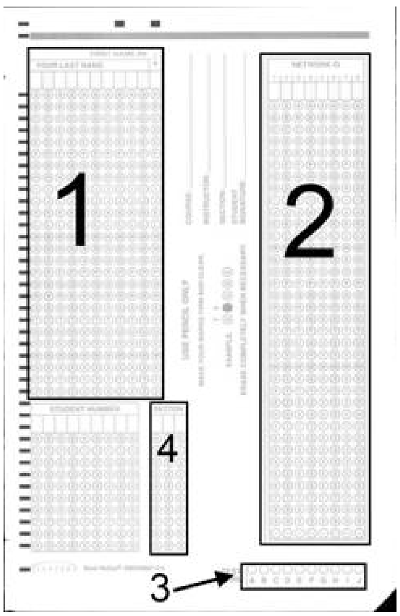
CS357 **Fall 2013**

Exam #1 A Thursday, October 3rd

READ and complete the following:

- Bubble your Scantron only with a No. 2 pencil.
- On your Scantron (shown in the figure below), bubble :

  1. Your Name
  2. Your NetID
  3. Form letter "**A**"



- No electronic devices or books are allowed while taking this exam.
- Please fill in the most correct answer on the provided Scantron sheet.
- We will not answer any questions during the exam.
- Each question has only ONE correct answer.
- You must stop writing when time is called by the proctors.
- Hand in both these exam pages and the Scantron.
- DO NOT turn this page UNTIL the proctor instructs you to.

1. Consider the matrix

$$A = \begin{bmatrix} 1 & 3 & 2 & 4 \\ 2 & 2 & 4 & 5 \\ 4 & 0 & 0 & 16 \\ 2 & 6 & 4 & 8 \end{bmatrix}$$

How many solutions will the linear system $Ax = b$ have?

(a) 1

(b) None

(c) It depends on b

(d) An infinite number

2. (True/False) Gaussian elimination is a method for solving a linear system of the form $Ax = b$ that requires computing A^{-1}.

(a) True

(b) False

3. Let A be an $n \times n$ matrix and z be an $n \times 1$ vector. What is the cost of calculating Az?

(a) $\mathcal{O}(n)$
(b) $\mathcal{O}(n^3)$
(c) $\mathcal{O}(1)$
(d) $\mathcal{O}(n^2)$

4. Which of the following statements is equivalent to "The determinant of A is 0"?

(a) The inverse of A exists.
(b) The rank of A is n.
(c) The rows of A are linearly independent.
(d) The columns of A are linearly dependent.

5. (True/False) An elementary elimination matrix, M_k, is singular and lower triangular.

(a) True

(b) False

6. **(True/False)** If A is an $n \times m$, then $A^T A$ is an $m \times m$ matrix.

(a) True

(b) False

7. Solve the following system of linear equations using Gaussian elimination without pivoting. Choose the correct solution from below.

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 2 \\ 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 12 \end{bmatrix}$$

(a) $\begin{bmatrix} -4 \\ 2 \\ -2 \end{bmatrix}$

(b) $\begin{bmatrix} -5 \\ 1 \\ -2 \end{bmatrix}$

(c) $\begin{bmatrix} -5 \\ 2 \\ 2 \end{bmatrix}$

(d) $\begin{bmatrix} 4 \\ -2 \\ 2 \end{bmatrix}$

8. **(True/False)** A small error means that the computed solution is close to the true solution.

(a) True

(b) False

9. Using Gaussian Elimination with partial pivoting to solve the following linear system, what is the first pivot element?

$$\begin{bmatrix} 1 & 0 & 0 & 1000 \\ 50 & 2 & 3 & 100 \\ 2 & 1 & 1 & 10^{-2} \\ 10^{-2} & 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

(a) Row 1, Column 1

(b) Row 3, Column 4

(c) Row 2, Column 1

(d) Row 3, Column 1

(e) Row 4, Column 1

10. **(True/False)** Well-conditioned matrices do not benefit from partial pivoting when solving with Gaussian Elimination.

(a) True

(b) False

11. Suppose the condition number of A is 10^{11} and A and b are stored in double precision. How many correct digits can we expect in the solution when using naive Gaussian Elimination?

