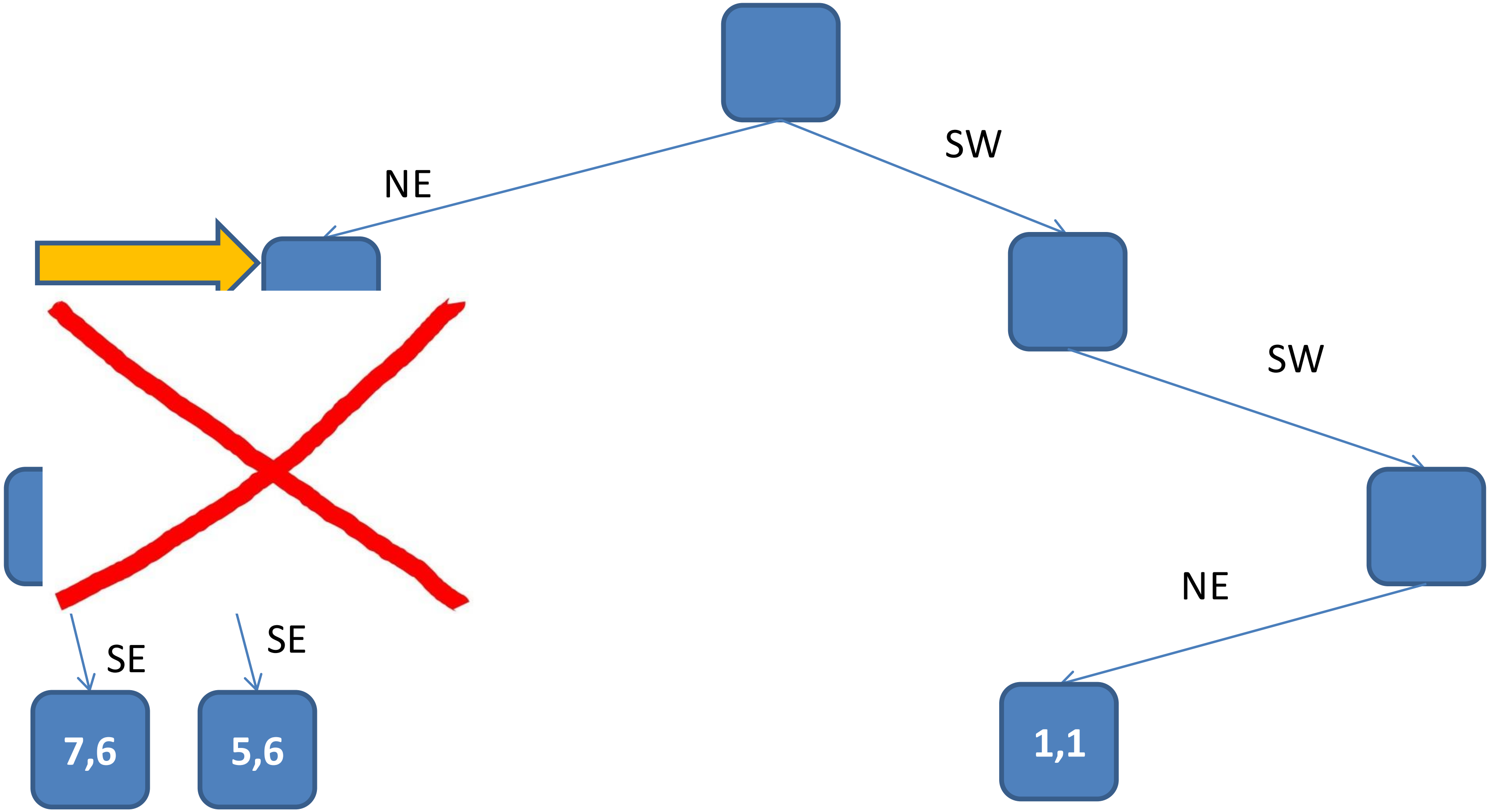
Does Q intersect the region of the root? Yes. So must search both children.

Example range search



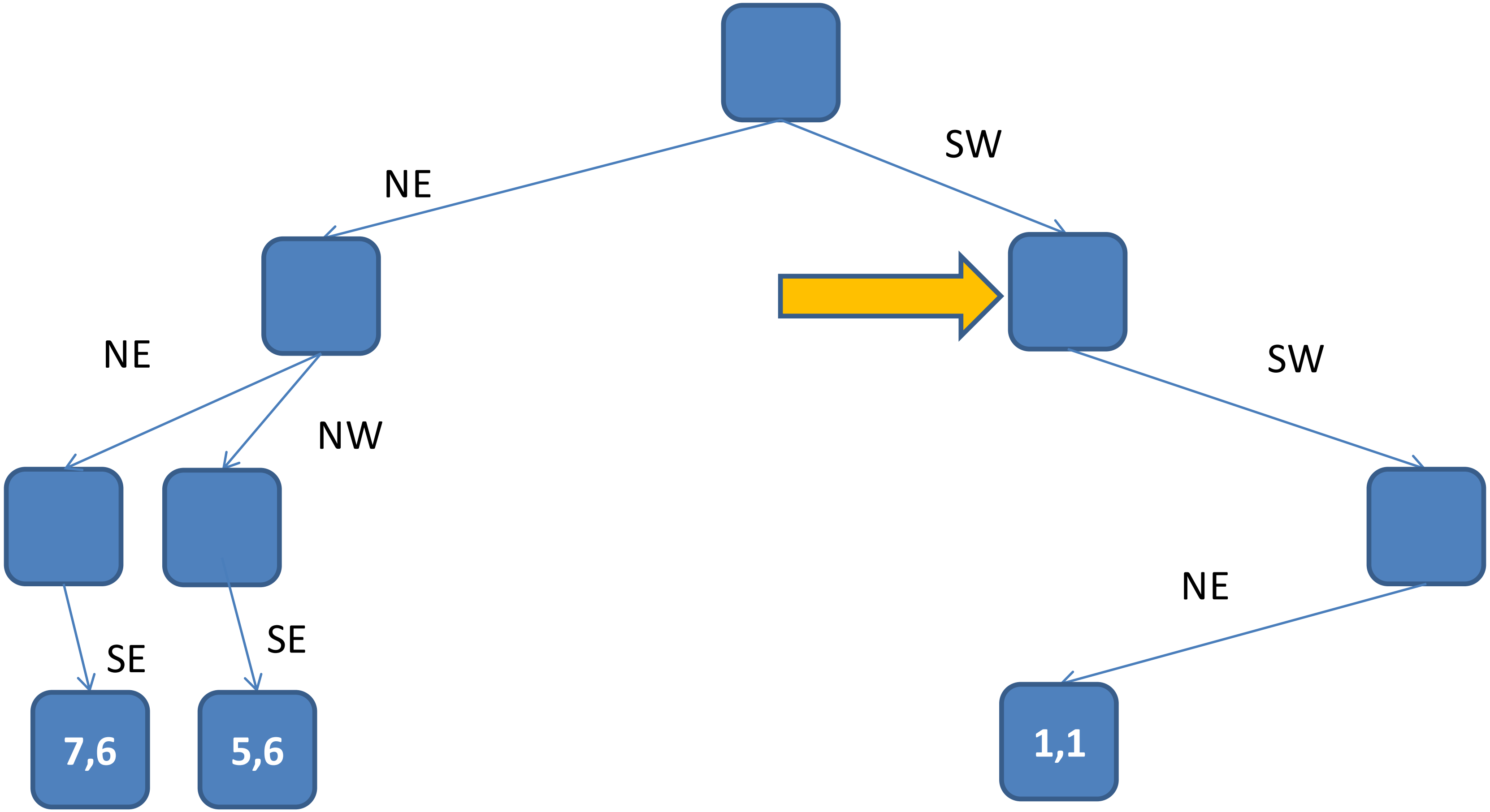
Does Q intersect the region of the root's NE child? No. ***PRUNE !***

