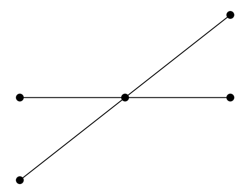
Exercise 3 *(skewed leapfrog)*

Suppose $a > 0$ and consider the following *skewed leapfrog* method for solving the advection equation $u_t + au_x = 0$:

$$U_j^{n+1} = U_{j-2}^{n-1} - \left(\frac{ak}{h} - 1\right)(U_j^n - U_{j-2}^n). \tag{Ex3a}$$

The stencil of this method is



Note that if $ak/h \approx 1$ then this stencil roughly follows the characteristic of the advection equation and might be expected to be more accurate than standard leapfrog. (If $ak/h = 1$ the method is exact.)

(a) What is the order of accuracy of this method?

(b) For what range of Courant number ak/h does this method satisfy the CFL condition?

(c) Show that the method is in fact stable for this range of Courant numbers by doing von Neumann analysis. **Hint:** Let $\gamma(\xi) = e^{i\xi h} g(\xi)$ and show that γ satisfies a quadratic equation closely related to the equation (10.34) that arises from a von Neumann analysis of the leapfrog method.

Exercise 4 *(modified equation for Lax-Wendroff)*

Derive the modified equation (10.45) for the Lax-Wendroff method.