CS 357 / MATH 357
Fall 2013

Numerical Methods – Homework 2
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Due: 09/17/2013

Note: Homework is due **5pm** on the due date. Please submit your homework through the dropbox in the Siebel Center basement. Make sure to include your name and **netid** in your homework.

Problem 1 **[5pt]** Let $\mathbf{A}$ and $\mathbf{B}$ be two $n \times n$ matrices.

(a) Is it necessarily true that $\mathbf{AB} = \mathbf{BA}$? Explain why or give an example where this does not hold.

(b) Is it necessarily true that $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$? Explain why or give an example where this does not hold.

Problem 2 **[5pt]** True/False questions

(a) **(True/False)** The forward elimination phase of naive Gaussian elimination produces an upper triangular matrix.

(b) **(True/False)** Every diagonal matrix is symmetric.

(c) **(True/False)** Every $n \times n$ matrix has an inverse.

(d) **(True/False)** The dot product of two vectors is a vector.

(e) **(True/False)** A system of linear equations always has either one unique solution or no solutions.

Problem 3 **[10pt]** Consider the following system of equations:

$$\begin{cases} .209x_1 + .113x_2 = .647 \\ .458x_1 + .237x_2 = .981 \end{cases}$$

(a) Rewrite the system in the form $\mathbf{Ax} = \mathbf{b}$ (matrix-vector form).

(b) Solve the system by hand using Gaussian elimination. At every step in the calculation, retain 3 significant figures. Provide your answer, $\hat{\mathbf{x}}$, in vector form.

(c) What is the exact solution? Use Python to compute the exact solution. Hint (Use the `numpy.linalg.solve()` command to compute the exact solution.

(d) What is the residual vector with the approximation, $\hat{\mathbf{x}}$, from above?

(e) What is the error vector with the approximation, $\hat{\mathbf{x}}$, from above?