Example range search



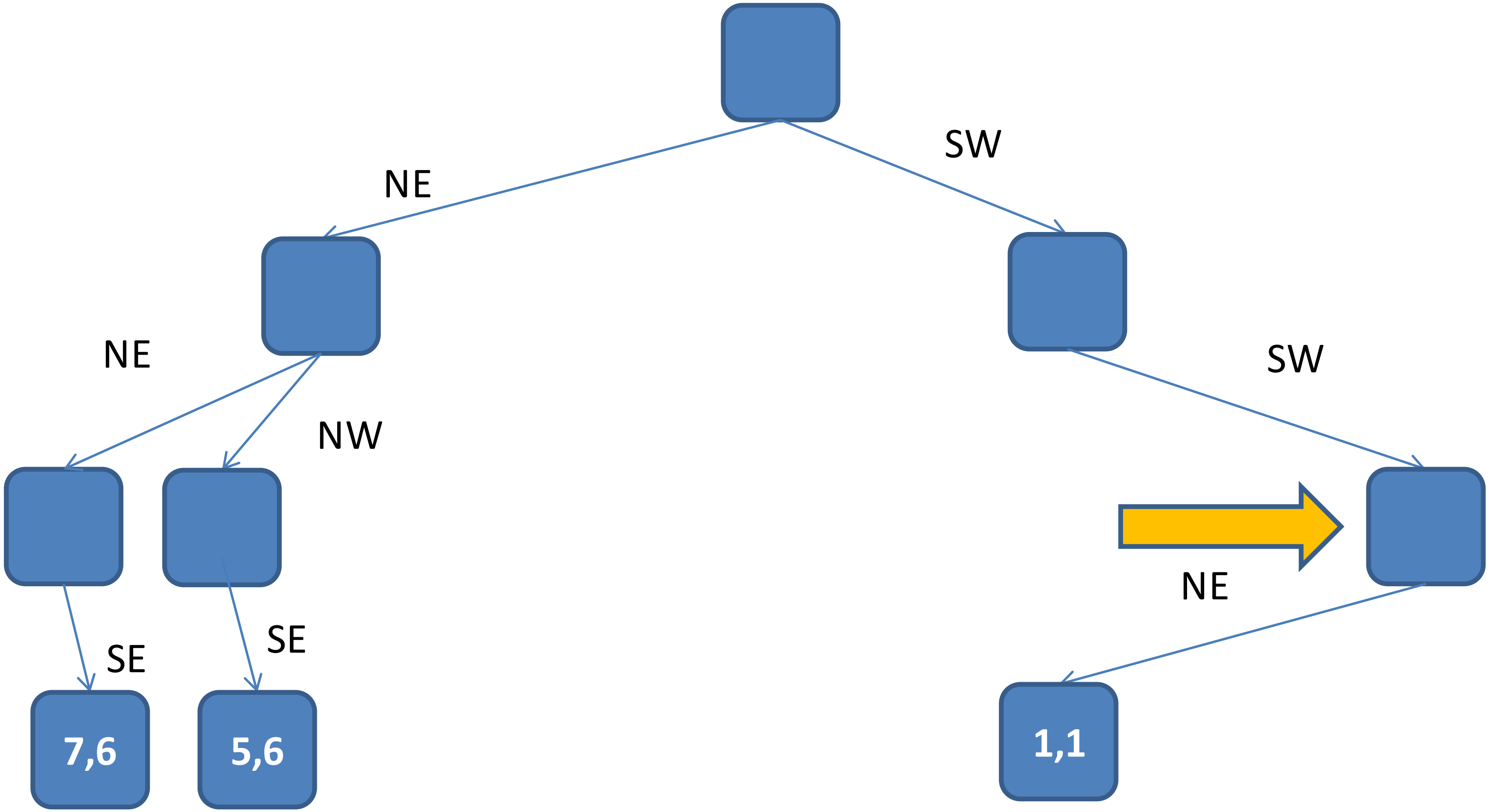
Does Q intersect the region of the root's NE child? No. ***PRUNE !***

Example range search



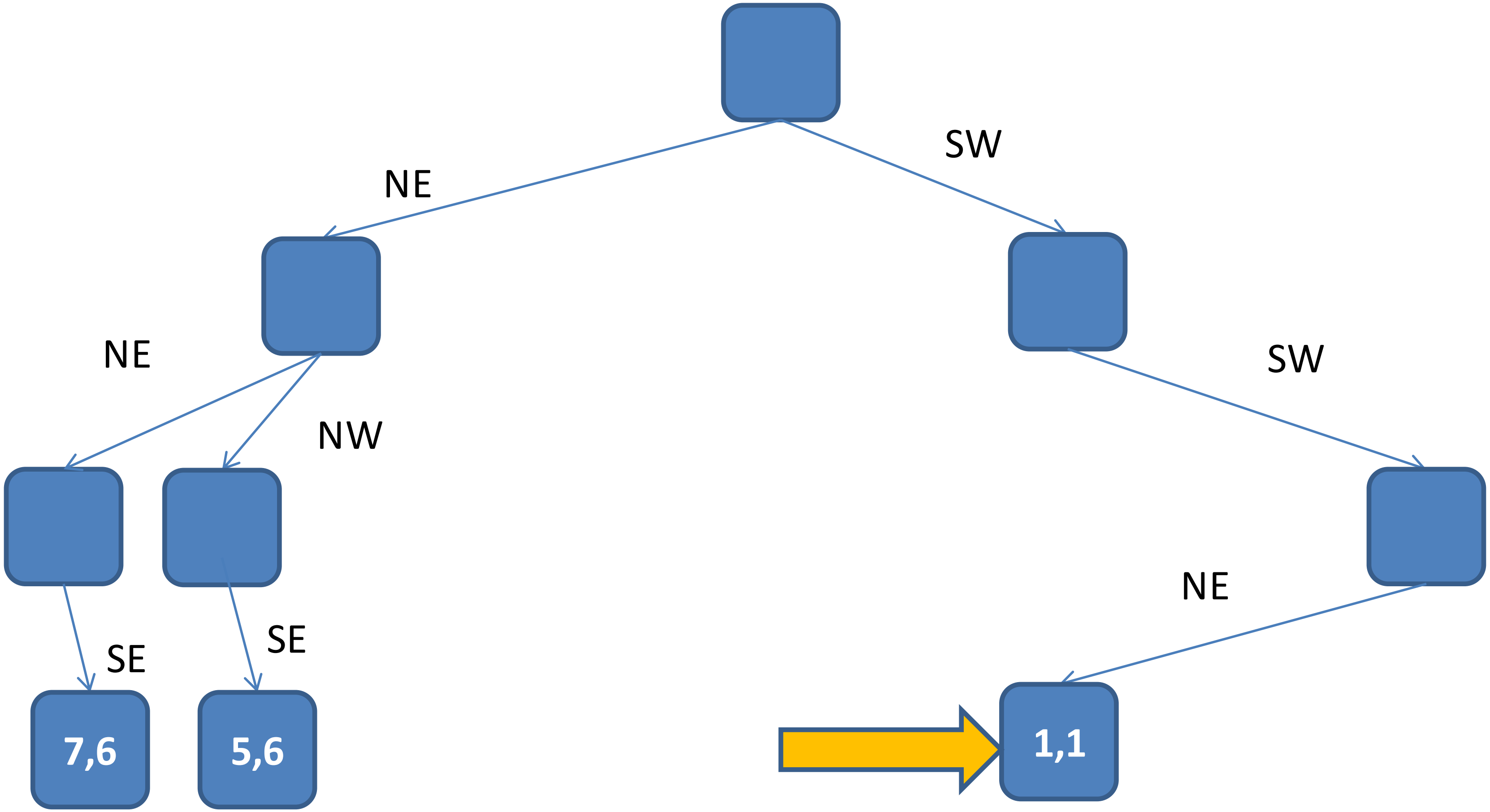
Does Q intersect the region of the root's SW child? Yes, so search children.

