CS 357 / MATH 357
Fall 2013

Numerical Methods – Homework 7
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Due: 10/05/2013

Note: Homework is due **5pm** on the due date. Please submit your homework through the dropbox in the Siebel Center basement. Make sure to include your name and **netid** in your homework.

Problem 1 [4pt] Consider the following pseudocode:

$x := 0.5, h := 1, tol := \frac{1}{2}10^{-8}$
$error := tol$
while $error \geq tol$
  $h := 0.25 * h$
  $y := [\sin(x+h) - \sin(x)]/h$
  $error := |cos(x) - y|$
end

How many iterations will it take for this method to converge? *Hint:* Find the truncation error term for this finite difference formula.

5 pts extra credit What will happen to the value of the error if the iteration is allowed to continue far past the number of iterations required for convergence? *Hint:* Remember cacnellation in subtraction of floating point numbers.

Problem 2 [4pt] Consider the following set of points

x	0	1	2	3	4
y	0	1	4	9	16

. Using the following formulas to approximate $y'(2)$.

(a) Forward difference

(b) Backward difference

(c) Central difference

(d) Fourth order Richardson extrapolation with the central difference

Problem 3 [9pt] Consider the linear system,

$$\begin{bmatrix} 10 & -2 & 1 \\ -2 & 10 & -2 \\ -2 & -5 & 10 \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} 9 \\ 12 \\ 18 \end{bmatrix}.$$

Find the value of $\boldsymbol{x}$ after two iterations of the following methods using $\boldsymbol{x_o} = [0\ 0\ 0]^T$. Retain at least 4 significant figures in your calculations.

(a) Jacobi iteration

(b) Gauss-Seidel iteration

(c) SOR iteration with $\omega = 1.1$

Problem 4 **[3pt]** True/False questions

(a) **(True/False)** Differentiation is inherently well-conditioned.

(b) **(True/False)** Diagonal dominance of $\boldsymbol{A}$ is a necessary and sufficient condition for the convergence of Jacobi and Gauss-Seidel methods for any starting vector.

(c) **(True/False)** The Jacobi method will converge for all initial iterates with $\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$.

Problem 5 **[15pt]** Implement each of the following methods in Python; then, evaluate the integral

$$\int_0^{\pi/2} \sin x \; dx$$

using $n = 120$ subintervals with each implementation.

(a) Composite trapezoid rule (uniform spacing).

(b) Composite Gaussian three-point rule:

$$\int_a^b f(x) \; dx \approx h \sum_{i=1}^{n/2} \left[\frac{5}{9} f\left(x_{2i-1} - h\sqrt{\frac{3}{5}} \right) + \frac{8}{9} f(x_{2i-1}) + \frac{5}{9} f\left(x_{2i-1} + h\sqrt{\frac{3}{5}} \right) \right]$$

(c) Composite Simpson's $\frac{1}{3}$ rule.