CS 357 / MATH 357
Fall 2013

Numerical Methods – Homework 6
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Due: 10/29/2013

Note: Homework is due **5pm** on the due date. Please submit your homework through the dropbox in the Siebel Center basement. Make sure to include your name and **netid** in your homework.

Problem 1 [4pt] Construct the formulas for $x(t)$ and $y(t)$ for the 5^{th} degree, $n = 5$, Bezier Polynomial.

Problem 2 [8pt] Consider the following set of points

x	-1	0	1
y	4	8	0

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(a) **[2pt]** Find the first degree spline function for the above points.

(b) **[4pt]** Find the natural cubic spline for the above points. Show your work.

(c) **[2pt]** Use matplotlib to plot your cubic spline and your first degree spline. You may choose to make one graph for each or put them both on the same graph. Turn in your source code and your graph(s).

Problem 3 [6pt] Consider the polynomial $x^3 - 5x^2 + 5x + 10$.

(a) **[2pt]** What is the companion matrix for the above polynomial?

(b) **[2pt]** Use Scipy to find the eigenvalues of the companion matrix.

(c) **[2pt]** How do these eigenvalues relate to the roots of the given polynomial?

Problem 4 [6pt] Consider the following integral

$$\int_0^2 (x^4 - 4x^2 + x + 4)dx.$$

(a) **[3pt]** Use the composite trapezoid rule with 5 uniform points to approximate the above integral. Show your work.

(b) **[3pt]** What value of n with uniform spacing would we have needed to use to have an error of at most 10^{-3}. Show your work.

Problem 5 [6pt] True/False questions

(a) **[2pt] (True/False)** A Bezier curve passes through all of its control points.

(b) **[2pt] (True/False)** Every first degree spline is also a second degree spline.

(c) **[2pt] (True/False)** The following function is a first-degree spline:

$$S(x) = \begin{cases} 1 & 0 \leq x \leq 5 \\ 1 + (x - 5) & 5 \leq x \leq 8 \\ 8 - \frac{1}{4} * x & 8 \leq x \leq 10 \end{cases} .$$

Problem 6 [5pt] Consider the Gaussian quadrature formula $\int_{-1}^{1} f(x)dx \approx \sum_{i=0}^{n} A_i f(x_i)$. We wish to find the Gaussian quadrature formula for $n = 3$.

(a) **[1pt]** Use the three term recurrence relation to find the $4^t h$ order Legendre polynomial.

(b) **[1pt]** Use python (np.roots) to find the roots of the $4^t h$ order Legendra polynomial.

(c) **[1pt]** Use the method of undetermined coefficients to find the weights of the Gaussian quadrature formula for $n = 3$

(d) **[2pt]** Write the Gaussian quadrature formula for the above integral for $n = 3$.

Problem 7 [5pt] EXTRA CREDIT. Use the formula from the problem above to approximate $\int_1^2 xe^{2x}dx$. (Hint: use change of variable to shift the interval of integration to $[-1, 1]$.)