Example range search



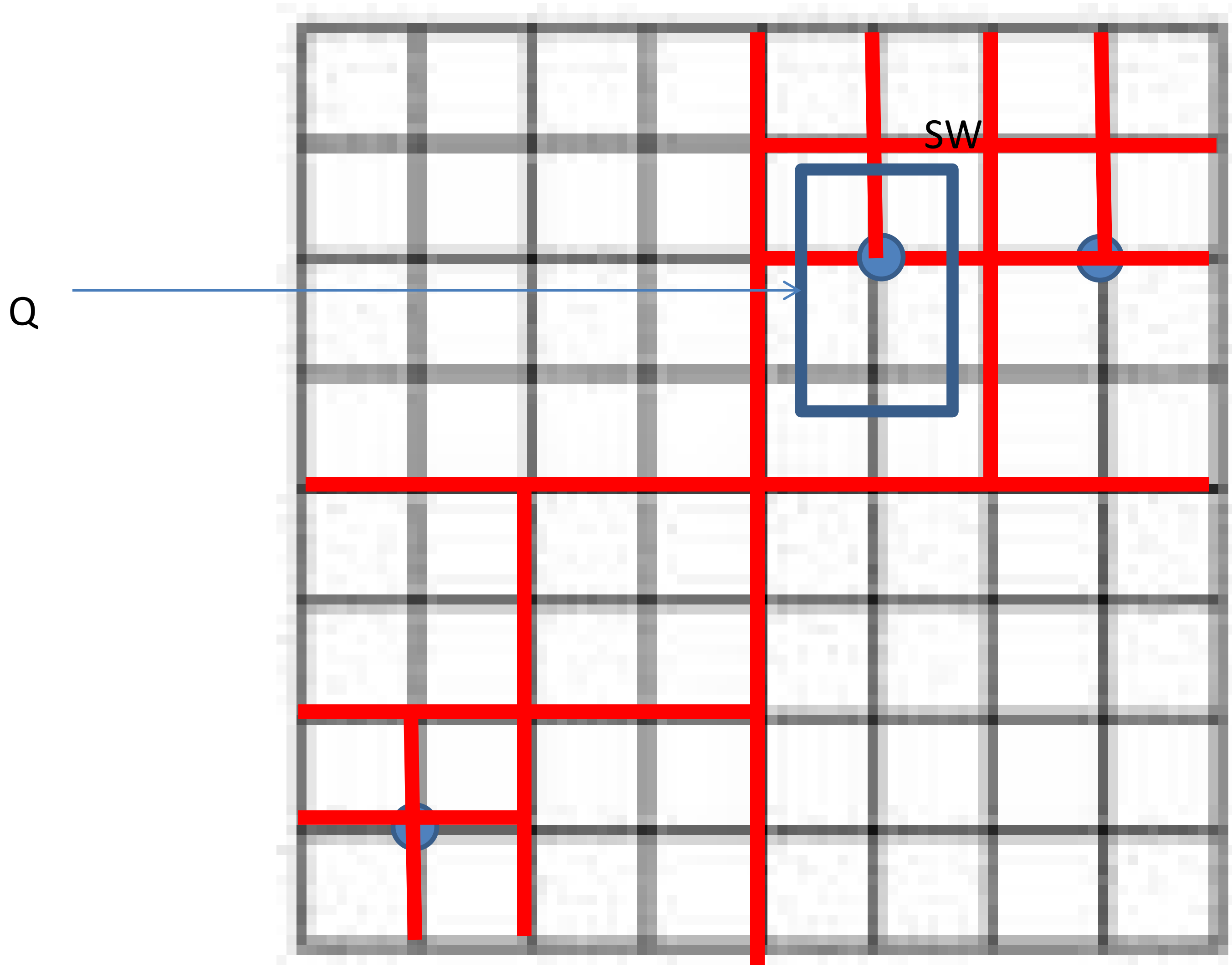
Does Q intersect the region of the root's SW child's SW child ? Yes, so search children.

Example range search



Does Q contain (1,1)? Yes. Return SOL = {(1,1)}. Done.

In Class Exercise: Example Range Search



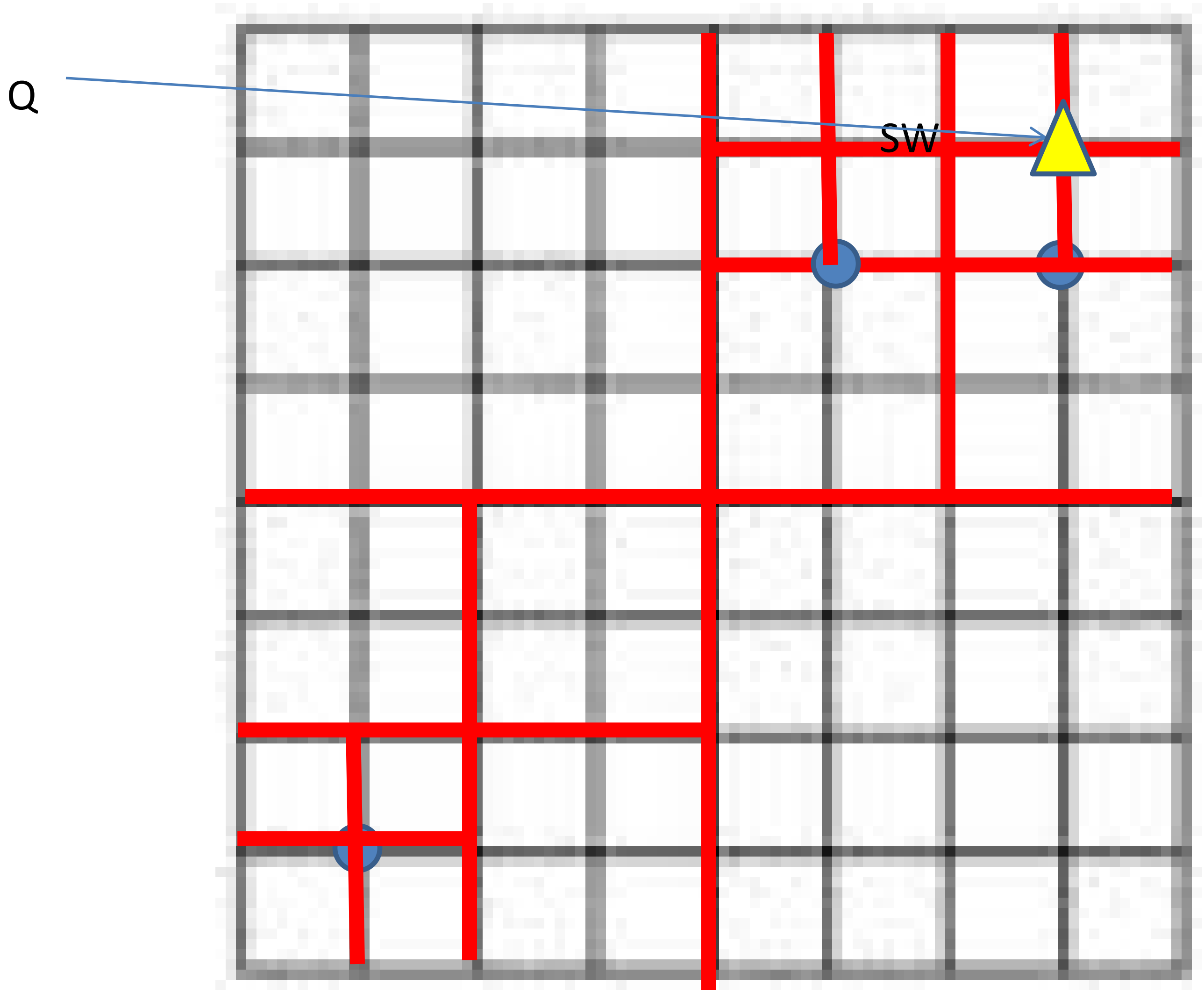
Nearest Neighbor Searches in MX-Quadtrees

- INPUT:
  - Pointer T to the root of an MX-quadtree.
  - Point Q (not necessarily with integer coordinates).
- OUTPUT:
  - Any nearest neighbor of Q.