(a) 5

(b) 7

(c) 13

(d) 16

(e) 21

12. What is the cost of back substitution in Gaussian Elimination?

(a) $\mathcal{O}(n)$

(b) $\mathcal{O}(n^2)$

(c) $\mathcal{O}(n^3)$

(d) $\mathcal{O}(2^n)$

(e) $\mathcal{O}(\log(n))$

13. The following matrix is (select all that apply)

$$\begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

(a) strictly diagonally dominant

(b) singular

(c) tridiagonal

(d) symmetric

(e) positive definite

14. Consider the matrix

$$\begin{bmatrix} 1 & 3 & 2 & 4 \\ 2 & 2 & 4 & 5 \\ 4 & 0 & 0 & 16 \\ 2 & 6 & 4 & 8 \end{bmatrix}$$

How many solutions will the linear system $Ax = b$ have?

(a) 1

(b) None

(c) It depends on b

(d) ∞

15. The condition number of the matrix $\begin{bmatrix} 8 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 10^{-8} \end{bmatrix}$ is

(a) 1
(b) 8
(c) 10^8
(d) $8 * 10^8$
(e) ∞

16. If ϵ_m is Machine epsilon, then $(2 + \epsilon_m) - 2$ is equal to

(a) 0
(b) $(1/2)\ \epsilon_m$
(c) ϵ_m
(d) $2\ \epsilon_m$

17. Which of the following real numbers can be exactly represented in IEEE-754 floating point:

(a) 0.1
(b) 0.2
(c) 0.25
(d) all of the above three numbers can be represented exactly in IEEE-754
(e) none of the above three numbers can be represented exactly in IEEE-754

18. Rounding error is the error committed in converting a real number into a floating point number.

(a) true
(b) false

19. Using a series approximation to $f(x) = \sin(3x)$ with the first two non-zero terms about the point $x = 0$, what is the best bound on the error in the approximation to $f(1/2)$?

(a) 10^{-1}
(b) 10^{-2}
(c) 10^{-3}
(d) 10^{-4}
(e) 10^{-5}

20. For the matrix problem $Ax = b$, which of the following is true about the condition number $\kappa(A)$:

(a) If any entry a_{ij} of the matrix is large, the condition number will always be large.
(b) If the condition number is large, the solution x is sensitive to small changes in A or b.
(c) The condition number is easy to compute for any matrix A

(d) If the determinant of A is large, the condition number of A will be large.

21. If the matrix is symmetric positive definite, pivoting is not required when using Gaussian Ellimination.

(a) true

(b) false

22. Computing $exp(x) - exp(2x)$ with the code

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a = exp(x) - exp(2*x)
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will provide an accurate value for a for x near zero.

(a) true

(b) false

23. What does multiplication of a 3×3 matrix A by the following permutation matrix do to A.

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(a) nothing

(b) swap rows 1 and 2

(c) swap rows 2 and 3

(d) swap rows 1 and 3

24. How many decimal digits of accuracy will the solution to $Ax = b$ given by a good variant of Gaussian elimination have if $\kappa(A) = 10^6$ and A and b are stored in IEEE double precision? (assume $\epsilon_m = 10^{-16}$)

(a) 0

(b) 8

(c) 10

(d) 6

25. Suppose that A is an $n \times n$ real matrix and that n is very large. Suppose that you decide to use LU factorization of the matrix A. What is the the total cost in floating point operations of solving $Ax = b_1$, $Ax = b_2$, $Ax = b_3$, $\ldots$, $Ax = b_m$ (all b_j are different)?

(a) $\mathcal{O}(n^2 + mn^2)$

(b) $\mathcal{O}(n^3)$

(c) $\mathcal{O}(n^3 + mn^2)$

(d) $\mathcal{O}(mn^3)$

(e) $\mathcal{O}(n^4)$

26. The cost of solving a banded linear system with m bands and n rows by Gaussian elimination is.

(a) $\mathcal{O}(m^2 n)$

(b) $\mathcal{O}(n^3)$

(c) $\mathcal{O}(n)$

(d) $\mathcal{O}(mn)$