MA 440 S. Schecter

Test 2 Answers

(1)

			q_1 e	B2 q_2 w	q_3 r
B1	p_1	e	(3,3)	(1,5)	(1,5)
	p_2	w	(5,1)	(0,0)	(5,1)
	p_3	r	(5,1)	(1,5)	(2,2)

15 a) We look for a Nash equilibrium (σ, τ),
$\sigma = p_1 e + p_2 w + p_3 r$, $\tau = q_1 e + q_2 w + q_3 r$,
all $p_i > 0$, all $q_i > 0$.

The following must be equal:

$\pi_1(e, \tau) = 3q_1 + q_2 + q_3$

$\pi_1(w, \tau) = 5q_1 + 5q_3$

$\pi_1(r, \tau) = 5q_1 + q_2 + 2q_3$

$\pi_1(e,\tau) = \pi_1(w,\tau) \Rightarrow 2q_1 - q_2 + 4q_3 = 0$

$\pi_1(e,\tau) = \pi_1(r,\tau) \Rightarrow 2q_1 + q_3 = 0$

Last equation: $q_1 + q_2 + q_3 = 1$

Solution : $(q_1, q_2, q_3) = (-\frac{1}{7}, \frac{6}{7}, \frac{2}{7})$

Because $q_1 < 0$, there is no Nash equilibrium of this type.

15 b) We look for a Nash equilibrium (σ, τ),

$\sigma = p_2 w + p_3 r$, $\tau = q_2 w + q_3 r$.

We must have

$\pi_1(w, \tau) = \pi_1(r, \tau) \Rightarrow$

$5q_3 = q_2 + 2q_3 \Rightarrow q_2 - 3q_3 = 0$

Other equation: $q_2 + q_3 = 1$

Solution : $q_2 = \frac{3}{4}$ $q_3 = \frac{1}{4}$

$\tau = \frac{3}{4} w + \frac{1}{4} r$

Similarly $\sigma = \frac{3}{4} w + \frac{1}{4} r$.

Now $\pi_1(w, \tau) = \pi_1(r, \tau) = \frac{5}{4}$.

For a Nash equilibrium, we need $\frac{5}{4} \geq \pi_1(e, \tau)$

$\pi_1(e, \tau) = 3 \cdot 0 + 1 \cdot \frac{3}{4} + 1 \cdot \frac{1}{4} = 1$

Since $\frac{5}{4} \geq 1$, (σ, τ) <u>is</u> a Nash equilibrium.

7 ② a) $(h, \dots, h)$ is not a Nash equilibrium. If one bank changes to d, its payoff increases from a to b.

7 b) It is a Nash equilibrium for one bank to be dishonest. If the dishonest bank switches to honest, its payoff falls from b to a. If an honest bank switches to dishonest, its payoff falls from a to $-c$.

15 c) Let $\sigma = ph + (1-p)d$. For a Nash equilibrium we need $\Pi_1(h, \sigma, \dots, \sigma) = \Pi_1(d, \sigma, \dots, \sigma)$

$$a = bp^{n-1} - c(1-p^{n-1})$$

$$a = bp^{n-1} - c + cp^{n-1}$$

$$a+c = (b+c)p^{n-1}$$

$$p^{n-1} = \frac{a+c}{b+c}$$

$$p = \left(\frac{a+c}{b+c}\right)^{\frac{1}{n-1}}$$

Since $0 < a+c < b+c$, we have $0 < p < 1$ as required.

15

(3) σ = trigger strategy

σ' = another strategy for Player 1, results in first use of s in period k.

Player 1 gets best payoff by using s in all subsequent periods.

For a Nash equilibrium, we need $\pi_1(\sigma,\sigma) \geq \pi_1(\sigma',\sigma)$.

Comparing payoffs from period k on:

$$4\delta^k + 4\delta^{k+1} + \dots \geq 5\delta^k + 3\delta^{k+1} + 3\delta^{k+2} + \dots$$

$$\frac{4\delta^k}{1-\delta} \geq 5\delta^k + \frac{3\delta^{k+1}}{1-\delta}$$

$$\frac{4}{1-\delta} \geq 5 + \frac{3\delta}{1-\delta}$$

$$4 \geq 5 - 5\delta + 3\delta$$

$$2\delta \geq 1$$

$$\delta \geq \frac{1}{2}$$

7 (4) a) If T orders steak, payoff is at least 2

If T orders quiche, payoff is at most -1

Since 2>-1, T orders steak

19 b) Tough customer:

	B: hl	B: lh
W: s	(3,0,-2)	(2,0,0)
W: q	(3,0,-2)	(2,0,0)

Wimp customer

	B: hl	B: lh
W: s	(0,-8,2)	(0,-2,0)
W: q	(0,2,0)	(0,-4,2)

$\frac{1}{2} \cdot \text{matrix } 1 + \frac{1}{2} \cdot \text{matrix } 2 =$

	B: hl	B: lh
W: s	$(\frac{3}{2},-4,0)$	(1,-1,0)
W: q	$(\frac{3}{2},1,-1)$	(1,-2,1)