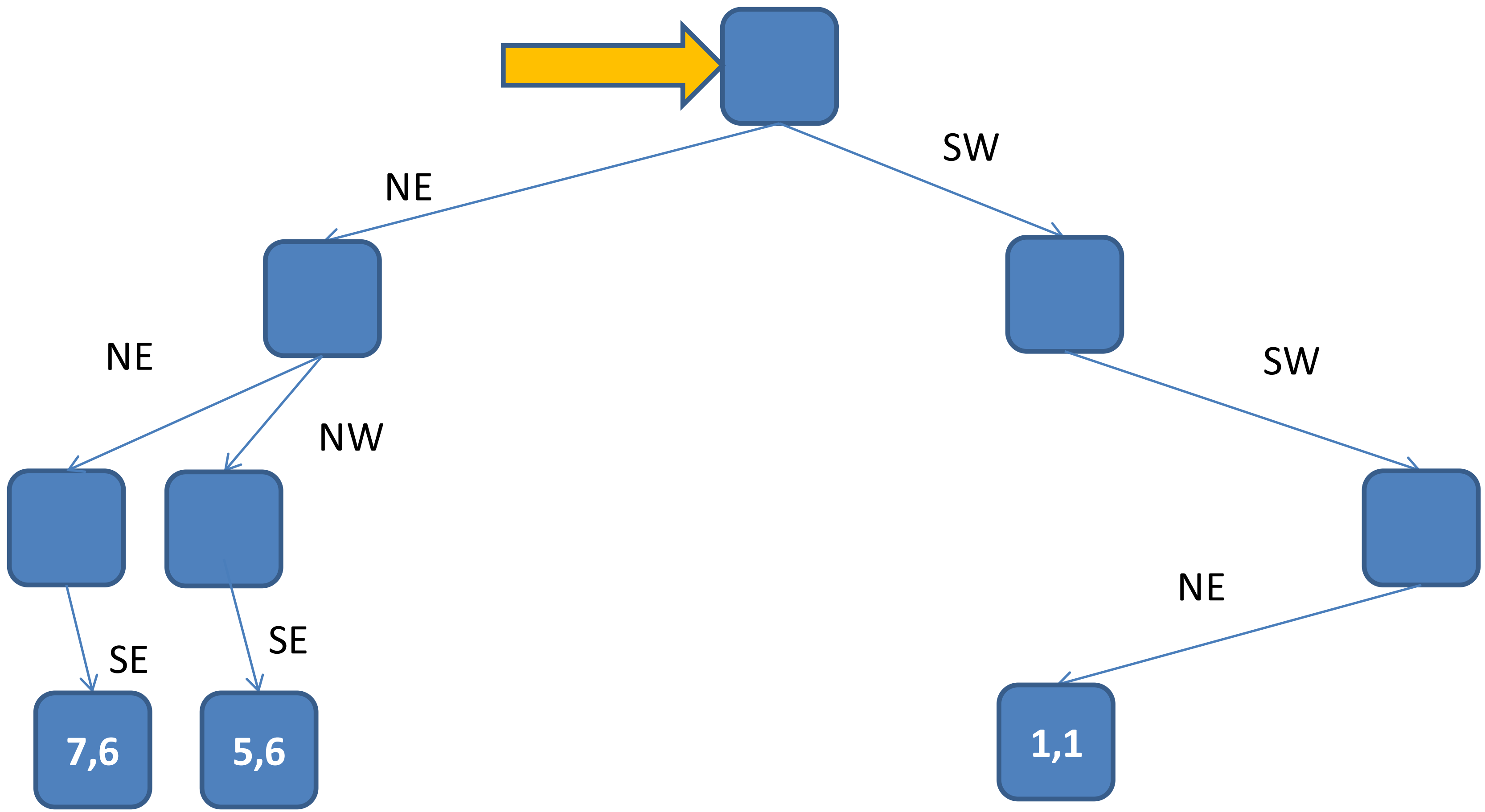
Nearest Neighbor Searches in MX-Quadtrees

- BestDist = infty, BestSOL = NIL.
- VISIT(N)
  - If N is not a leaf
    - If d(Reg(N),Q) < BestDist then
      - Visit all of N's children
    - Else prune
  - Else
    - If d(N.Point,Q) < BestDist then
      - BestSol = N.Point, BestDist = d(N.Point,Q).
  - Return BestSOL.

