CS 357 / MATH 357
Fall 2013

Numerical Methods – Homework 9
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Due: 12/3/2013

Note: Homework is due **5pm** on the due date. Please submit your homework through the dropbox in the Siebel Center basement. Make sure to include your name and **netid** in your homework.

Problem 1 [5pt] Write a python program to verify numerically that $\pi = 4\int_0^1 (1-x^2)^{(1/2)}$. Use the Monte Carlo method and 10^6 random numbers.

Problem 2 [10pt] We wish to perform a least-squares best fit of a quadratic polynomial $a_0 + a_1 x + a_2 x^2 = y$ to the data in the following table.

x	0	1	2	3
y	0	2	10	5

(a) **[2pt]** Write the overdetermined system of equations in the form for this least squares problem.

(b) **[2pt]** If the condition number for A is α, then what is the condition number for $A^T A$?

(c) **[6pt]** Find the best fit quadratic polynomial $y = a_0 + a_1 x + a_2 x^2$ with the above points.

Problem 3 [5pt] Compute the Gram-Schmidt QR factorization for the following matrix. Show work.

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 1 & 3 & 2 \end{bmatrix}$$

Problem 4 [10pt] Extra Credit. In this problem we use linear least squares to fit a line to some observed climate data. The data represents the anomaly in the global temperature from the late 1800s to 2012. The anomaly is represented as the difference between the average yearly surface temperature and the 20th century average (1901 - 2000). The data is contained in an ascii file on piazza called climate.txt. This file contains a data point per line of the form "year anomaly". That is each line is a string with two "words" in it separated by spaces. The first word is the year, 1997 for example, and the second word is the anomaly in degrees C. You are to read the data and fit a line to it by using the least squares method.

a. Write a python program to read the climate data and fit a line to it using the least squares methd. The first attempt should use the numpy function "numpy.linalg.lstsq" to solve for the coeficients of the linear polynomial. Plot the data as points marked by the symbol of your choice. Use the output of lstsq to plot a solid line through the data.
b. Repeat the above exercise only this time instead of numpy.linalg.lstsq use the numpy.linalg.qr to perform a qr factorization. Use the factors to solve the least squares formulation and plot the dat a second time using the solution derived from the QR factorization of the overdetermined system.

Hand in the following:

Part a.

- ☐ Turn in your python program for part a.
- ☐ Hand in a graph of your linear polynomial on top of the actual data.

Part b.

- ☐ Turn in your python program for part b.
- ☐ Hand in a graph of your linear polynomial on top of the actual data.