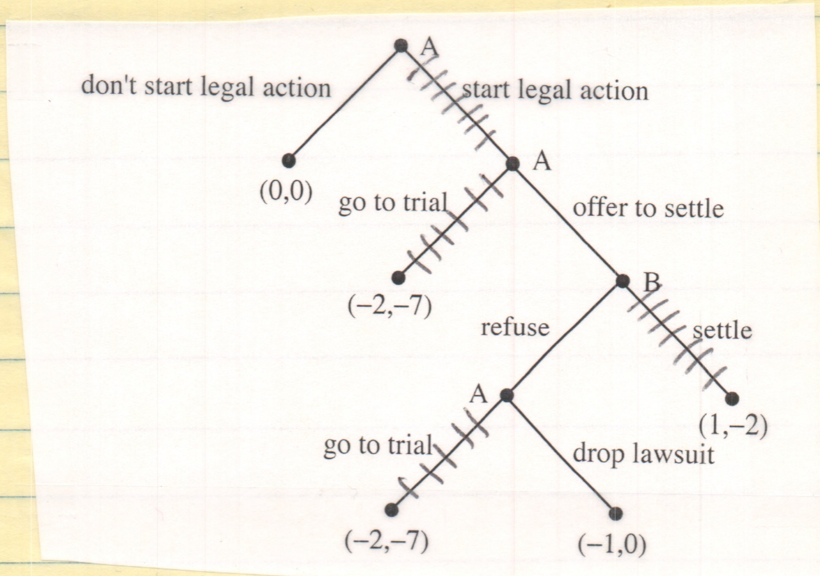
MA 440 S. Schecter

Test 1 Answers

15 (1)



A will not start legal action.

18 (2)

		Defense		
		counter run	counter pass	blitz
Offense	run	~~(3, −3)~~	~~(6, −6)~~ (2)	(15, −15)
	pass	(10, −10)	(7, −7)	(9, −9)
		(3)		(1)

(1) Eliminate blitz: s.d. by counter pass.
(2) Eliminate run: s.d. by pass in reduced game
(3) Eliminate counter run: s.d. by counter pass in reduced game.

Dominant strategy equilibrium: (pass, counter pass)

(3) $s < t$: $\Pi_1(s,t) = -s$, $\Pi_2(s,t) = 1-s$

$s > t$: $\Pi_1(s,t) = 1-t$, $\Pi_2(s,t) = -t$

$s = t$: $\Pi_1(s,t) = \frac{1}{2} - s$, $\Pi_2(s,t) = \frac{1}{2} - s$.

12 a) $(s,t) = (0,1)$: $\Pi_1(0,1) = 0$, $\Pi_2(0,1) = 1$.

P1 cannot improve his payoff. If he changes to s' with $0 < s' < 1$, he gets $\Pi_1(s',1) = -s' < 0$. If he changes to $s' = 1$, he gets $\Pi_1(1,1) = \frac{1}{2} - 1 = -\frac{1}{2} < 0$.

P2 cannot improve his payoff, since 1 is the best possible payoff.

This is a Nash equilibrium.

10 b) (s,t), $0 < s < t < 1$. Not a Nash equilibrium.
$\Pi_1(s,t) = -s < 0$. If P1 changes to s' with $t < s' \leq 1$, his new payoff is $\Pi_1(s',t) = 1-t > 0$. *

10 c) $s = t < 1$: Not a Nash eq. $\Pi_2(s,s) = \frac{1}{2} - s$.

If P2 changes to t' with $s < t' \leq 1$, his new payoff is $\Pi_2(s,t') = 1-s$, which is greater than $\frac{1}{2} - s$.

* OR: P1 can improve payoff by changing to s', $0 \leq s' < s < t$. New payoff is $-s'$, better than $-s$.

(4) $\Pi_1(x_1, x_2) = 24x_1 - x_1^2 - 6x_1x_2 - 9x_2^2$

$\Pi_2(x_1, x_2) = 24x_2 - 9x_1^2 - 6x_1x_2 - x_2^2$.

15 a) For a Nash equilibrium we need $\frac{\partial \Pi_1}{\partial x_1} = 0$ and $\frac{\partial \Pi_2}{\partial x_2} = 0$

$$\frac{\partial \Pi_1}{\partial x_1} = 24 - 2x_1 - 6x_2 = 0.$$

$$\frac{\partial \Pi_2}{\partial x_2} = 24 - 6x_1 - 2x_2 = 0.$$

Solve simultaneously: $(x_1, x_2) = (3, 3)$

20 b) First we find C2's best response $x_2 = b(x_1)$ by solving $\frac{\partial \Pi_2}{\partial x_2} = 0$ for x_2:

$$\frac{\partial \Pi_2}{\partial x_2} = 24 - 6x_1 - 2x_2 = 0 \Rightarrow x_2 = 12 - 3x_1$$

Then we calculate $\Pi_1(x_1, 12 - 3x_1) =$

$24x_1 - x_1^2 - 6x_1(12 - 3x_1) - 9(12 - 3x_1)^2 =$

$24x_1 - x_1^2 - 72x_1 + 18x_1^2 - 9(144 - 72x_1 + 9x_1^2) =$

$-9 \cdot 144 + (24 - 72 + 9 \cdot 72)x_1 + (-1 + 18 - 9 \cdot 9)x_1^2 =$

$-1296 + 600x_1 - 64x_1^2$.

$$\frac{d}{dx_1}\Pi_1(x_1, 12 - 3x_1) = 600 - 128x_1 = 0 \text{ if } x_1 = \frac{600}{128} = 4.6875$$