Nearest Neighbor Search Example

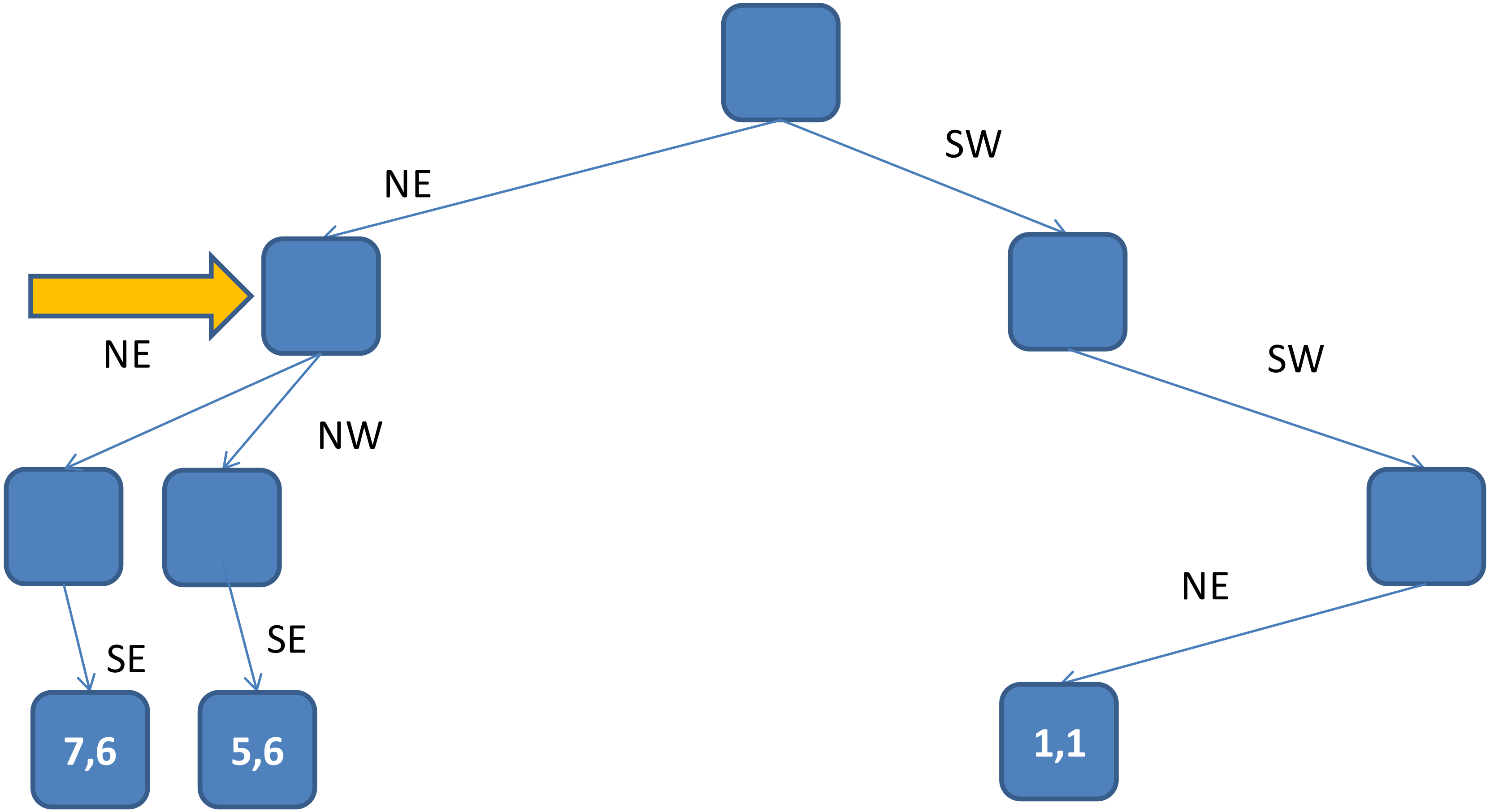


Example range search



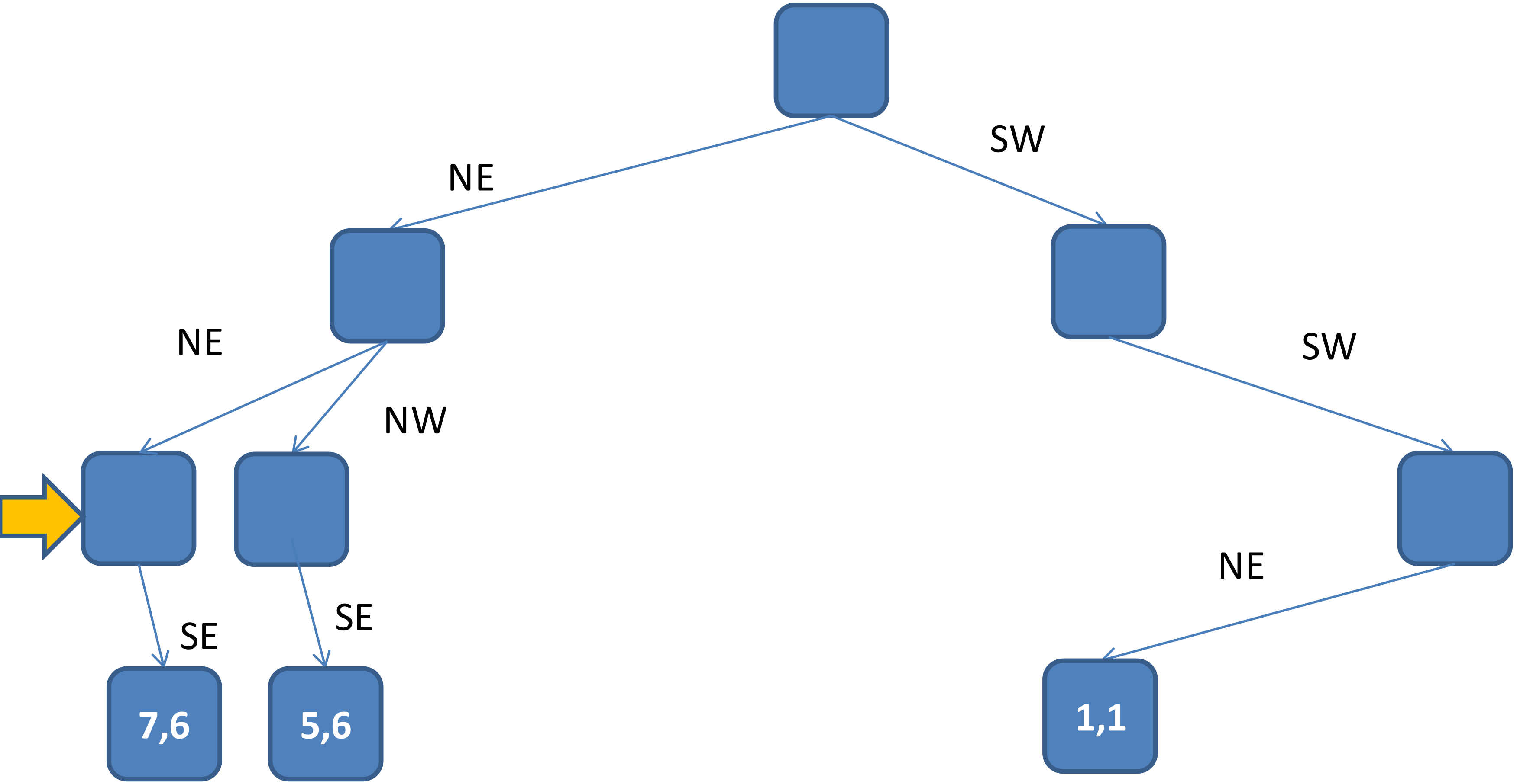
Is d(Reg(Root),Q) < BestDist? Yes, so must visit children. Suppose we visit children in depth-first order.

Example range search



Is d(Reg(N),Q) < BestDist? Yes, so must visit children. Suppose we visit children in depth-first order.

Example range search



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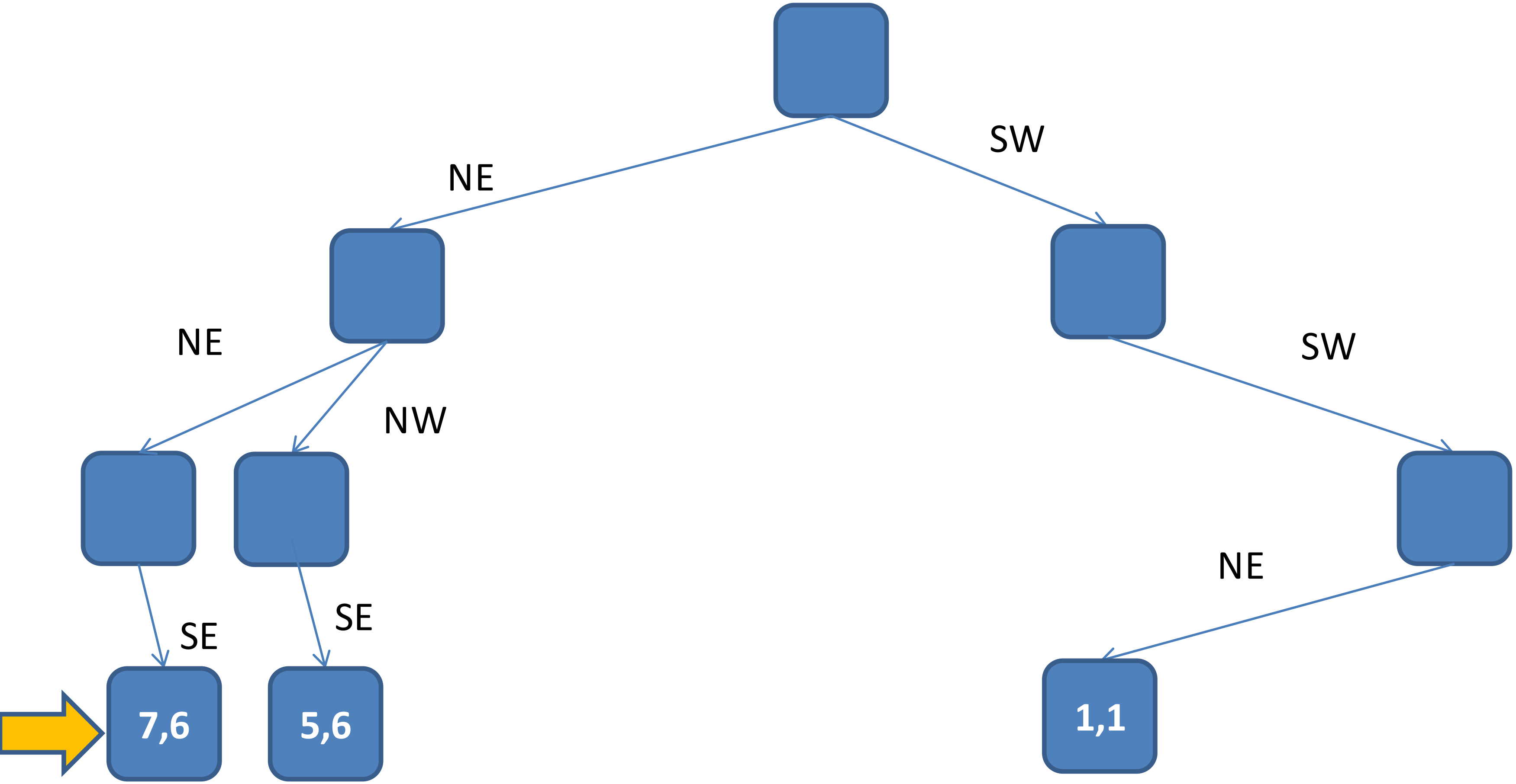
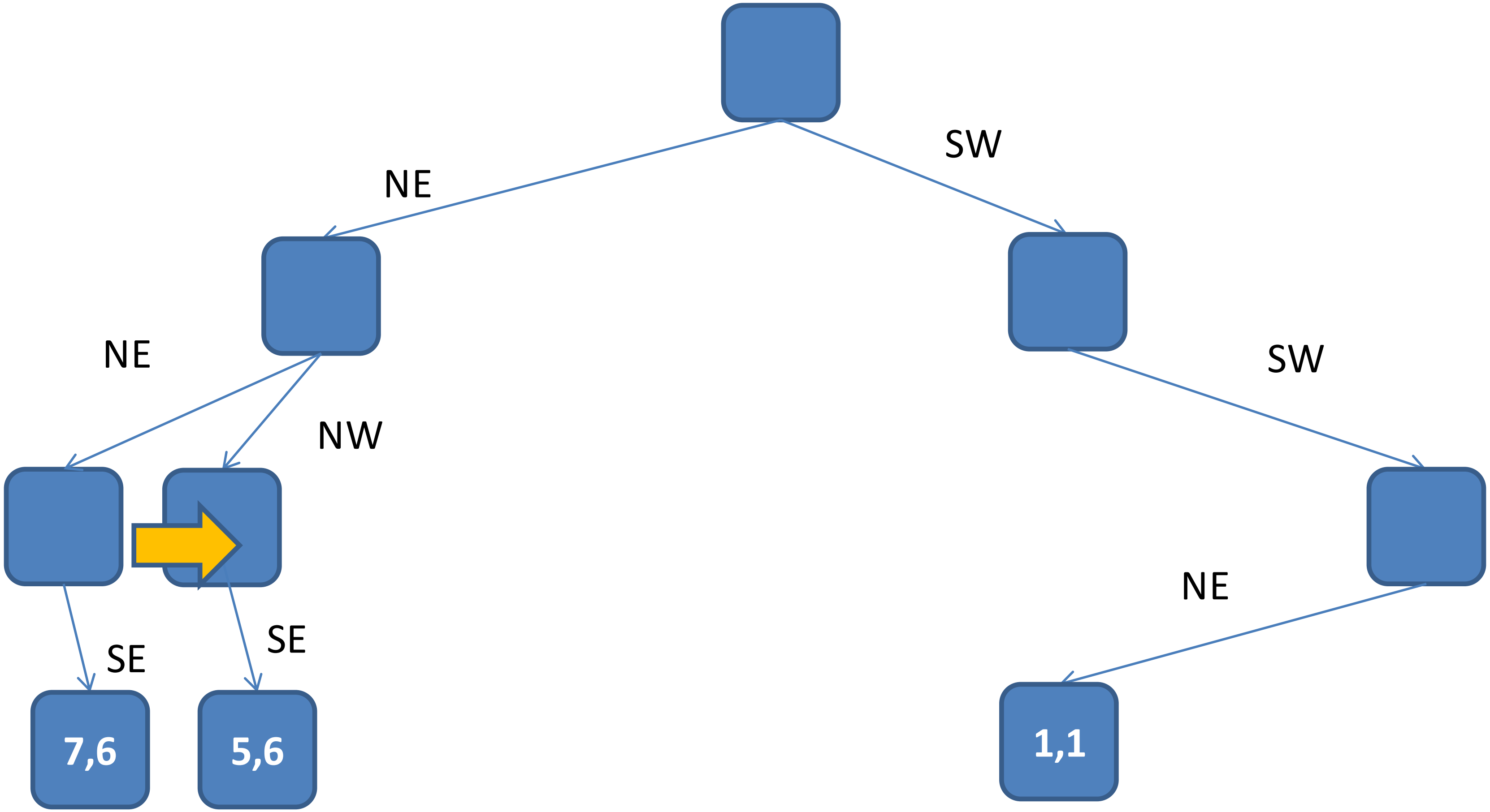


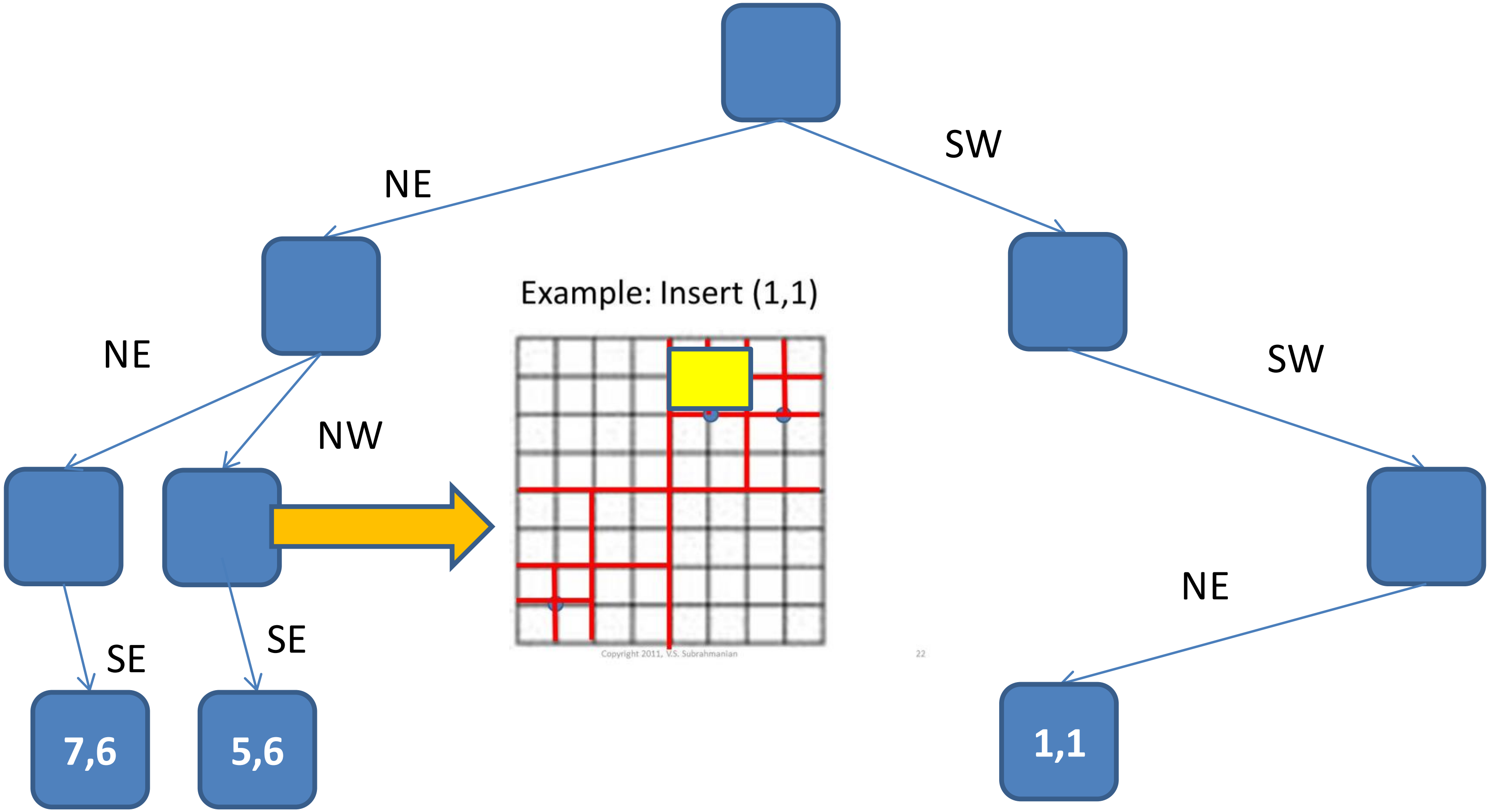
Is d((7,6),Q) < infty? Yes, so set BestSOL = (7,6), BestDist=1.

Example range search

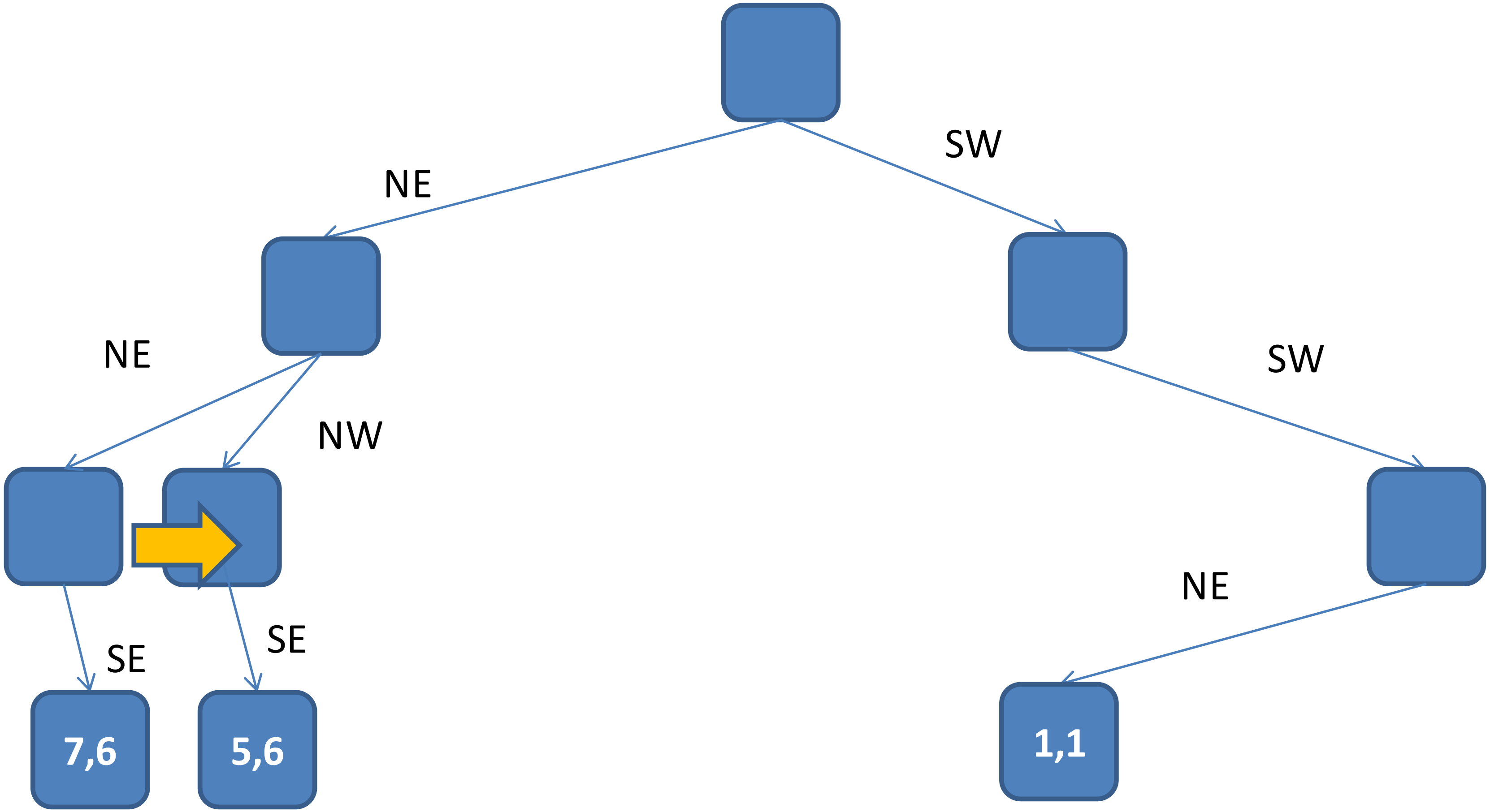


Is d(Reg(N),Q) < 1? Yes, so set BestSOL = (7,6), BestDist=1.

Regions associated with nodes

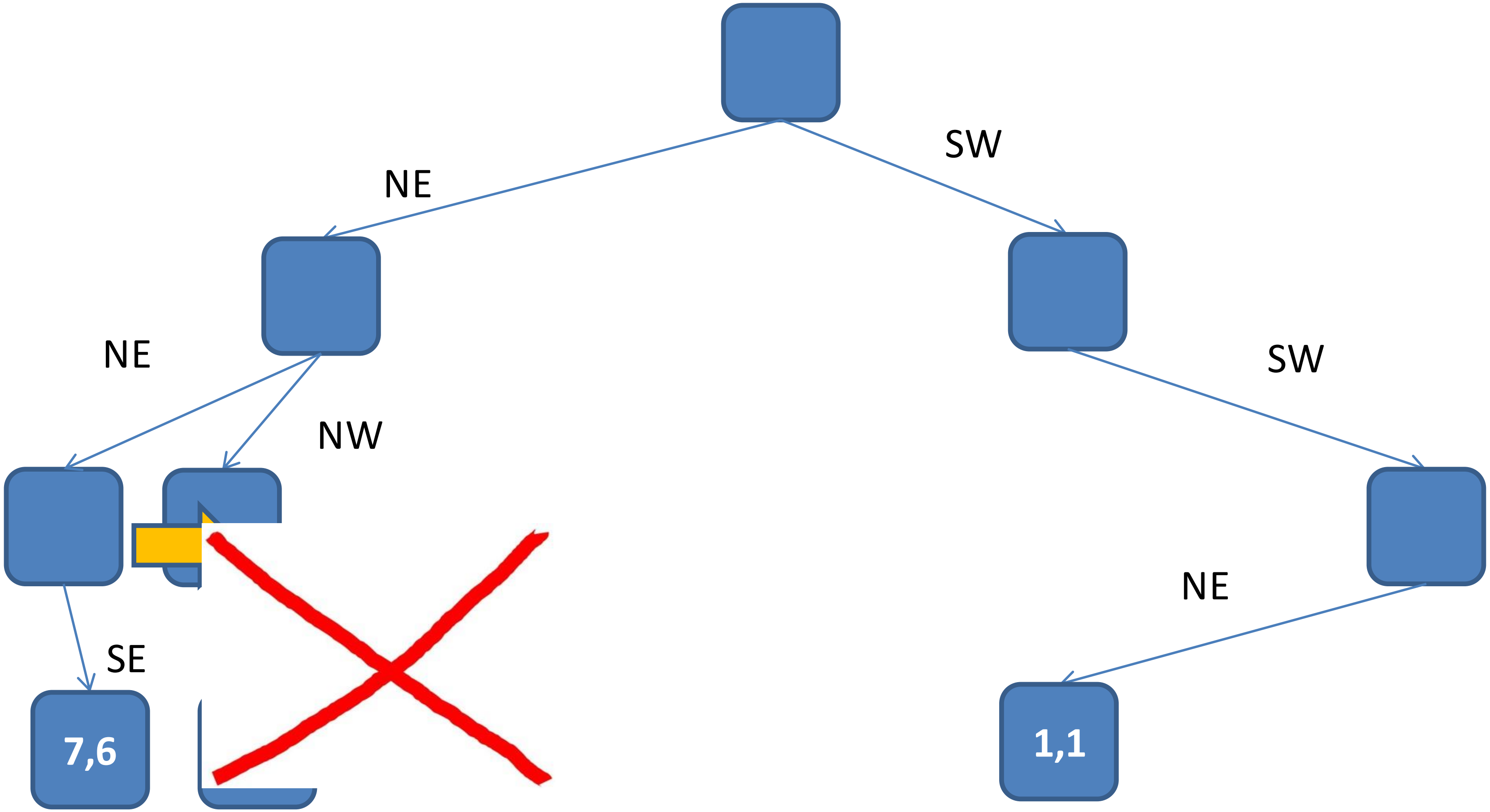


Example range search



Is d(Reg(N),Q) < 1? No, so prune.

Example range search



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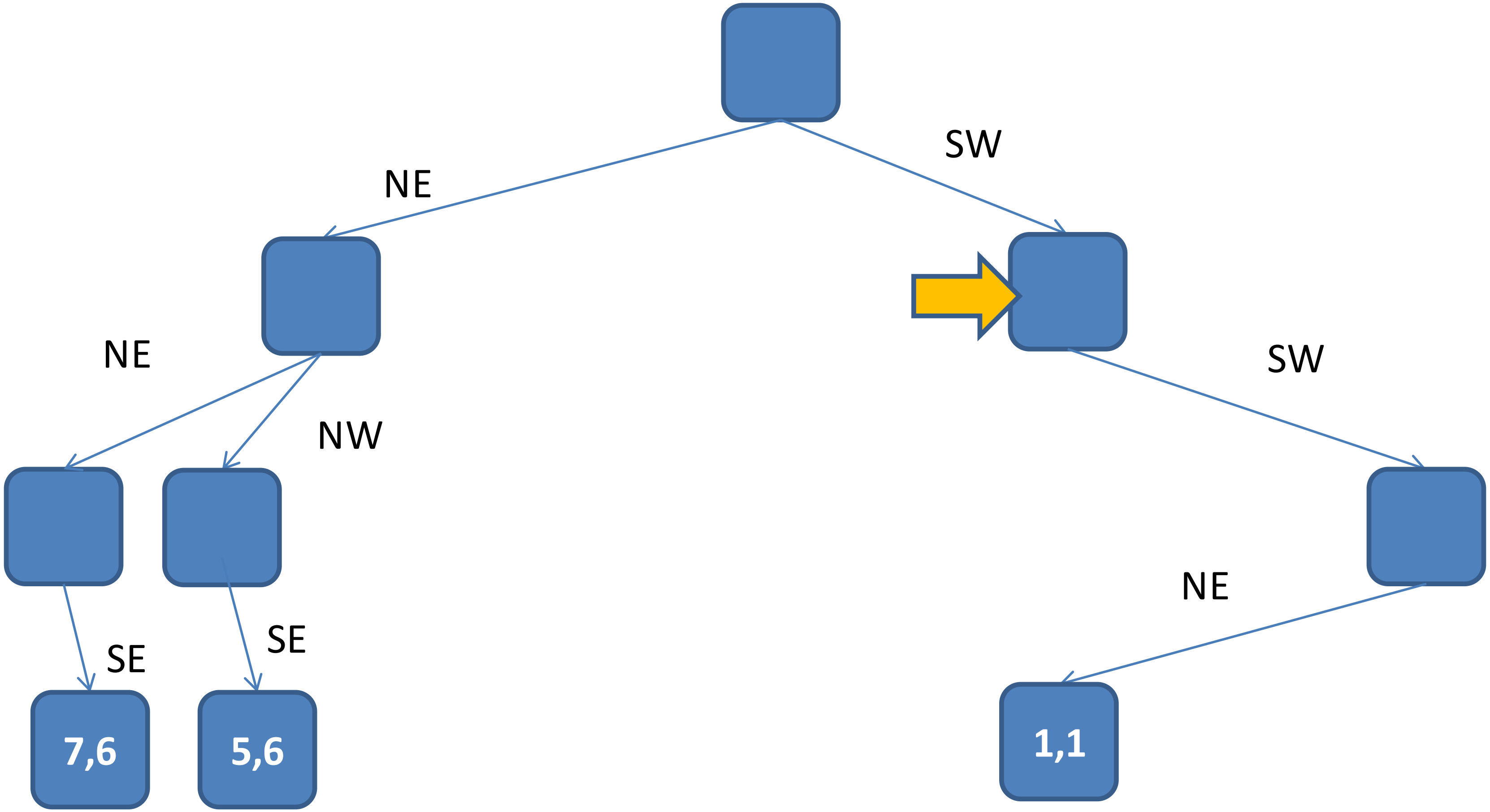
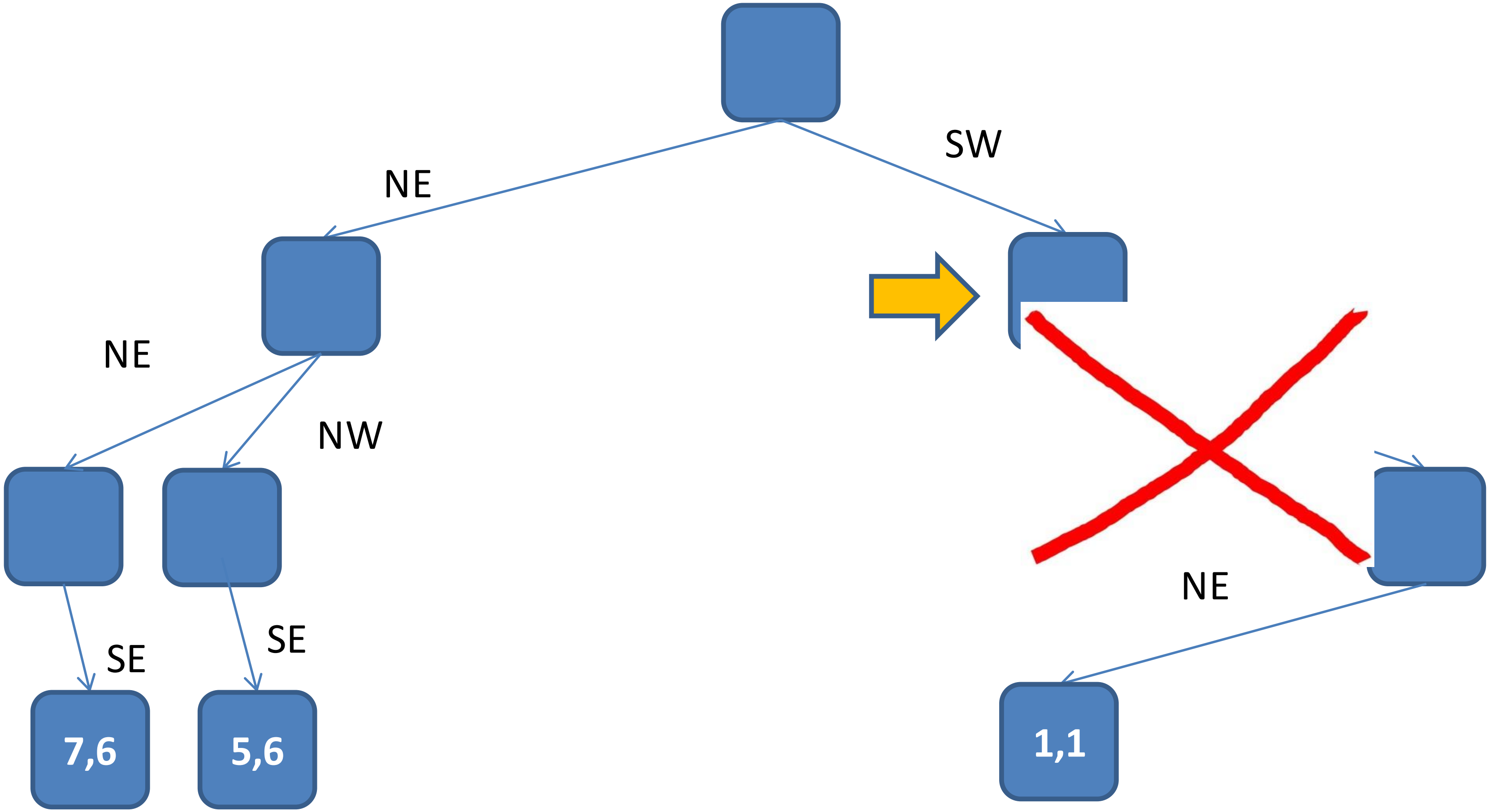
Example range search



Is d(Reg(N),Q) < 1? No, so prune. Return BestSOL = (7,6)

Example range search



Is d(Reg(N),Q) < 1? No, so prune.

What about Deletion?

- Really easy.
- All points are stored at leaves.
- Deletion algorithm sketch:
  - Search for point
  - If point is found
    - Set appropriate link of its parent to NIL
    - If all 4 of the parent's child links are NIL, then set the appropriate link of the parent's parent to NIL. ("Collapsing step").
    - Repeat.